Moveout approximation for a converted wave in a moderately anisotropic homogeneous DTI layer

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Outline

Introduction

Weak-anisotropy parameters

Approximate traveltime formula

Tests of the formula

Conclusions

Introduction

Moveout approximations

common

- Taylor series expansion of T^2 in terms of x^2

hyperbolic, non-hyperbolic, ...

alternative

- expansion of T^2 in terms of the deviations

of anisotropy from isotropy $=>$

weak anisotropy (WA) parameters

Introduction

Moveout approximations with WA parameters

- replacement of the actual ray
  by a reference ray in a reference isotropic medium

- approximation of the exact ray velocity
  by the first-order phase velocity

- possible approximate determination of the conversion point
  of the reference ray in the reference medium

Weak-anisotropy parameters

- 21 weak-anisotropy (WA) parameters

  slightly modified *Mensch & Rasolofosaon (1997)*

- generalization of *Thomsen's (1986)* parameters

- characterize deviations from an isotropic reference

- zero WA parameters $\rightarrow$ moveout in the isotropic reference medium

- applicable to anisotropy of any type, strength and orientation

- an alternative to stiffness tensor or Voigt's $C_{\alpha\beta}$ or $A_{\alpha\beta}$

- can describe exactly any wave attribute

Weak-anisotropy parameters

- natural combinations of $C_{\alpha\beta}$ or $A_{\alpha\beta}$ taken into account

- linear relation of WA to $C_{\alpha\beta}$ or $A_{\alpha\beta}$ parameters

- simple linear transformation of WA parameters
  from one coordinate system to another

- definable in coordinate systems independent of symmetry elements
  of studied anisotropy symmetry

- optional choice of the reference velocities α, β

- all 21 WA parameters dimensionless, of comparable size

Weak-anisotropy parameters

$$\epsilon_{\mathbf{x}} = \frac{\mathbf{A_{11}} - \alpha^{\mathbf{2}}}{\mathbf{2}\alpha^{\mathbf{2}}}, \quad \epsilon_y = \frac{A_{22} - \alpha^2}{2\alpha^2}, \quad \epsilon_{\mathbf{z}} = \frac{\mathbf{A_{33}} - \alpha^{\mathbf{2}}}{\mathbf{2}\alpha^{\mathbf{2}}}$$

$$\delta_x = \frac{A_{23} + 2A_{44} - \alpha^2}{\alpha^2}, \quad \delta_{\mathbf{y}} = \frac{\mathbf{A_{13}} + \mathbf{2A_{55}} - \alpha^{\mathbf{2}}}{\alpha^{\mathbf{2}}}, \quad \delta_z = \frac{A_{12} + 2A_{66} - \alpha^2}{\alpha^2}$$

$$\epsilon_{15} = \frac{A_{15}}{\alpha^2}, \quad \epsilon_{16} = \frac{A_{16}}{\alpha^2}, \quad \epsilon_{24} = \frac{A_{24}}{\alpha^2}, \quad \epsilon_{26} = \frac{A_{26}}{\alpha^2}, \quad \epsilon_{34} = \frac{A_{34}}{\alpha^2}, \quad \epsilon_{35} = \frac{A_{35}}{\alpha^2}$$

$$\chi_x = \frac{A_{14} + 2A_{56}}{\alpha^2}, \quad \chi_y = \frac{A_{25} + 2A_{46}}{\alpha^2}, \quad \chi_z = \frac{A_{36} + 2A_{45}}{\alpha^2}$$

$$\gamma_x = \frac{A_{44} - \beta^2}{2\beta^2}, \quad \gamma_{\mathbf{y}} = \frac{\mathbf{A_{55}} - \beta^{\mathbf{2}}}{\mathbf{2}\beta^{\mathbf{2}}}, \quad \gamma_{\mathbf{z}} = \frac{\mathbf{A_{66}} - \beta^{\mathbf{2}}}{\mathbf{2}\beta^{\mathbf{2}}}, \quad \epsilon_{46} = \frac{A_{46}}{\beta^2}, \quad \epsilon_{56} = \frac{A_{56}}{\beta^2}, \quad \epsilon_{45} = \frac{A_{45}}{\beta^2}$$

DTI WA parameters in the local reflector coordinate system

α, β - P-, S-wave reference velocities

Weak-anisotropy parameters

DTI medium $(\epsilon_x,\ \epsilon_z,\ \delta_y,\ \gamma_y,\ \gamma_z)$

WA parameters related to the local reflector coordinate system

$$\epsilon_x = \epsilon_y = \tfrac{1}{2}\delta_z \quad , \qquad \delta_x = \delta_y \quad , \qquad \gamma_x = \gamma_y \quad ,$$

$$\epsilon_{15} = \epsilon_{16} = \epsilon_{24} = \epsilon_{26} = \epsilon_{34} = \epsilon_{35} = \epsilon_{45} = \epsilon_{46} = \epsilon_{56} = \chi_x = \chi_y = \chi_z = 0$$

Weak-anisotropy parameters

Possible use of the estimates of ϵ_x, ϵ_z, δ_y, γ_y
in approximate reconstruction of phase-velocity surfaces:

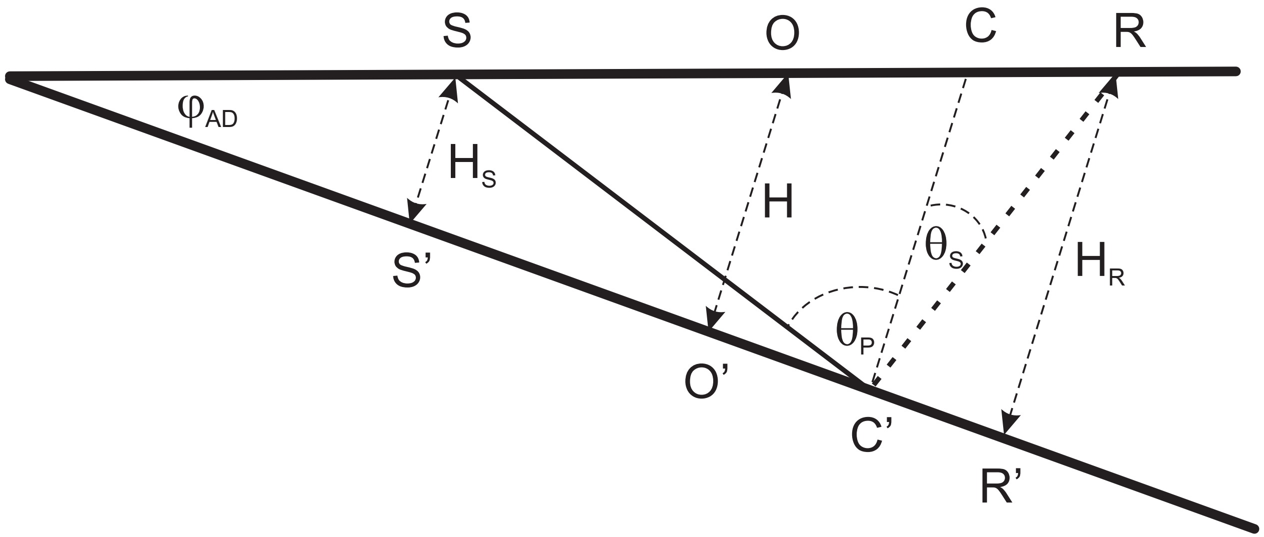
$$\widetilde{c_P^2}(\mathbf{N}) \sim \alpha^2 \left\{ 1 + 2[\epsilon_x N_1^4 + \delta_y N_1^2 N_3^2 + \epsilon_z N_3^4] \right\}$$

$$\widetilde{c_{SV}^2}(\mathbf{N}) \sim \beta^2 [1 + 2\gamma_y + 2r^{-2}(\epsilon_x + \epsilon_z - \delta_y) N_1^2 N_3^2]$$

$\widetilde{c_P^2}$, $\widetilde{c_{SV}^2}$ - approximate squares of P- and S-wave phase velocities
α, β - P- and S-wave WA parameters reference velocities
$r = \beta/\alpha$, $\mathbf{N}$ - unit source-receiver vector

Approximate traveltime formula

Configuration



S, R, O, C' - source, receiver, midpoint, conversion point

$\overline{SR} \equiv x$ - offset, $\overline{SC} \equiv x_C$ - offset of the conversion point

H_S, H_R, H - orthogonal distances of S, R, O from the reflector

φ_{AD}, θ_P, θ_S - apparent dip angle, angles of incidence and reflection

Approximate traveltime formula

Reflection traveltime

$$T(x) = T_{HS}(1 + \check{x}_C^2)^{3/2} P_P^{-1/2}(\check{x}_C) + T_{HR}[1 + (\hat{x} - \hat{x}_C)^2]^{3/2} P_{SV}^{-1/2}(\hat{x} - \hat{x}_C)$$

$$T_{HS} = \frac{H_S}{\bar{\alpha}}, \quad T_{HR} = \frac{H_R}{\bar{\beta}}, \quad \check{x}_C = \frac{x_C \cos\varphi_{AD}}{H_S}, \quad \hat{x} - \hat{x}_C = \frac{(x - x_C)\cos\varphi_{AD}}{H_R}$$

$$P_P(x) = (1 + x^2)^2 + 2\epsilon_x x^4 + 2\delta_y x^2 + 2\epsilon_z$$

$$P_{SV}(x) = (1 + x^2)^2(1 + 2\gamma_y) + 2r^{-2}(\epsilon_x + \epsilon_z - \delta_y)x^2$$

$r = \beta/\alpha$; α, β - P- and S-wave WA parameters reference velocities

Approximate traveltime formula

Offset of the conversion point x_C
in a reference isotropic medium:
solution of the quartic equation

$$x_C^4 - 2x_C^3 x + x_C^2\left(x^2 + \frac{H_R^2 \bar{r}^2 - H_S^2}{Q(\bar{r},\varphi_{AD})}\right) + 2x_C \frac{H_S^2 x}{Q(\bar{r},\varphi_{AD})} - \frac{H_S^2 x^2}{Q(\bar{r},\varphi_{AD})} = 0$$

x, x_C - offset, offset of the conversion point

$Q(\bar{r},\varphi_{AD}) = (\bar{r}^2 - 1)\cos^2\varphi_{AD}$

$\bar{r} = \bar{\beta}/\bar{\alpha}$; $\bar{\alpha}$, $\bar{\beta}$ - P- and S-wave reference velocities

$\bar{\alpha}$, $\bar{\beta}$ may differ from α, β specifying WA parameters

Determination of x_C: exact or approximate (Thomsen, 1999)

Approximate traveltime formula

Approximate determination of the conversion point x_C

$$x_C \sim x[C_0 + \frac{C_1(x/H)+C_2(x/H)^2}{1+C_3(x/H)^2}]$$

$$C_0 = \frac{1}{1+\bar{r}} \,, \quad C_1 = -\frac{\bar{r}\sin\varphi_{AD}}{(1+\bar{r})^2} \,, \quad C_2 = \frac{\bar{r}}{2}\frac{1-\bar{r}}{(1+\bar{r})^3}\cos 2\varphi_{AD} \,,$$

$$C_3 = \frac{1}{2}\frac{C_1|\sin\varphi_{AD}|+2C_2}{\bar{x}_{Clim}-C_0} - \frac{1}{4}\sin^2\varphi_{AD}$$

$\bar{r} = \bar{\beta}/\bar{\alpha}$, $\bar{x}_{Clim}$ - limiting value of x_C, $0 \leq \bar{x}_{Clim} \leq 1$

Horizontal reflector

$C_0 = \frac{1}{1+\bar{r}}$, $C_1 = 0$, $C_2 = \frac{\bar{r}}{2}\frac{1-\bar{r}}{(1+\bar{r})^3}$, $C_3 = \frac{1}{2}\frac{1-\bar{r}}{(1+\bar{r})^2}$ Thomsen (1999)

Tests of formula

Strength of anisotropy

$2(c_{max} - c_{min})/(c_{max} + c_{min}) \times$ 100%

Exact reference - ANRAY program package

Relative traveltime errors: $(T - T_{ex})/T_{ex} \times 100\%$

T - approximate traveltime, T_{ex} - exact (ANRAY) traveltime

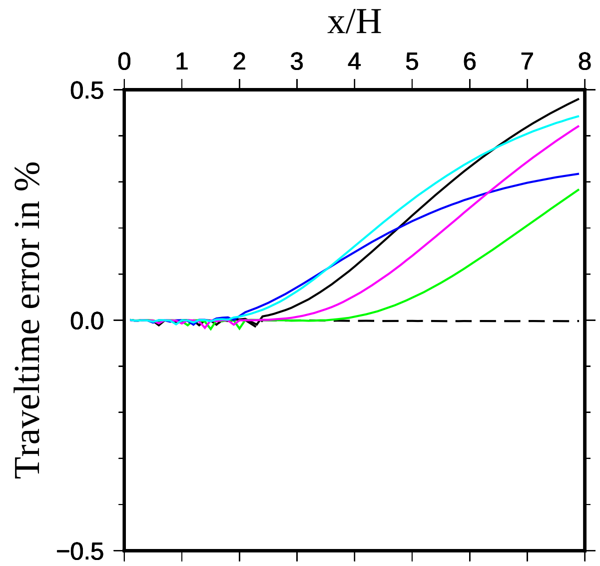
Tests of formula

Isotropy (α=2.54 km/s) - horizontal reflector

$\bar{r} = r = \beta/\alpha =$ 0.2 (blue), 0.3 (light blue), 0.4 (black), 0.5 (pink), 0.6 (green)

black dashed curve - WA formula, $r =$0.4; numerical determination of x_C

coloured solid curves - WA formula, approximate determination of x_C



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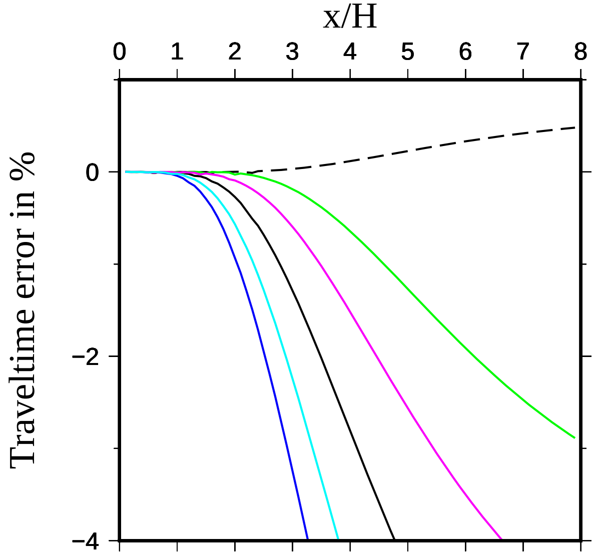
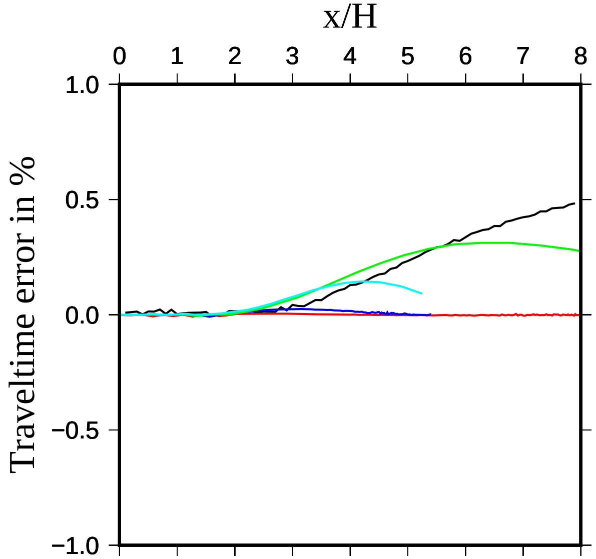
coloured solid curves -Tsvankin & Thomsen (1994)



Tests of formula

Isotropy (α=2.54 km/s, $\bar{r} = r$=0.4) - dipping reflector

$\varphi_{AD} = 0°$ (black), $10°$ (red), $20°$ (blue), $-10°$ (green), $-20°$ (light blue)

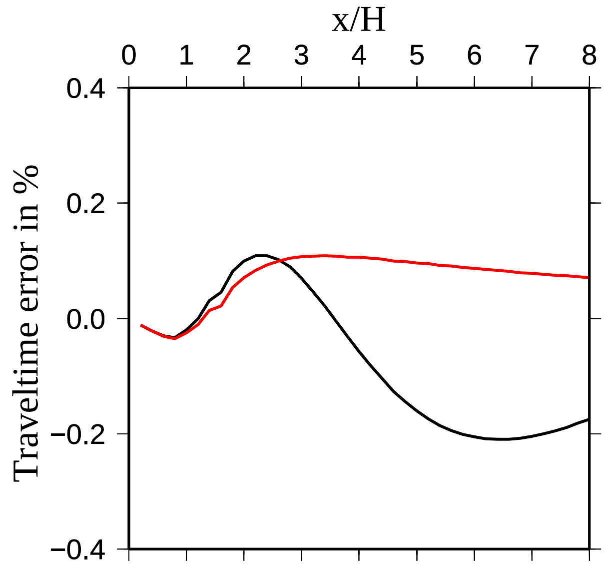


x_C determined from approximate formula (modified Thomsen 1999)

Tests of formula

Limestone; $\sim$ 8% (P), $\sim$ 5% (SV) - horizontal reflector

ϵ_x=0.076, δ_y=0.133, ϵ_z=0, γ_y=0, ($\bar{r} = r = 0.569$)



x_C **determined by numerical solution of quadric equation**

x_C determined from approximate formula of Thomsen (1999)

Tests of formula

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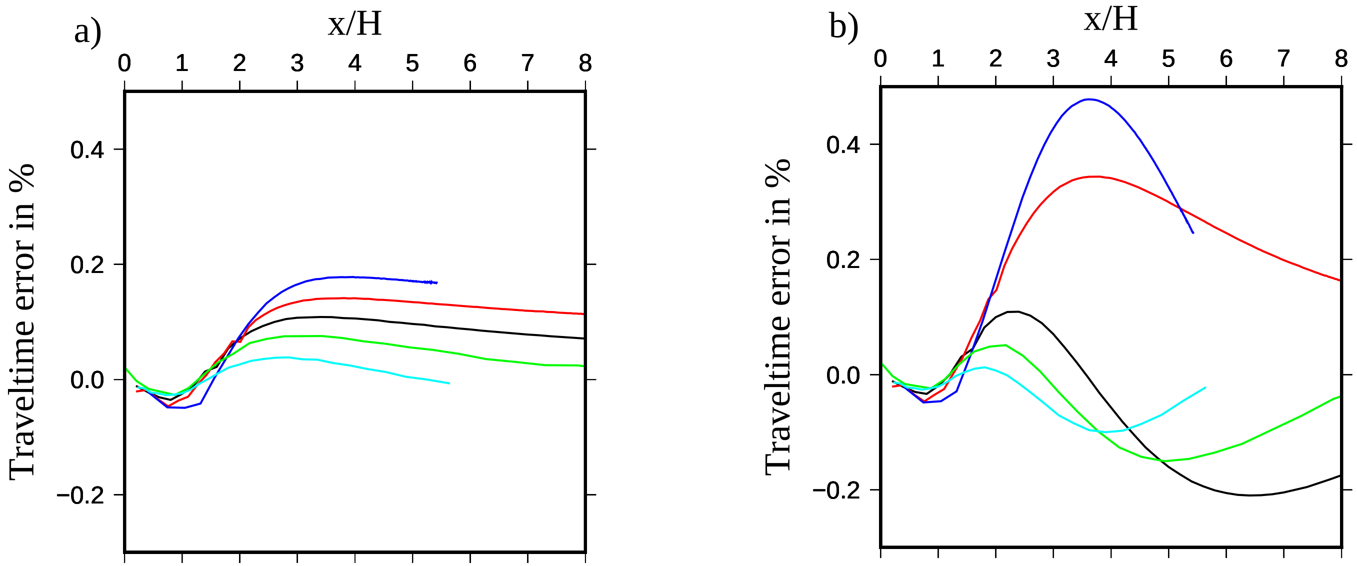


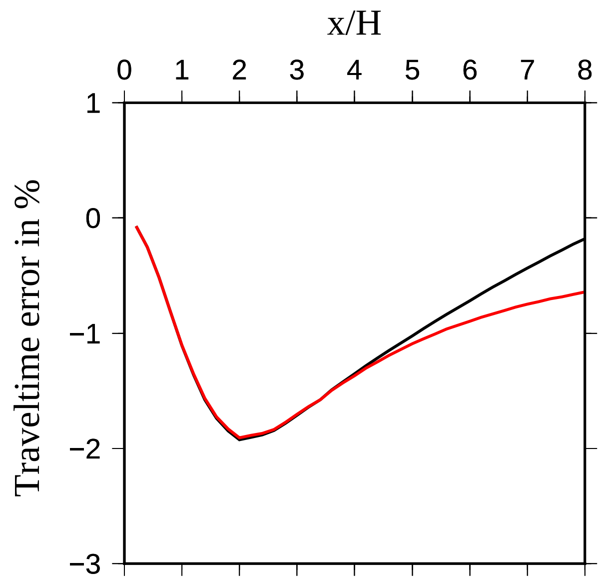
a) x_C determined by numerical solution of quadric equation

b) x_C determined from approximate formula (modified Thomsen 1999)

Tests of formula

Hard shale; ∼ 25% (P), ∼ 12% (SV) - horizontal reflector

ϵ_x=0.252, δ_y=0.034, ϵ_z=0, γ_y=0, $(\bar{r} = r = 0.638)$



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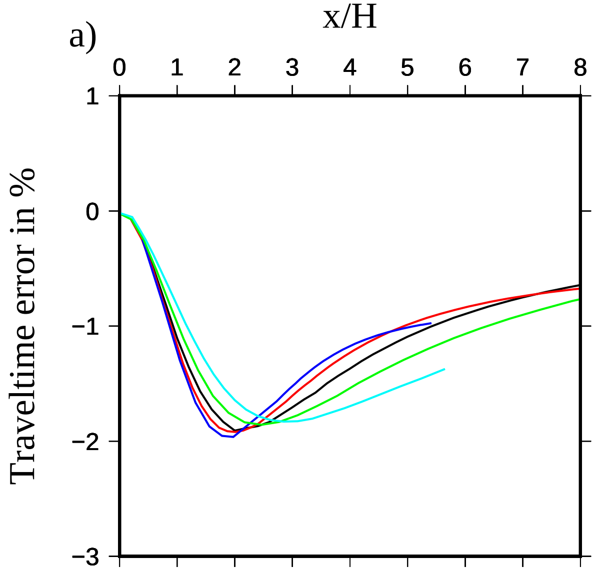
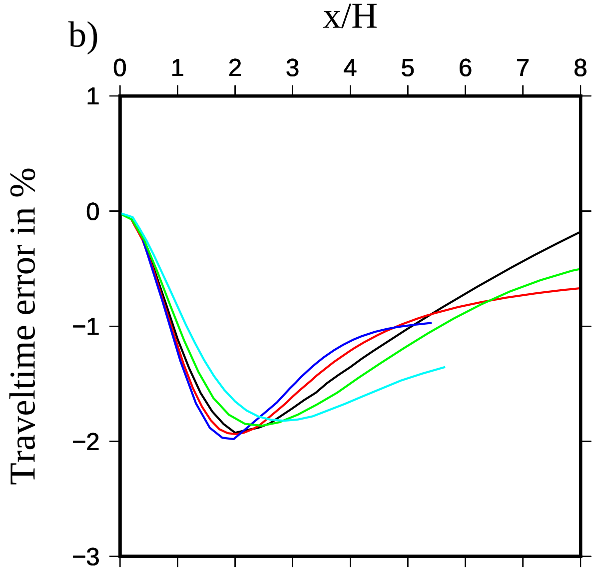




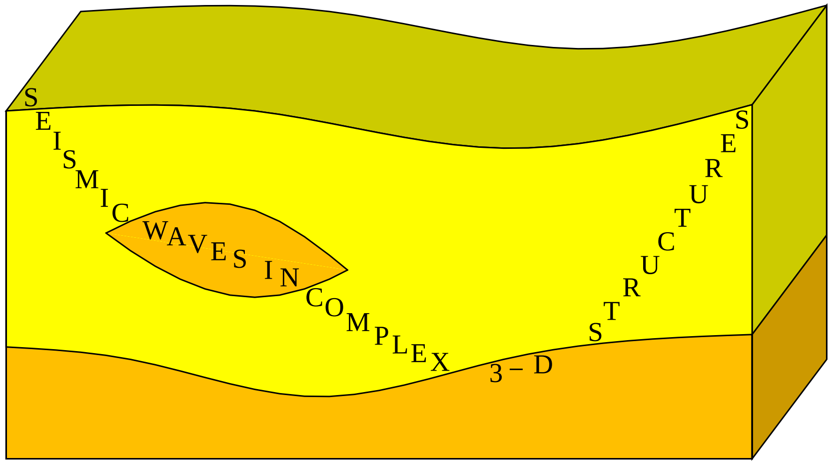
a) x_C determined by numerical solution of quadric equation

b) x_C determined from approximate formula (modified Thomsen 1999)

Conclusions

- moveout formula based on WA approximation
- weak or moderate anisotropy
- simple and transparent, applicable to any offset
- accuracy within the first-order WA approximation
- inaccuracies caused by deviations of $\mathbf{n}$ and $\mathbf{N}$
- comparable accuracy with unconverted waves
- formula holds for P-SV as well as SV-P waves
- possible approximate reconstruction of phase-velocity surfaces
- horizontal reflector: possible to express T_0, v_{NMO}, A_4 in terms of WA parameters

Acknowledgement



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