

3D and 2D computations of 3D synthetic
seismograms using the ray-based Born
approximation in simple models

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Structure of the speech

1 Models

Structure of the speech

- 1 Models
- 2 Numerical 3D computations of 3D seismograms

Structure of the speech

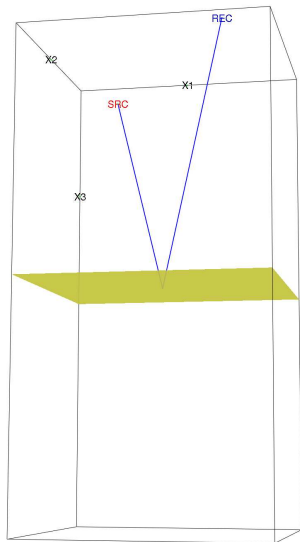
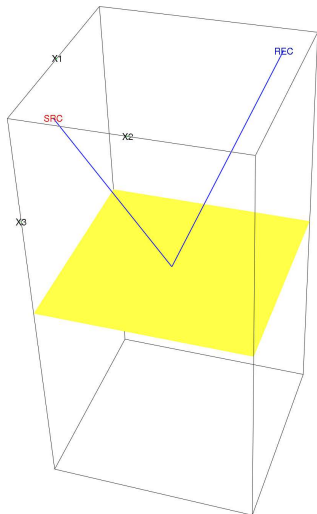
- 1 Models
- 2 Numerical 3D computations of 3D seismograms
- 3 Numerical 2D computations of 3D seismograms

Current section

- 1 Models
- 2 Numerical 3D computations of 3D seismograms
- 3 Numerical 2D computations of 3D seismograms

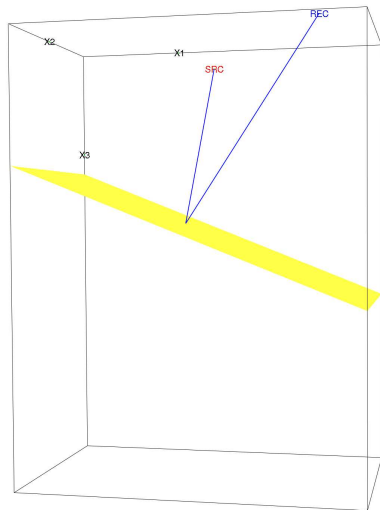
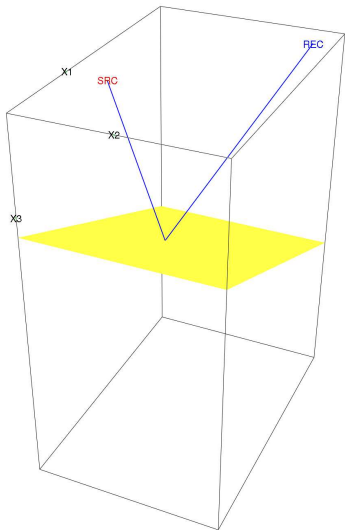
Model 1

Horizontal interface in the depth 10 km.



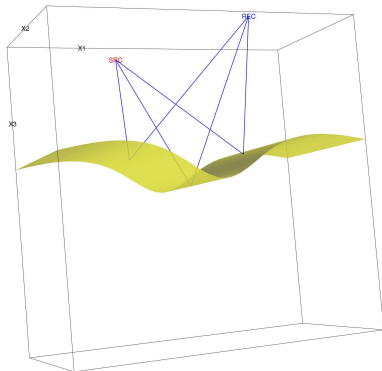
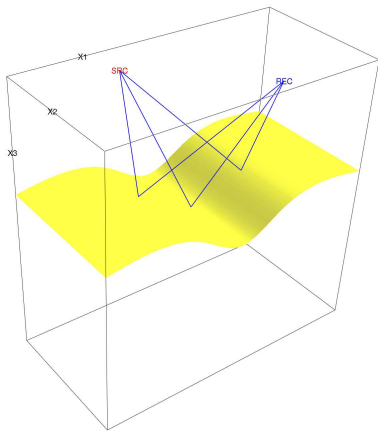
Model 2

Inclined interface with slope 2/5.



Model 3

Curved interface.



Perturbed models

- 3 perturbed models, each model:
 - Two homogenous layers.
 - Upper layer:

$$v_p = 6 \text{ km/s}, \quad v_s = 3 \text{ km/s}, \quad \rho = 2000 \text{ kg/m}^3.$$

- Lower layer:

$$v_p = 6.01 \text{ km/s}, \quad v_s = 3.01 \text{ km/s}, \quad \rho = 2010 \text{ kg/m}^3.$$

- Models differ by the shape of the interface (and model volume).

Unperturbed models

Homogenous without interface. Elastic parameters:

$$v_p = 6 \text{ km/s}, \quad v_s = 3 \text{ km/s}, \quad \rho = 2000 \text{ kg/m}^3.$$

Elastic parameters=perturbed m. upper layer.

Current section

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The purpose of the modelling?

Comparison of the seismograms, computed using

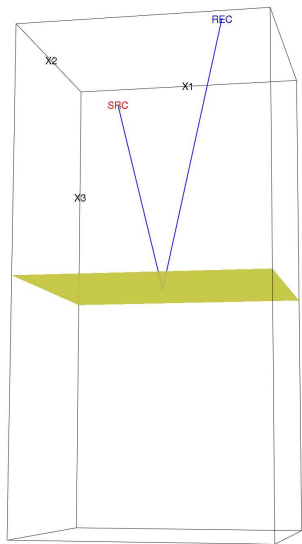
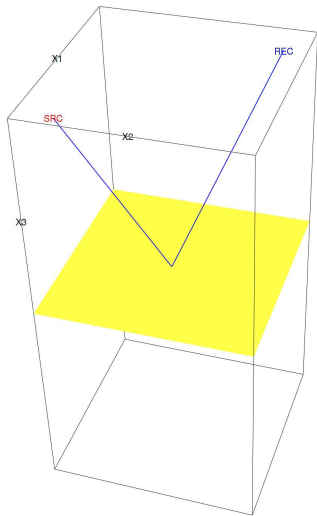
- ray theory in the unperturbed model + the Born approximation (reflected waves and diffracted waves)
- the ray theory in the perturbed model (reflected waves)

In the computations...

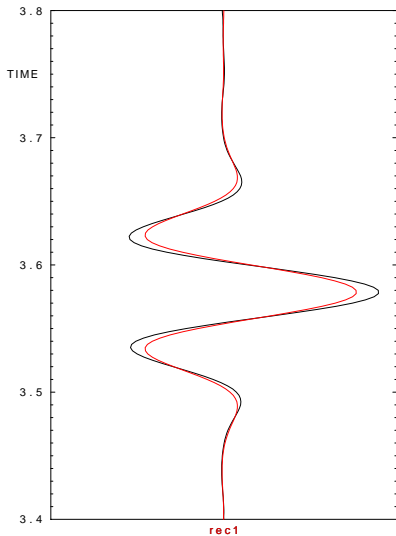
- We use the first-order ray-based Born approximation in an isotropic medium with the high frequency approximation of the spatial derivatives.
- The source time function is a Gabor signal with prevailing frequency 10 Hz filtered by frequency filter.
- P waves except one numerical example.

Model 1

Horizontal interface in the depth 10 km.

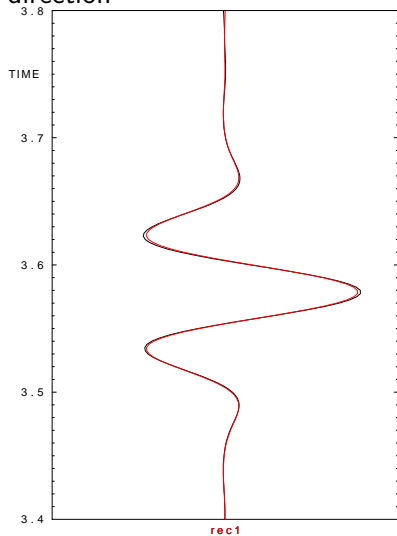


Grid $100 \times 100 \times 100$ gridpoints



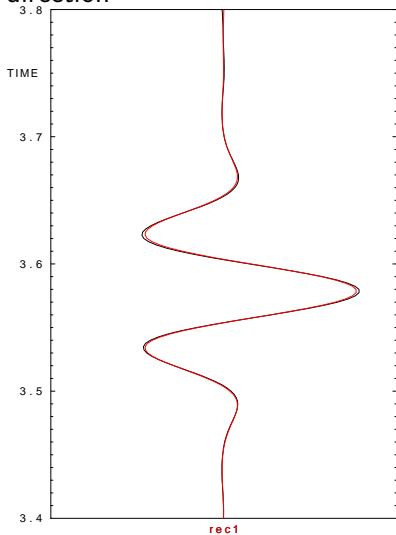
BLACK...Born approximation
RED...Ray theory

Grid **twice** denser in the **vertical** direction



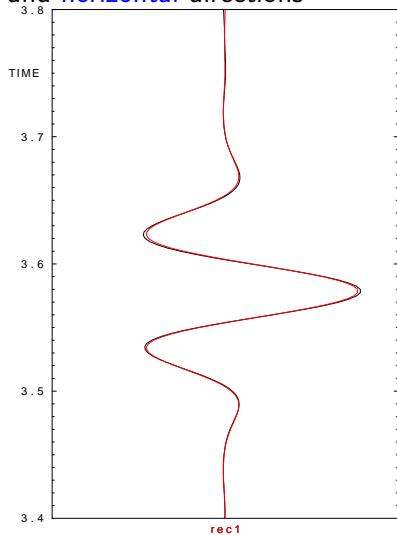
BLACK...Born approximation
RED...Ray theory

Grid twice denser in the vertical direction



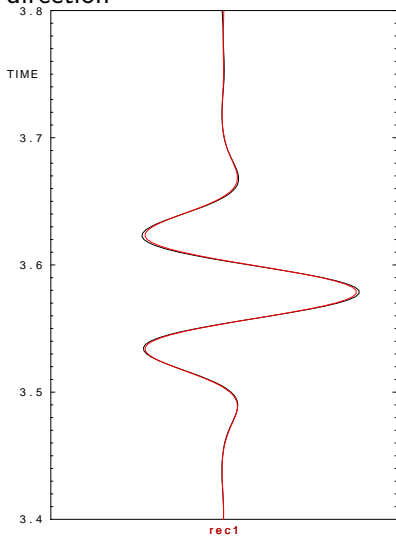
BLACK...Born approximation
RED...Ray theory

Grid **twice** denser in the **vertical** and **horizontal** directions



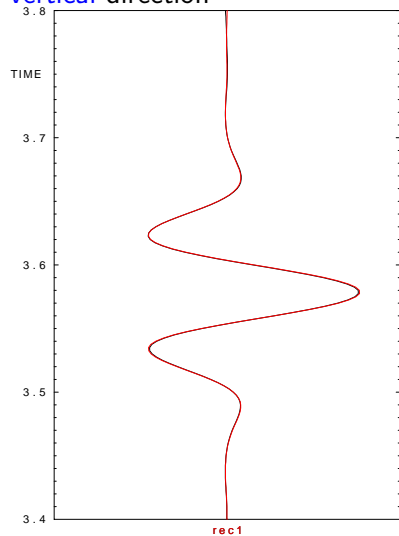
BLACK...Born approximation
RED...Ray theory

Grid twice denser in the vertical direction



BLACK...Born approximation
RED...Ray theory

Grid **four-times** denser in the vertical direction



BLACK...Born approximation
RED...Ray theory

Theoretical description of the discretization error

We assume

- paraxial approximation with the first-order Taylor expansion of travel with respect to the spatial coordinates,
- the central point is the point of reflection.

Then discretization error is approximately scaled with $\frac{h}{\lambda} \cos \alpha$,
where

h ... discretization in the vertical direction

α ... angle between the slowness vector of the incident wave and the normal to the interface

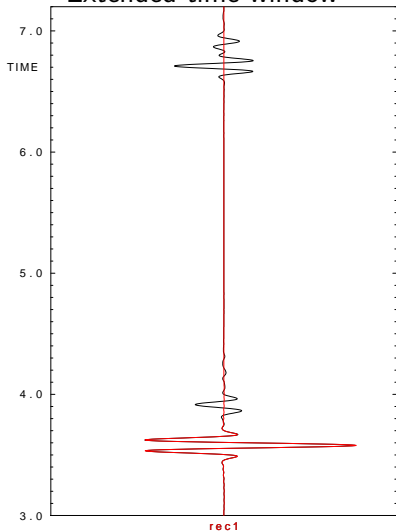
λ ... wavelength

Comparison with the numerical results

$h = D_3$ [km]	Theory [%]	Measured [%]
0.1	16	17
0.05	3.7	2.2
0.025	0.90	0.90

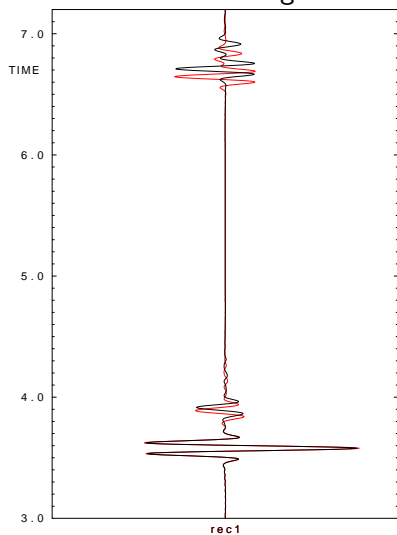
$D_3 \dots$ the grid interval in the direction of the third coordinate axis.

Extended time window



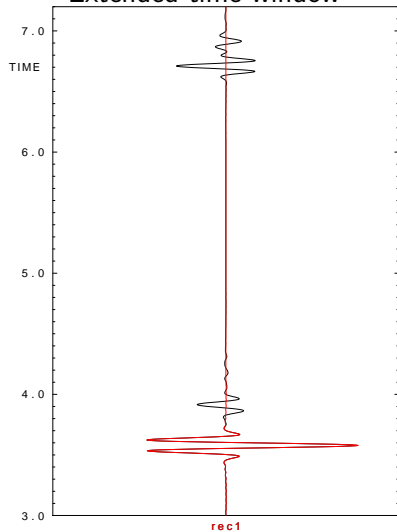
BLACK...Born approximation
RED...Ray theory

Grid vs. smaller grid



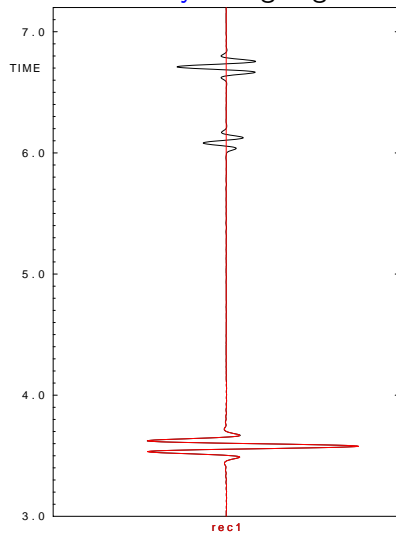
BLACK...Born approximation
RED...Born approximation, smaller grid

Extended time window



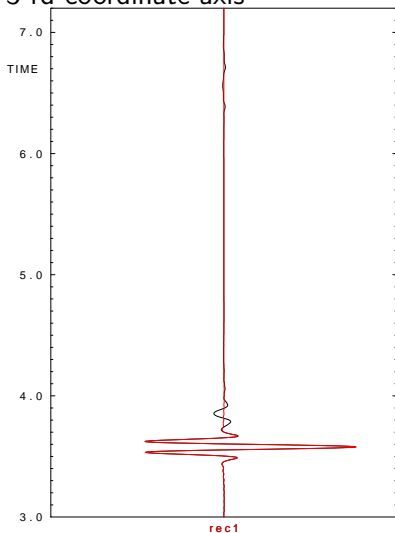
BLACK...Born approximation
RED...Ray theory

Horizontally enlarged grid



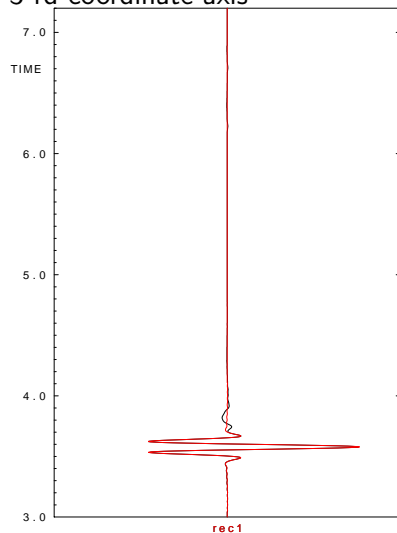
BLACK...Born approximation
RED...Ray theory

Cosine window of length 1 km
applied in all directions except
the negative direction of the
3-rd coordinate axis



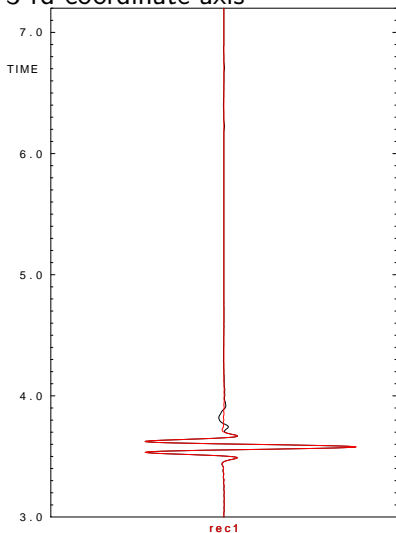
BLACK...Born approximation
RED...Ray theory

Cosine window of length 1.5 km
applied in all directions except
the negative direction of the
3-rd coordinate axis



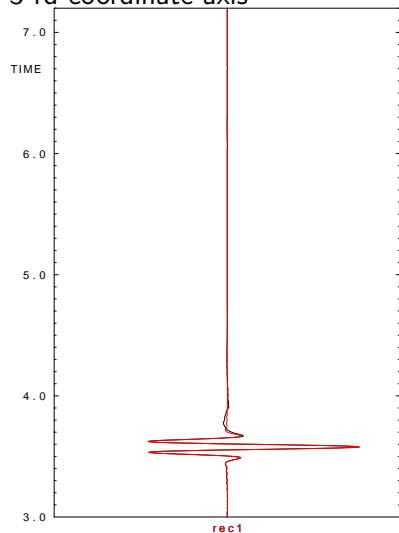
BLACK...Born approximation
RED...Ray theory

Cosine window of length 1.5 km
applied in all directions except
the negative direction of the
3-rd coordinate axis



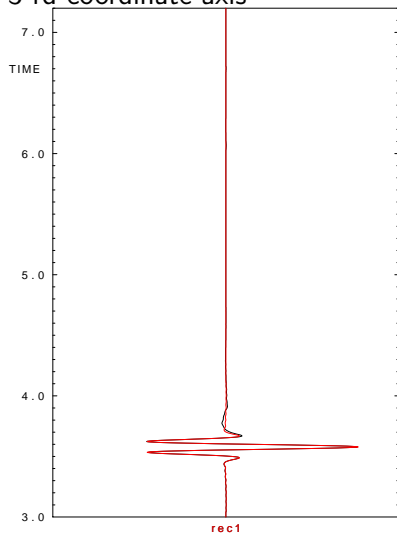
BLACK...Born approximation
RED...Ray theory

Cosine window of length 2 km
applied in all directions except
the negative direction of the
3-rd coordinate axis



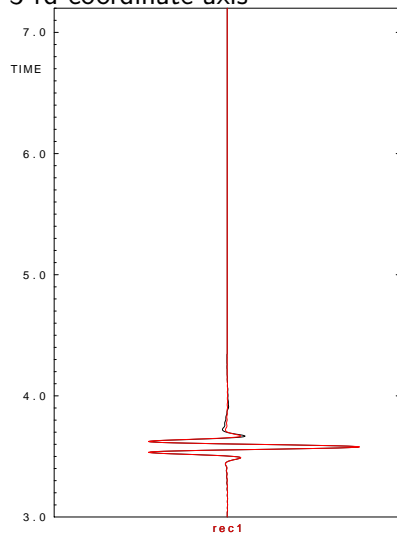
BLACK...Born approximation
RED...Ray theory

Cosine window of length 2 km
applied in all directions except
the negative direction of the
3-rd coordinate axis



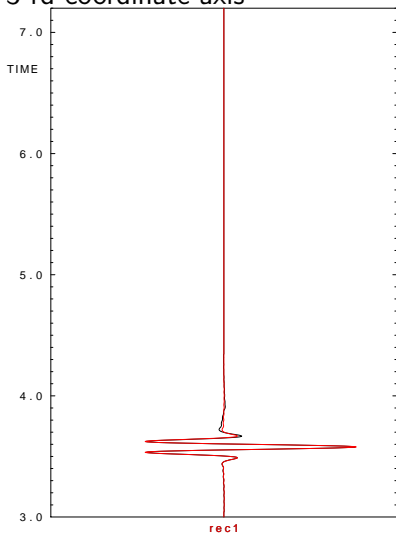
BLACK...Born approximation
RED...Ray theory

Cosine window of length 2.5 km
applied in all directions except
the negative direction of the
3-rd coordinate axis



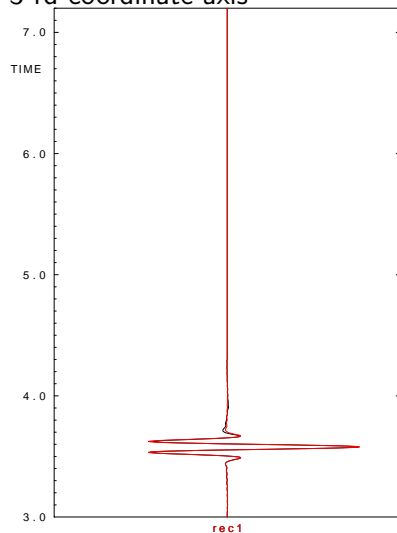
BLACK...Born approximation
RED...Ray theory

Cosine window of length 2.5 km applied in all directions except the negative direction of the 3-rd coordinate axis



BLACK...Born approximation
RED...Ray theory

Cosine window of length 3 km applied in all directions except the negative direction of the 3-rd coordinate axis



BLACK...Born approximation
RED...Ray theory

Theoretical description of the effect of cosine window

Formula

$$\left| \frac{I_1}{I_0} \right| < \left(\frac{\lambda}{4L \cos \alpha} \right)^2.$$

I_1 ... amplitude of spurious reflection **with** cosine window,

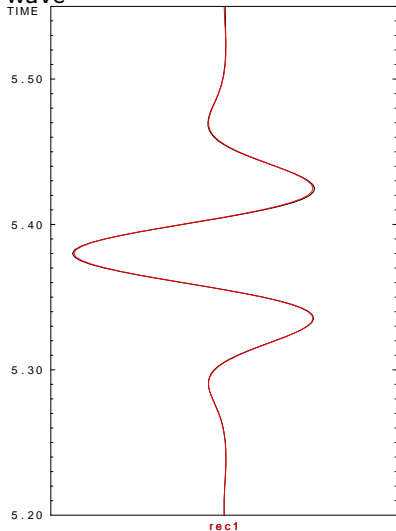
I_0 ... amplitude of spurious reflection **without** cosine window.

The angle of incidence is important!

- Normal incidence \Rightarrow wave is well damped.
- Not normal incidence \Rightarrow a damping may be problematic.

Incident P wave, reflected S

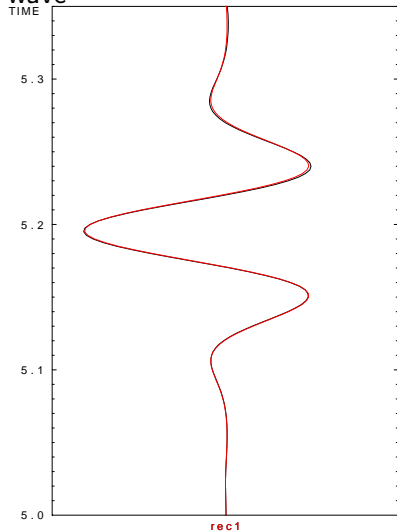
wave



BLACK...Born approximation
RED...Ray theory

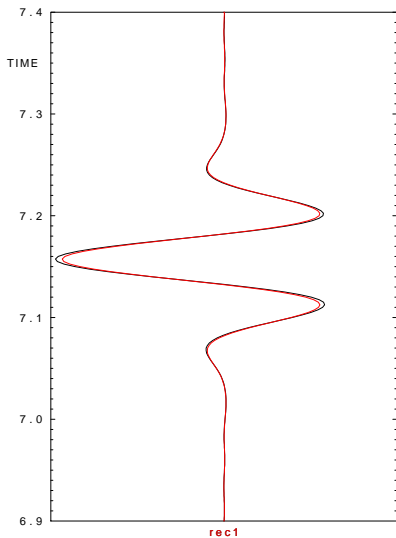
Incident S wave, reflected P

wave



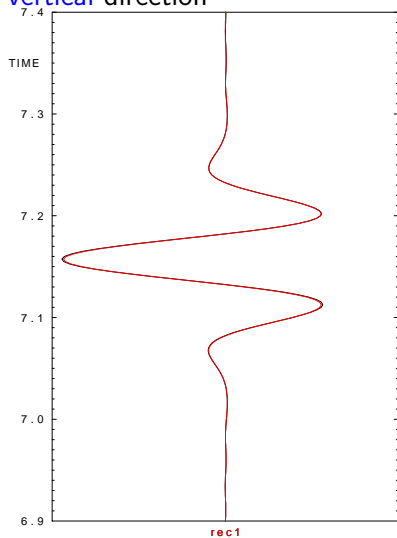
BLACK...Born approximation
RED...Ray theory

Incident S wave, reflected S wave



BLACK...Born approximation
RED...Ray theory

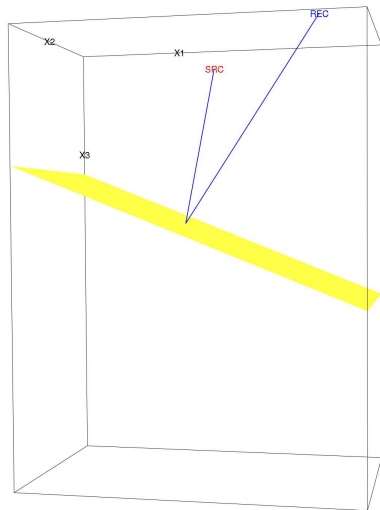
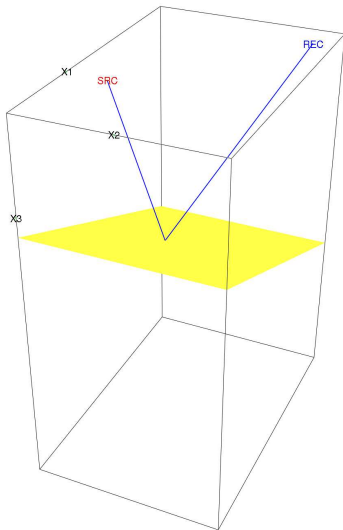
Incident S wave, reflected S wave, grid **twice** denser in the vertical direction



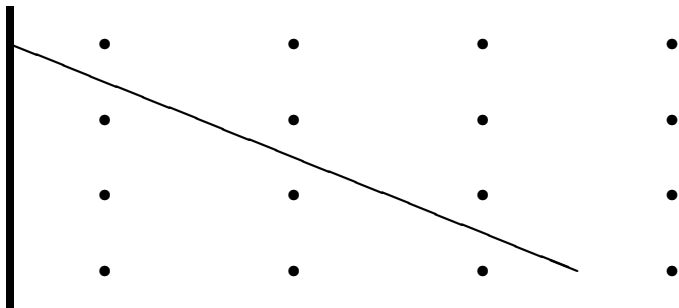
BLACK...Born approximation
RED...Ray theory

Model 2

Inclined interface with slope 2/5

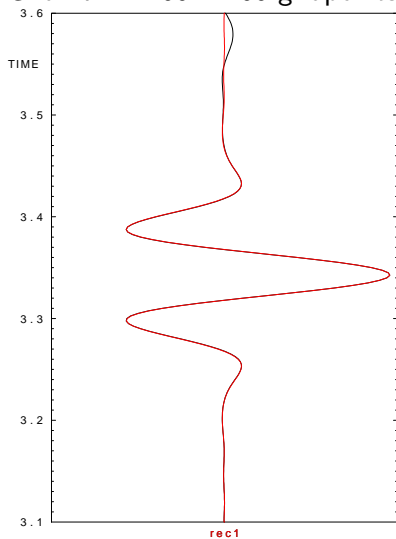


Model 2 - grid



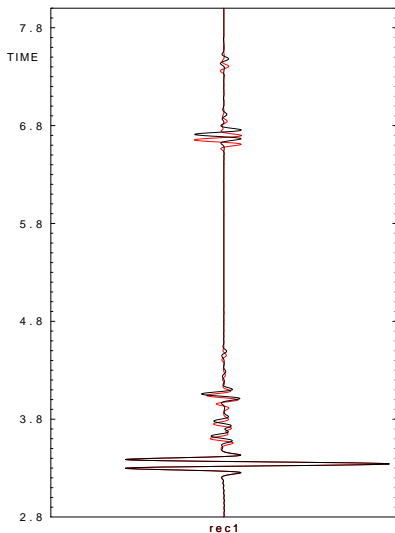
Grid which fits the shape and position of the interface in model 2.
Cross section in $x_1 - x_3$ plane. Bold line: Model boundary. Thin line: Interface.

Grid $171 \times 100 \times 400$ gridpoints



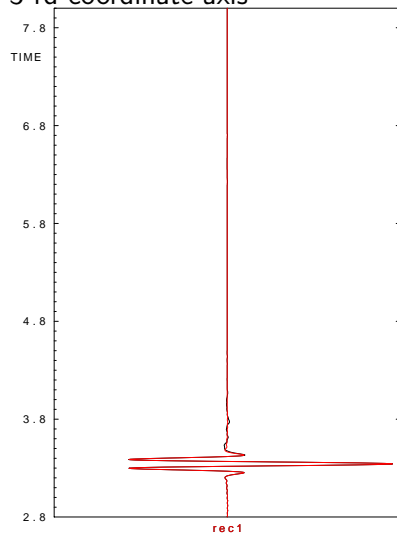
BLACK...Born approximation
RED...Ray theory

Grid vs. smaller grid



BLACK...Born approximation
RED...Born approximation, smaller grid

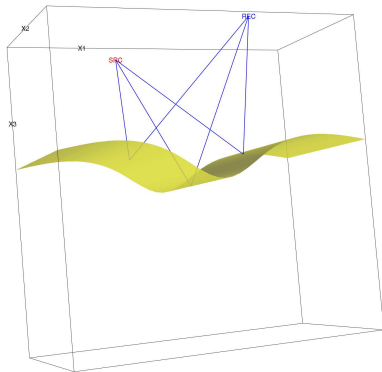
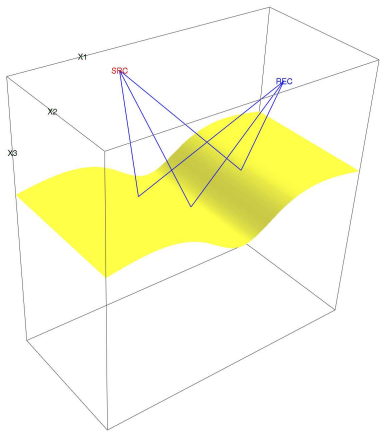
Cosine window of length 1.5 km applied in all directions except the negative direction of the 3-rd coordinate axis



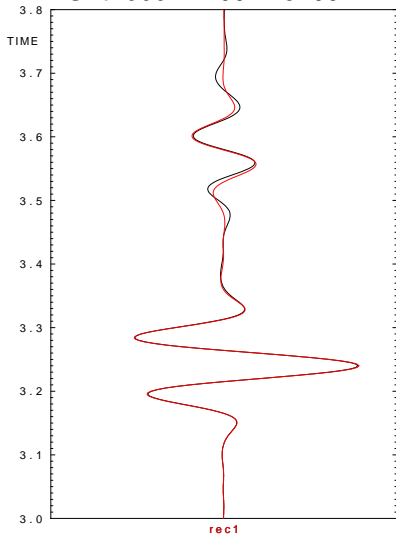
BLACK...Born approximation
RED...Ray theory

Model 3

Curved interface.

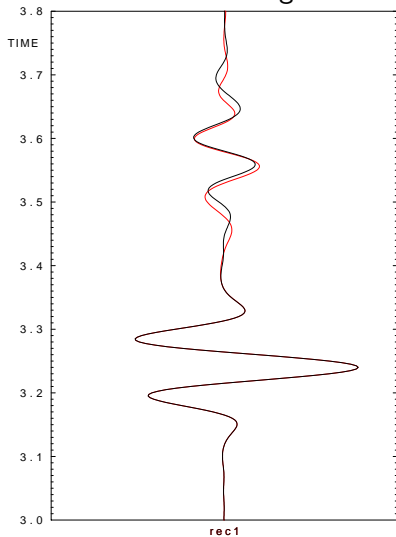


Grid 666 × 100 × 3200



BLACK...Born approximation
RED...Ray theory

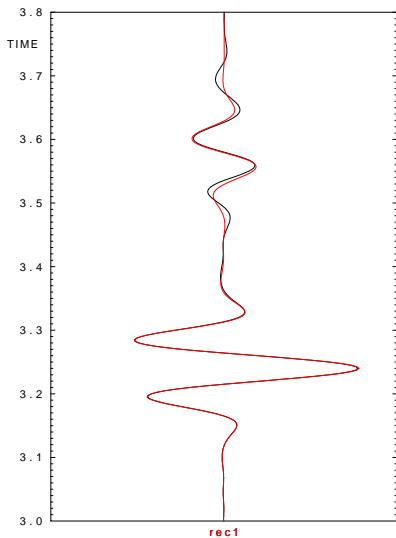
Grid vs. smaller grid



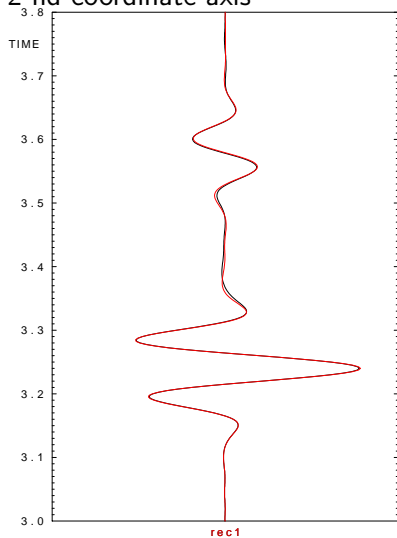
BLACK...Born approximation
RED...Born approximation, smaller grid

Grid $666 \times 100 \times 3200$

Cosine window of length 1.5 km
applied in the direction of the
2-nd coordinate axis



BLACK...Born approximation
RED...Ray theory



BLACK...Born approximation
RED...Ray theory

Current section

- 1 Models
- 2 Numerical 3D computations of 3D seismograms
- 3 Numerical 2D computations of 3D seismograms

A few words about the theory.

2D model + the source and receiver are situated in a symmetry plane \Rightarrow simplification of the Born approximation:

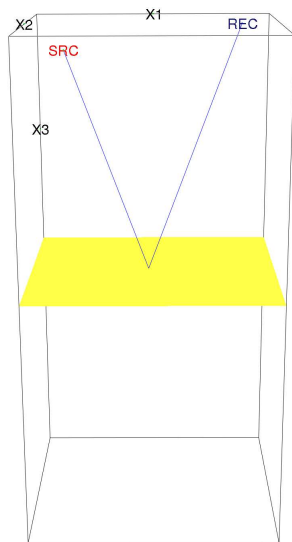
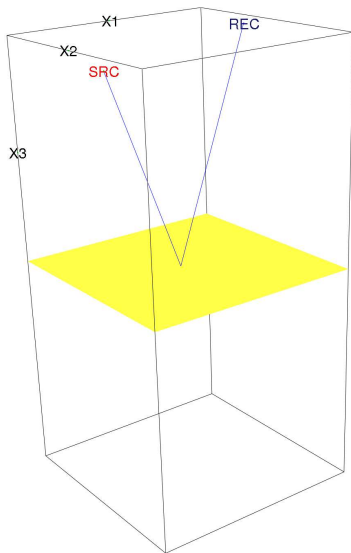
3D Born integral \rightarrow 2D Born integral with correction term

$$I_{cor1} = \sqrt{\frac{\pi}{\omega(\tau_{,\perp\perp} + T_{,\perp\perp})}}(1 + i).$$

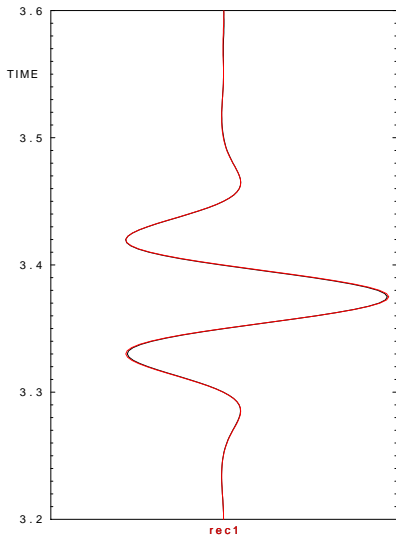
$\tau_{,\perp\perp}, T_{,\perp\perp} \dots$ derivatives of travel times in the direction perpendicular to the symmetry plane

Model 1

Horizontal interface in the depth 10 km.

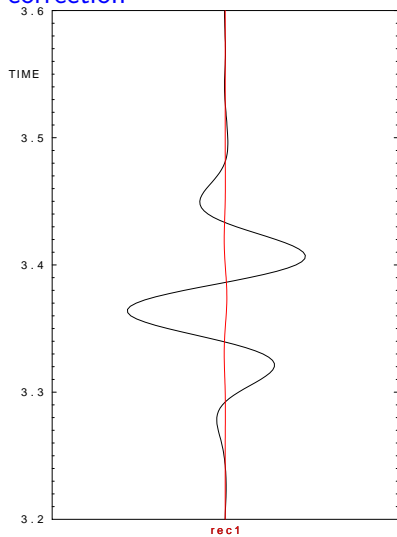


Grid 100×400



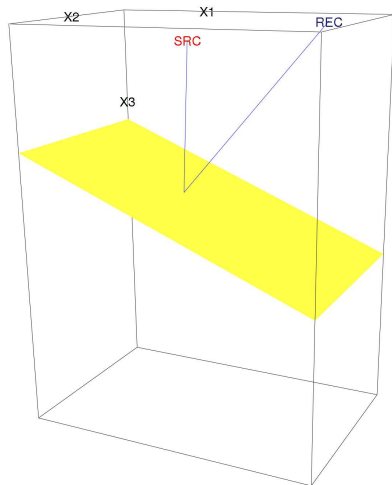
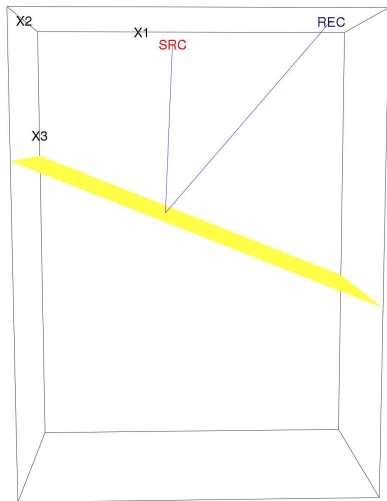
BLACK...Born approximation
RED...Ray theory

Grid 100×400 without correction

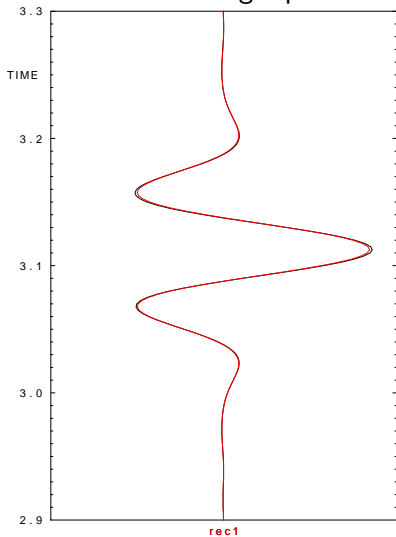


BLACK...Born approximation
RED...Ray theory

Model 2

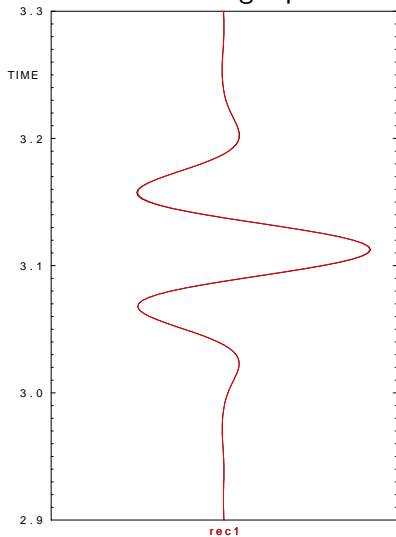
Inclined interface with slope $2/5$.

Grid 171×400 gridpoints



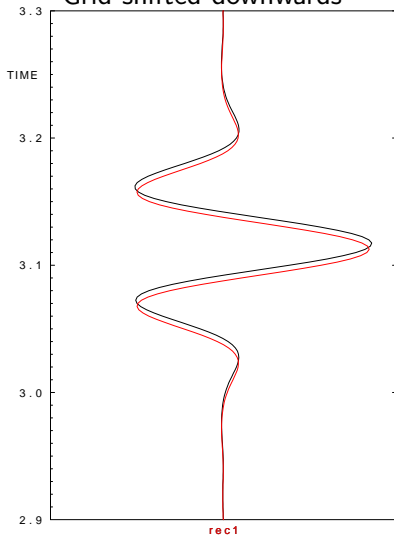
BLACK...Born approximation
RED...Ray theory

Grid 343×800 gridpoints



BLACK...Born approximation
RED...Ray theory

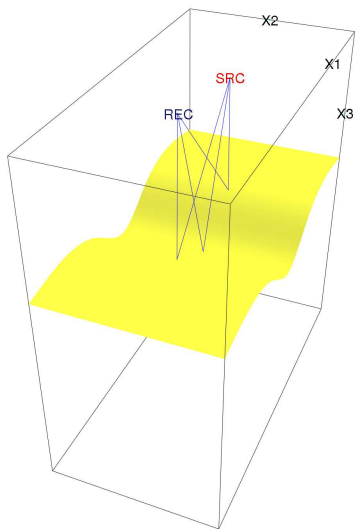
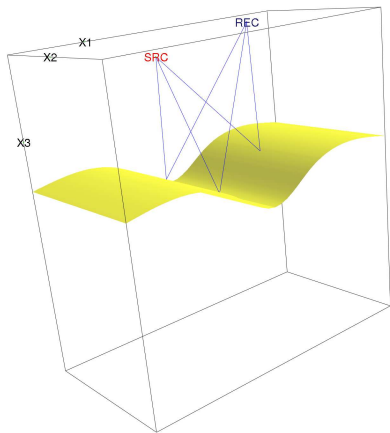
Grid shifted downwards



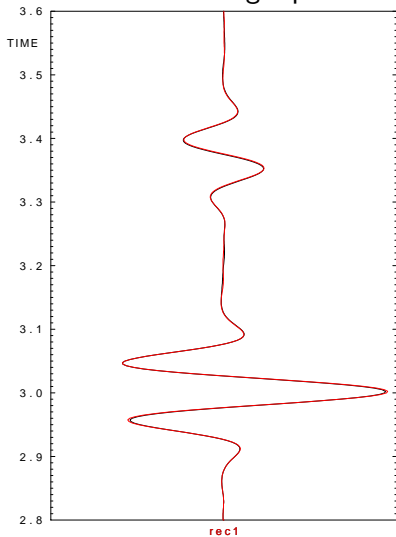
BLACK...Born approximation
RED...Ray theory

Model 3

Curved interface.

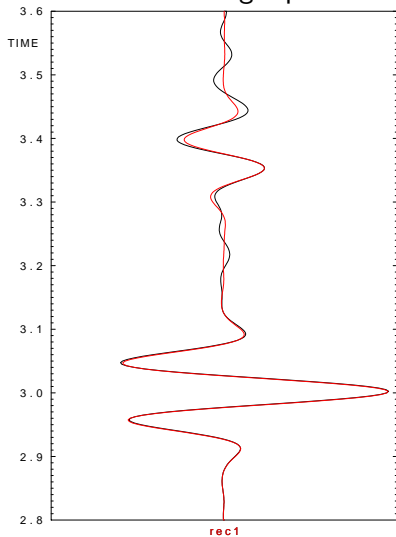


Grid 600×3200 gridpoints



BLACK...Born approximation
RED...Ray theory

Grid 600×800 gridpoints



BLACK...Born approximation
RED...Ray theory

Concluding remarks

- We tested the ray-based first-order Born approximation in homogenous background models. Both 3D and 2D modelling.
- Problems
 - 1 Discretization error (observed+theoretically explained)
 - 2 Spurious waves (observed+theoretically explained)