Perturbation expansions of complex-valued travel time along real-valued reference rays

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Reference Hamiltonian function for real-valued reference rays in attenuating media

Given complex-valued Hamiltonian function $H(x^m, p_n)$ is a holomorphic function of complex slowness vector p_n.

We need reference Hamiltonian function $\widetilde{H}(x^m, p_n)$ to be a holomorphic function of p_n.

We want $\widetilde{H}(x^m, p_n)$ to be equal to the real part $\mathrm{Re}[H(x^m, p_n)]$ for real p_n.

These two conditions determine $\widetilde{H}(x^m, p_n)$ uniquely:

$$\widetilde{H}(x^m, p_n) = \sum_{\Omega=0}^{+\infty} \frac{\mathrm{i}^{\Omega}}{\Omega!} \mathrm{Re}[H^{,k_1 k_2 \ldots k_\Omega}(x^m, \mathrm{Re}\, p_n)] \, \mathrm{Im}(p_{k_1}) \, \mathrm{Im}(p_{k_2}) \ldots \mathrm{Im}(p_{k_\Omega})$$

where

$$H^{,k_1 k_2 \ldots k_\Omega}(x^m, p_n) = \frac{\partial}{\partial p_{k_1}} \frac{\partial}{\partial p_{k_2}} \ldots \frac{\partial}{\partial p_{k_\Omega}} H(x^m, p_n)$$

The most accurate perturbations of travel time are usually obtained if $H(x^m, p_n)$ is a homogeneous function of degree -1 with respect to p_n.

Perturbation Hamiltonian function
for the perturbation expansion of complex-valued travel time along real-valued reference rays in attenuating media

For a convenient perturbation from the reference Hamiltonian function $\widetilde{H}(x^m, p_n)$ to the given complex-valued Hamiltonian function $H(x^m, p_n)$, we define the one-parametric perturbation Hamiltonian function

$$H(x^m, p_n, \alpha) = \widetilde{H}(x^m, p_n) + [H(x^m, p_n) - \widetilde{H}(x^m, p_n)]\, \alpha$$

linear with respect to perturbation parameter α.

All perturbation derivatives of travel time and of its spatial derivatives can be calculated using equations of Klimeš (2002) and Klimeš (2010).

References

Klimeš, L. (2002): Second–order and higher–order perturbations of travel time in isotropic and anisotropic media. *Stud. geophys. geod.*, **46**, 213–248, online at "http://sw3d.cz".

Klimeš, L. (2010): Transformation of spatial and perturbation derivatives of travel time at a general interface between two general media. In: *Seismic Waves in Complex 3–D Structures, Report 20*, pp. 103–114, Dep. Geophys., Charles Univ., Prague.