

RESEARCH PROGRAMME ON SEISMIC WAVES IN COMPLEX 3-D STRUCTURES (SW3D)

The research is focused primarily on the fundamental issues of high-frequency wave propagation in complex 3-D isotropic and anisotropic structures, which go beyond the traditional approaches. The ray method and its extensions, as well as its combination with other methods are mainly applied and investigated. The emphasis is put on new, stable, more efficient and flexible algorithms for both forward numerical modelling and inversion of wave fields in 3-D inhomogeneous, isotropic or anisotropic, non-attenuating or attenuating structures. Considerable attention is devoted to applications involving S waves or electromagnetic waves, converted waves, wave splitting and coupling in anisotropic media, particle ground motions, etc. Much more detailed information can be obtained at “<http://sw3d.cz>”. The research programme was begun on October 1, 1993, and was devoted to seismic waves as suggested by its title. In the recent years, the research has been extended to electromagnetic waves.

RESEARCH PROGRAMME FOR THE TWENTY-FOURTH YEAR

October 1, 2016 — September 30, 2017

1. *Ray histories and two-point ray tracing in complex 3-D structures*

Ray histories are of principal importance not only for two-point ray tracing and for wavefront tracing, but also for the summation of Gaussian beams. Properties of the projection of ray coordinates onto Cartesian coordinates within individual ray histories will be studied. New applications of ray histories to numerical algorithms will be proposed. The two-point ray tracing code will further be tested and applied to various velocity models. The “velocity model” represents the spatial distribution of the stiffness tensor for elastic waves or of the constitutive tensor for electromagnetic waves.

2. *Fast computation of multivalued ray-theory travel times and Green functions at the nodes of 3-D grids*

Ray histories enable to use controlled initial-value ray tracing for covering the velocity model by ray tubes and ray cells. Multivalued ray-theory travel times and other quantities at the nodes of a 3-D grid are then obtained by interpolation within ray cells. This algorithm is especially useful for the Born approximation and for Kirchhoff migrations in heterogeneous 3-D velocity models, and may be used for nonlinear hypocentre determination. The current computer code can be applied to P, S and converted waves in isotropic velocity models with interfaces, and to P waves in smooth anisotropic velocity models. It will further be tested in various velocity models, including the velocity models delivered by the consortium members.

We shall continue in extending the algorithm and computer code of interpolating the Green function within ray cells to S waves or electromagnetic waves in smooth anisotropic velocity models by including the coupling-ray-theory travel times and amplitudes calculated using the prevailing-frequency approximation of the coupling ray theory along common anisotropic rays.

Algorithms of fast calculation of ray-theory travel times in dense rectangular grids will be investigated further. The accuracy and efficiency of the interpolation of ray-theory travel times within ray cells in 3-D velocity models will be studied, and the relevant numerical algorithms will be improved, or new ones will be proposed. Attention will also be devoted to the interpolation between different source and receiver positions.

3. *Synthetic seismograms in 3-D isotropic and anisotropic complex structures*

Methods to calculate synthetic seismograms in complex structures will be studied, mutually compared and combined. The synthetic seismograms will also be compared with synthetic seismograms generated by non-ray methods. For velocity models suggested by the consortium members, we are ready to perform ray-synthetic studies illustrating how the wave responses differ in heterogeneous and anisotropic media from homogeneous or isotropic media.

4. *Gaussian beams and Gaussian packets in inhomogeneous isotropic or anisotropic media*

Paraxial Gaussian beams and paraxial Gaussian packets in smoothly varying anisotropic media, and in laterally varying layered anisotropic structures, including coupling-ray-theory Gaussian beams and packets. Paraxial Gaussian beams and packets in smoothly varying anisotropic attenuating media.

Integral superposition of paraxial Gaussian beams. Integral superposition of paraxial Gaussian packets. Applications of the integral superpositions.

Optimization of the shape of Gaussian beams.

5. *Wave propagation in inhomogeneous weakly anisotropic media*

In the prevailing-frequency approximation of the coupling ray theory for S waves or electromagnetic waves, the wave field is composed of two waves expressed in terms of their frequency-independent coupling-ray-theory travel times and amplitudes. We shall investigate possibilities to include the prevailing-frequency approximation of the coupling ray theory into the interpolation within ray cells in anisotropic media, which could facilitate the common-source Kirchhoff prestack depth migration with coupling-ray-theory S waves in the future.

Anisotropic common ray approximation of the coupling ray theory: extension of the anisotropic-ray-theory P-wave and anisotropic common S-wave and electromagnetic-wave ray tracing towards general initial conditions and velocity models with structural interfaces.

A quantitative criterion whether the SH and SV reference rays are better than the anisotropic common reference rays in a given velocity model, or vice versa. Tracing the SH and SV reference rays in velocity models with generally heterogeneous dependence of the reference symmetry vector on the spatial coordinates. The SH and SV reference rays for the coupling ray theory: initial conditions, structural interfaces.

Study of the problems related to anisotropic-ray-theory ray tracing in a vicinity of S-wave or electromagnetic-wave singularities. Theoretical and numerical studies of the phase-slowness and ray-velocity surfaces.

Derivation of coupling ray theory from the elastodynamic or Maxwell equations concentrated on the study of errors due to neglected terms in order to estimate the accuracy of coupling ray theory. Study of coupling ray series.

Numerical comparison of various approximations of coupling ray theory with more accurate methods.

6. *Wave propagation in anisotropic attenuating media*

Investigation of plane waves propagating in anisotropic attenuating media. Both homogeneous and inhomogeneous waves will be considered. For weakly attenuating media, the perturbation methods will be used.

The results obtained for plane waves propagating in homogeneous media will be generalized to non-planar waves propagating in smoothly varying anisotropic weakly attenuating media using perturbation methods based on the ray theory. A special attention will be devoted to waves generated by point sources. Together with the attenuation properties, also the dispersion properties of waves propagating in attenuating media will be studied.

7. *Computation of two-point paraxial travel times*

Algorithms and codes for fast computation of approximate two-point paraxial travel times calculated in the vicinities of reference rays will be studied and tested numerically. Inhomogeneous, isotropic or anisotropic velocity models with curved interfaces will be considered. Principal attention will be devoted to the study of the accuracy and efficiency of the proposed algorithms. We shall study calculation of two-point paraxial travel times from perturbed sources to perturbed receivers and their use in various applications.

8. *Perturbation derivatives and spatial derivatives of travel time*

Theoretical and numerical applications of the equations for transforming the perturbation derivatives and the third-order and higher-order spatial derivatives of travel time at structural interfaces. Generalization of spatial and perturbation derivatives of travel time to Hamilton–Jacobi equation with right-hand sides dependent on ray parameters. Spatial and perturbation derivatives of two-point travel time (characteristic function).

9. *Accuracy of wave modelling*

The research will be concentrated mainly on the accuracy of travel-time calculations, on the accuracy of finite-difference modelling of wave fields, and on the accuracy of other modelling methods designed or studied within the framework of the project. The main attention will be devoted to the estimation of the feasibility and costs of ray tracing, and to the definition and high-frequency validity of velocity models.

10. *Sensitivity of waves to the structure (sensitivity Gaussian packets)*

Continuing investigation how the perturbations of a generally inhomogeneous isotropic or anisotropic structure manifest themselves in the wave field, and which perturbations can be detected within a limited aperture and a limited frequency band.

Developing the algorithms and computer programs for calculating the sensitivity Gaussian packets. Developing the corresponding algorithm for linearized inversion based on wave-field sensitivity to structural Gabor functions. Sensitivity Gaussian packets should offer explicit correspondence between the time and depth sections. Attention is paid to the optimization of the shape of Gaussian packets.

11. *Lyapunov exponents and velocity model smoothing*

Construction and smoothing of velocity models will further be studied, with emphasis on the application of Sobolev scalar products and Lyapunov exponents. Attention will be paid to a possible extension of the estimation of Lyapunov exponents to smooth 3-D velocity models and to 2-D velocity models with structural interfaces. Sobolev scalar products with spatially variable weights will also be studied. We are ready to smooth the velocity models delivered by the consortium members for application of ray theory methods.

12. *Travel-time tomography and related problems*

Development of theory, algorithms and programs applicable in refraction travel-time tomography, with emphasis on the estimation of the accuracy of the resulting velocity model compared to the geological structure.

Stochastic travel-time tomography: Developing an algorithm for calculating geometrical covariances of travel times. Developing an algorithm for calculating geometrical covariances between rays and B-splines.

Determination of the medium correlation functions from well logs. Calculation of sonic-log travel times in anisotropic media. Estimation of uncertainty of sonic-log travel times. Estimation of attenuation from vertical-seismic-profiling travel times.

Estimation of anisotropy parameters from various experiments (vertical seismic profiling, reflection seismics, etc.): travel-time inversion, inversion of moveout formulae, local parameter estimation from vertical-seismic-profiling measurements. Use of P-wave and, possibly, S-wave data (travel times, polarizations, etc.). Tests with synthetic and, if available, real data sets.

13. *Migrations*

Resolution and accuracy of migrations will be studied. Attention will be paid to the physical meaning of the migrated sections and to their sensitivity to the velocity model, including its anisotropy.

The numerical algorithm of the common-source Kirchhoff prestack depth migration will be extended to three-component seismograms.

Possibilities to include coupling ray theory in seismic imaging will be studied. Kirchhoff migration will be generalized to multivalued travel times.

Amplitude preserving Kirchhoff migration in anisotropic media: Attention will be paid to the synthetic study of possibilities and limitations to recover reflection coefficients from data measured in inhomogeneous anisotropic media. Common-source Kirchhoff prestack depth migration with S waves will be generalized from isotropic to anisotropic velocity models. Estimation of the misplacement of the interface due to an incorrectly known medium using the travel-time perturbations will be compared numerically with migrated sections.

14. *Nonlinear hypocentre determination*

Calculation and interpolation of the geometrical crossvariances of theoretical travel times for the nonlinear kinematic hypocentre determination.

15. *Electromagnetic ray theory for heterogeneous bianisotropic media*

Developing the ray theory for electromagnetic waves in heterogeneous bianisotropic media, including the application of the above-mentioned topics to electromagnetic waves.

16. *Sample data for the program packages*

Examples of the input data describing or approximating velocity models delivered by the consortium members or other typical velocity models can be prepared. Examples of the input data to perform calculations in such velocity models can also be prepared.

17. *Concluding remarks*

In addition to this programme, we will certainly be responsive to specific technical suggestions and recommendations of the consortium members within the general framework of the project. The research in most directions listed above will continue into the future years of the project.

SW3D PROGRAM PACKAGES

Package CRT:

Velocity model: Using package MODEL (see below). Isotropic velocity models with attenuation, anisotropic velocity models without attenuation.

Type of waves: Arbitrary type of elementary seismic body wave corresponding to the zero-order ray theory (P, S, converted, coupled S waves).

Computations: Arbitrary position and shape of the source, initial-value ray tracing by numerical integration of ray equations, general isotropic-ray-theory rays, anisotropic-ray-theory P-wave rays and anisotropic common S-wave rays in smooth velocity models, two-point ray tracing by the shooting method based on ray histories, travel-time computation, dynamic ray tracing, paraxial-ray propagator matrix, geometrical spreading, vectorial amplitudes, polarization vectors. The package may be applied to the evaluation of the elastodynamic ray-theory Green function, and to the computation of synthetic seismograms, including the coupling ray theory along isotropic or anisotropic common S-wave rays and the response of fine layers at receiver sites (program package RMATRIX by C.J. Thomson, linked to the CRT package). Fast computation of multivalued ray-theory travel times, Green functions and other quantities at the nodes of 3-D grids by interpolation within ray cells (P, S and converted waves in general isotropic velocity models, P waves in smooth anisotropic velocity models). Least-square travel-time tomography with smoothing using Sobolev scalar products.

Acquisition schemes: Surface seismics (land and marine), vertical seismic profiling, cross-hole, ocean bottom.

Planned innovations: Fast computation of multivalued ray-theory Green functions at the nodes of 3-D grids by interpolation within ray cells for coupled S waves in smooth anisotropic velocity models. Extension of the anisotropic-ray-theory P-wave and anisotropic common S-wave ray tracing towards general initial conditions and velocity models with structural interfaces. Testing and improving anisotropic-ray-theory S-wave ray tracing in smooth velocity models without interfaces. Coupling ray theory along the anisotropic-ray-theory rays in smooth velocity models without interfaces. The package will be extended to solve various inverse problems, stochastic travel-time tomography in particular.

Package ANRAY:

Velocity model: 3-D laterally varying structures containing isotropic and anisotropic non-vanishing layers. Specification of elastic parameters inside individual layers either by linear interpolation between isosurfaces of elastic parameters, or by B-spline interpolation within a 3-D rectangular grid of elastic parameters. VRML and GOCAD visualization.

Types of waves: Arbitrary type of elementary seismic body wave (P, S, any converted wave, coupled S waves).

Computations: Numerical integration of ray tracing and dynamic ray tracing equations, calculation of ray vectorial amplitudes, ray-theory Green function including the Green function in the quasi-isotropic approximation for S waves, ray synthetic seismograms, particle ground motions.

Acquisition schemes: Surface seismics (land and marine), vertical seismic profiling, cross-hole, ocean bottom.

Planned innovations: (a) Incorporation of effects of weak attenuation. (b) Ray tracing and dynamic ray tracing in media with rotated higher-symmetry anisotropy (transverse isotropy, orthorhombic symmetry). (c) First-order ray tracing of P waves and coupled S waves in layered inhomogeneous weakly anisotropic media. (d) Simplified dynamic

ray tracing in Cartesian coordinates. (e) Calculation of KMAH index in anisotropic media. (f) Removal of problems of P-wave reflections/transmissions in a vicinity of S-wave singularities. (g) Extension of applicability of the ANRAY package to Gaussian beam summation or Maslov method. (h) Further debugging, completion, and removal of inconsistencies in the description of the package.

Package SEIS:

Velocity model: 2-D laterally varying isotropic structures composed of layers separated by curved interfaces. Any interface may form edges. It may also coincide with a neighbouring interface(s) in some region. Thus, the velocity models with isolated bodies and pinchouts can be considered. Inside the layers, the velocities of P and S waves may vary in two directions. Weak attenuation and non-planar topography can be considered.

Types of waves: Arbitrary type of elementary seismic body wave (P,S, any converted or multiply reflected wave).

Computations: Arbitrary position of a point source, numerical integration of 2-D ray tracing and dynamic ray tracing equations, computation of ray vectorial amplitudes or Green functions of individual elementary waves, ray synthetic seismograms, particle ground motions.

Acquisition schemes: Surface seismics (land and marine), vertical seismic profiling, cross-hole.

Planned innovations: Alternative computation of synthetic seismograms in the frequency domain. Ocean bottom configuration. Extended visualization. Extension of test examples.

Package MODEL:

Velocity model: General 3-D layered and block isotropic or anisotropic structures, containing isolated bodies, pinchouts, etc. Inside the layers and blocks, the elastic parameters may vary in all three dimensions. Attenuation and non-planar topography can be considered. Possibility of velocity model smoothing, data fitting by inversion including fitting and smoothing GOCAD models, conversion of velocity model parametrization, triangulation of structural interfaces, VRML and GOCAD visualization.

Types of waves: Seismic waves.

Package NET:

Velocity model: Isotropic without attenuation, using package MODEL or using gridded velocities.

Types of waves: Wave-independent, first arrivals, constrained first arrivals.

Computations: Arbitrary position and shape of the source. First-arrival travel times in the whole velocity model are computed. The algorithm of computation is independent of the velocity model's complexity.

Acquisition schemes: Surface seismics (land and marine), vertical seismic profiling, cross-hole, ocean bottom.

Package FORMS:

Types of waves: Wave-independent or seismic waves.

Computations: Subroutines used by other program packages including data input and output subroutines, management and plotting of synthetic seismograms, 2-D and 3-D graphics including 3-D virtual reality with VRML and GOCAD visualization, manipulation and calculation with gridded data (data cubes), programs for matrix and vector operations necessary for inversion, other general-purpose seismic software.

Planned innovations: Nonlinear hypocentre determination. Program for computation of plane-wave reflection/transmission coefficients at planar interfaces separating arbitrary anisotropic media.