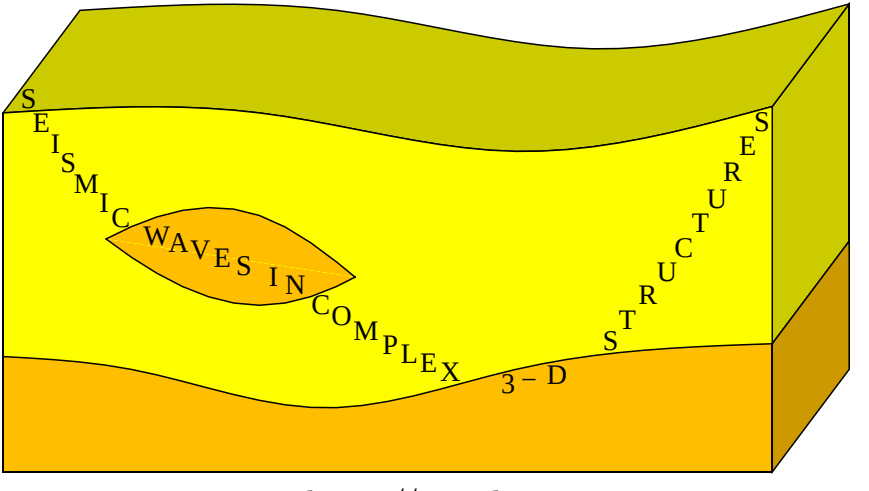
Single-frequency approximation of the coupling ray theory

Luděk Klimeš & Petr Bulant

Department of Geophysics
Faculty of Mathematics and Physics
Charles University in Prague



http://sw3d.cz

Idea

Anisotropic-ray-theory Green tensor:

$$G_{ij}^{\mathrm{ART}}(\mathbf{x},\mathbf{x}_0,\omega) = \sum_{K=1}^{2} A^{(K)}(\mathbf{x},\mathbf{x}_0)\; g_i^{(K)}(\mathbf{x})\, g_j^{(K)}(\mathbf{x}_0)\; \exp[\mathrm{i}\omega\tau^{(K)}(\mathbf{x},\mathbf{x}_0)]$$

Coupling-ray-theory Green tensor:

$$G_{ij}^{\mathrm{CRT}}(\mathbf{x},\mathbf{x}_0,\omega)$$

Single-frequency approximation of the coupling-ray-theory Green tensor:

$$G_{ij}^{\mathrm{CRT}}(\mathbf{x},\mathbf{x}_0,\omega) \approx G_{ij}^{\mathrm{SF}}(\mathbf{x},\mathbf{x}_0,\omega)$$

Single-frequency Green tensor:

$$G_{ij}^{\mathrm{SF}}(\mathbf{x},\mathbf{x}_0,\omega) = \sum_{K=1}^{2} A^{(K)}(\mathbf{x},\mathbf{x}_0)\, e_i^{(K)}(\mathbf{x},\omega_0)\, e_j^{(K)}(\mathbf{x}_0,\omega_0)\, \exp\big[\mathrm{i}\omega T^{(K)}(\mathbf{x},\mathbf{x}_0,\omega_0)\big]$$

ω_0... prevailing circular frequency.

$T^{(K)}(\mathbf{x},\mathbf{x}_0,\omega_0)$... coupling-ray-theory travel times.
$e_i^{(K)}(\mathbf{x},\omega_0)$, $e_j^{(K)}(\mathbf{x_0},\omega_0)$... coupling-ray-theory polarization vectors.

Single-frequency Green tensor — conditions:

$$G_{ij}^{\mathrm{SF}}(\mathbf{x}, \mathbf{x}_0, \omega_0) = G_{ij}^{\mathrm{CRT}}(\mathbf{x}, \mathbf{x}_0, \omega_0)$$

$$\frac{\partial G_{ij}^{\mathrm{SF}}}{\partial \omega}(\mathbf{x}, \mathbf{x}_0, \omega_0) = \frac{\partial G_{ij}^{\mathrm{CRT}}}{\partial \omega}(\mathbf{x}, \mathbf{x}_0, \omega_0)$$

These conditions uniquely determine coupling-ray-theory travel times $T^{(K)}(\mathbf{x}, \mathbf{x}_0, \omega_0)$ and coupling-ray-theory polarization vectors $e_i^{(K)}(\mathbf{x}, \omega_0)$ and $e_j^{(K)}(\mathbf{x_0}, \omega_0)$.

We numerically calculate $G_{ij}^{\mathrm{CRT}}(\mathbf{x}, \mathbf{x}_0, \omega_0)$ using the algorithm by Bulant & Klimeš (2002).

We calculate $\frac{\partial G_{ij}^{\mathrm{CRT}}}{\partial \omega}(\mathbf{x}, \mathbf{x}_0, \omega_0)$ using the derivative of this algorithm.

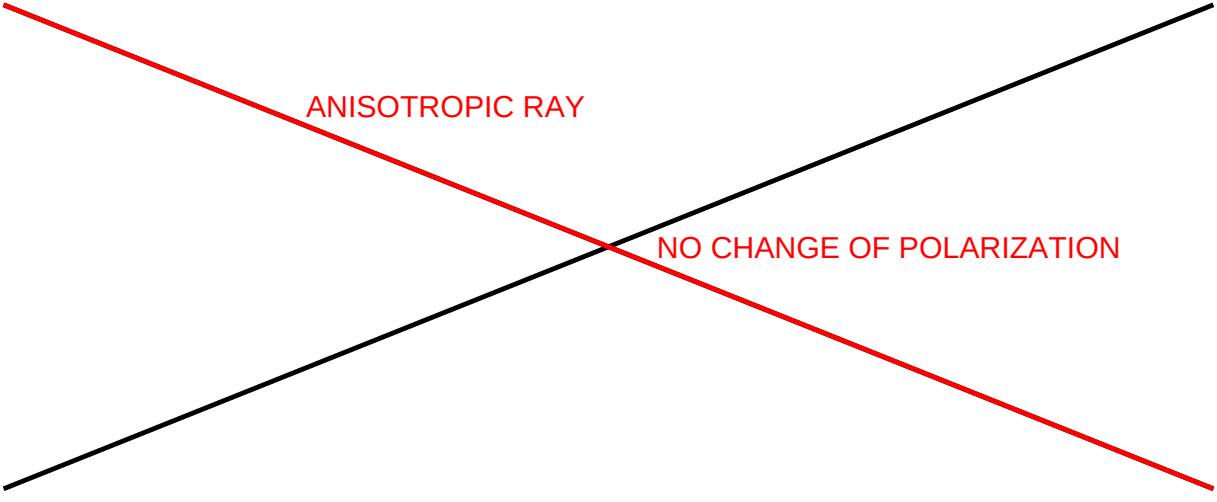
Examples of singularities at the slowness surface

Along the reference ray, the slowness vector smoothly rotates according to Hamilton's equations of rays, and thus moves along the slowness surface.

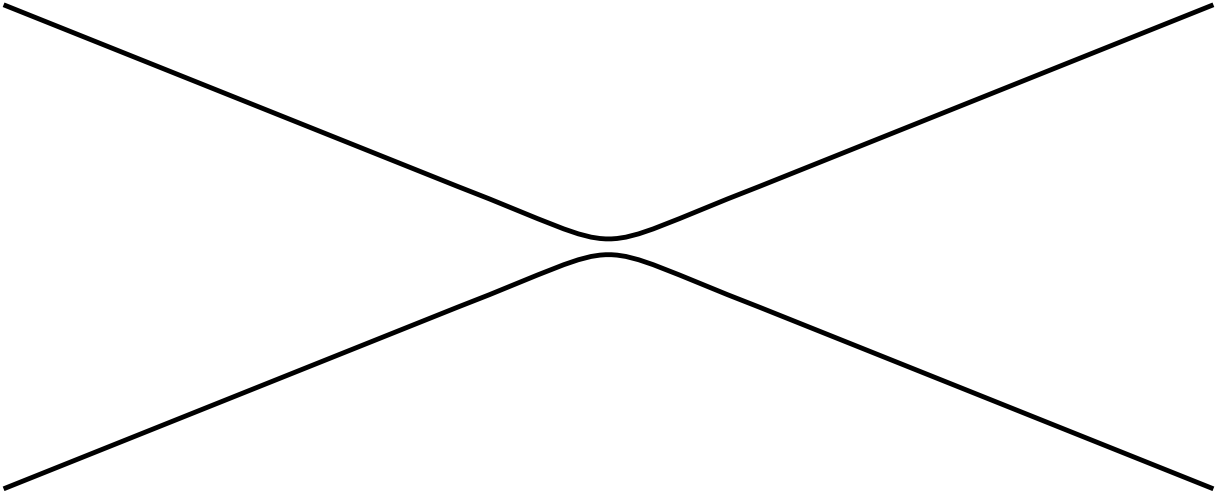
Let us see how the slowness vector moves along the slowness surface.

INTERSECTION SINGULARITY AT THE SLOWNESS SURFACE

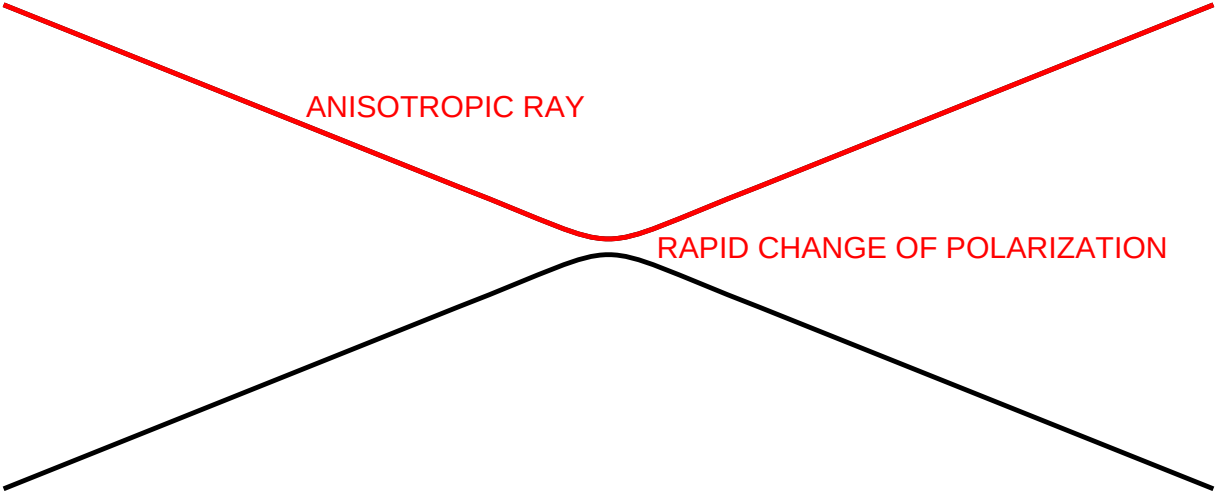
INTERSECTION SINGULARITY AT THE SLOWNESS SURFACE



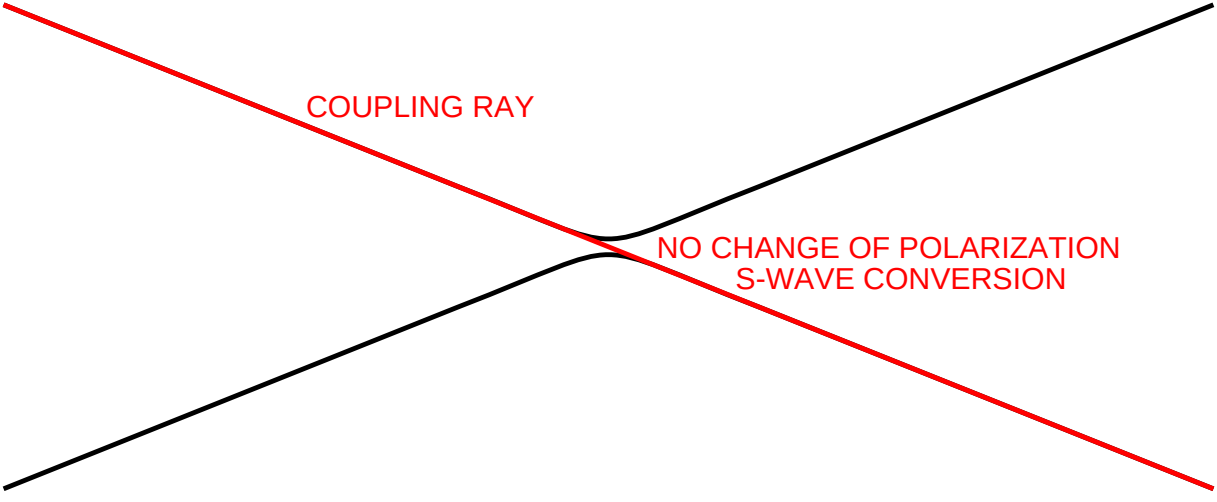
SPLIT INTERSECTION SINGULARITY AT THE SLOWNESS SURFACE



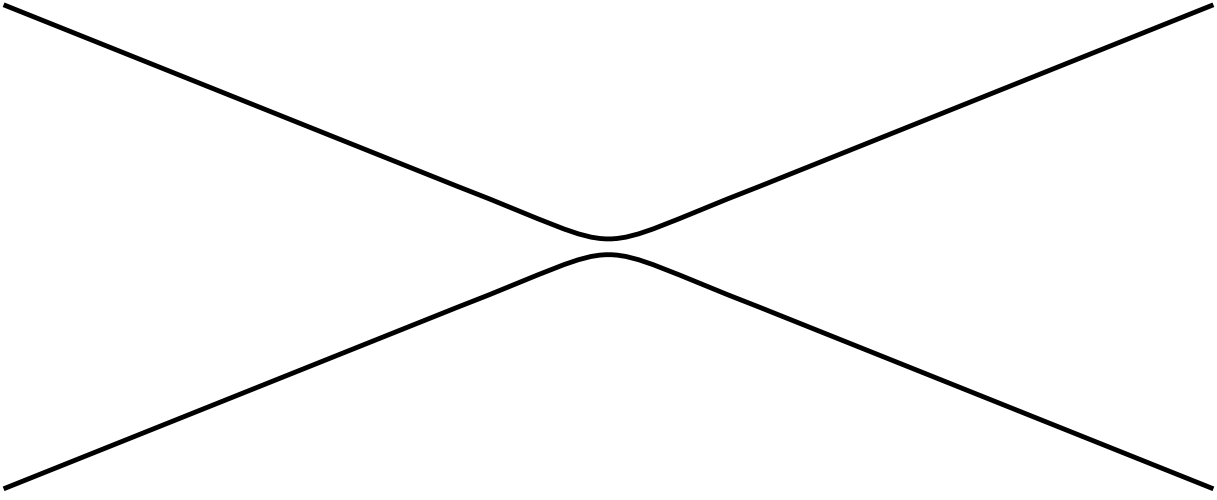
SPLIT INTERSECTION SINGULARITY AT THE SLOWNESS SURFACE



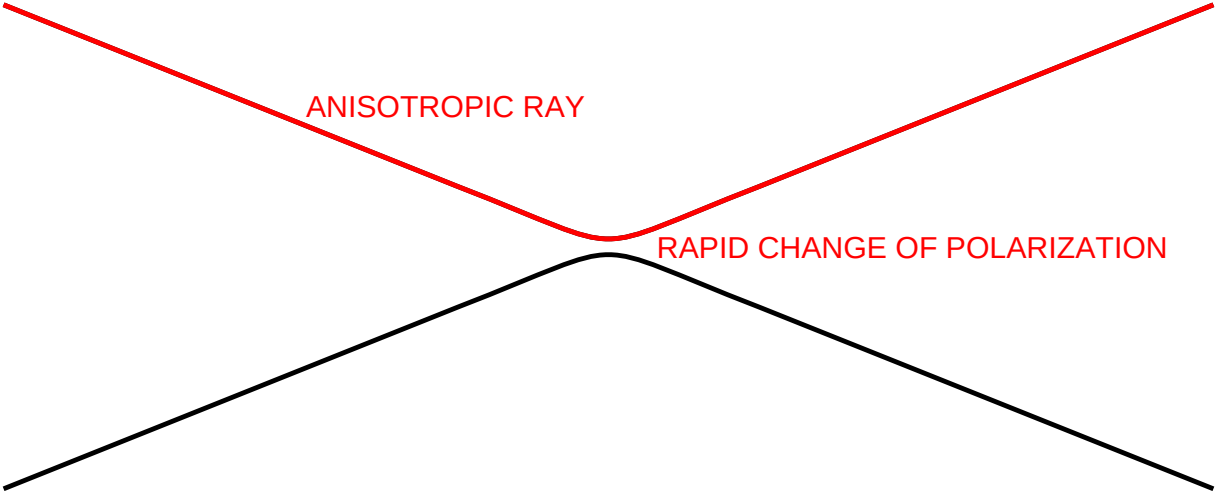
SPLIT INTERSECTION SINGULARITY AT THE SLOWNESS SURFACE



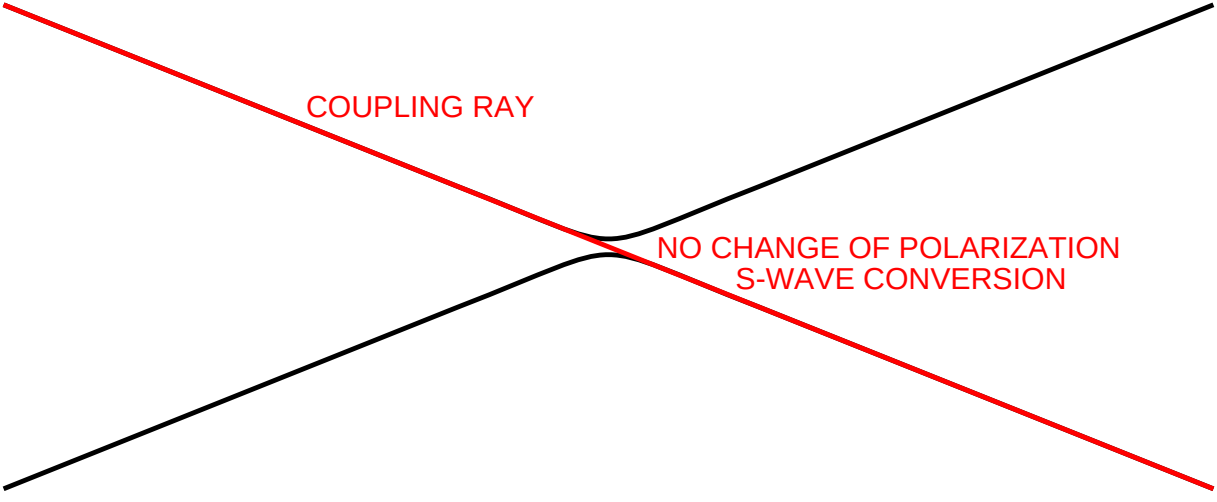
VICINITY OF CONICAL SINGULARITY AT THE SLOWNESS SURFACE



VICINITY OF CONICAL SINGULARITY AT THE SLOWNESS SURFACE



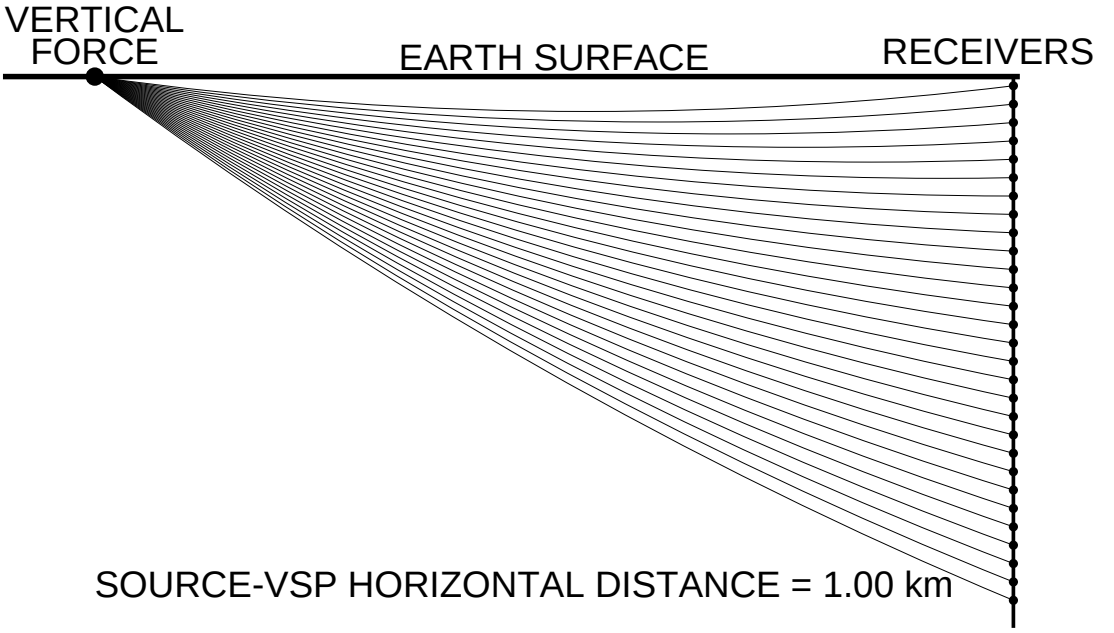
VICINITY OF CONICAL SINGULARITY AT THE SLOWNESS SURFACE

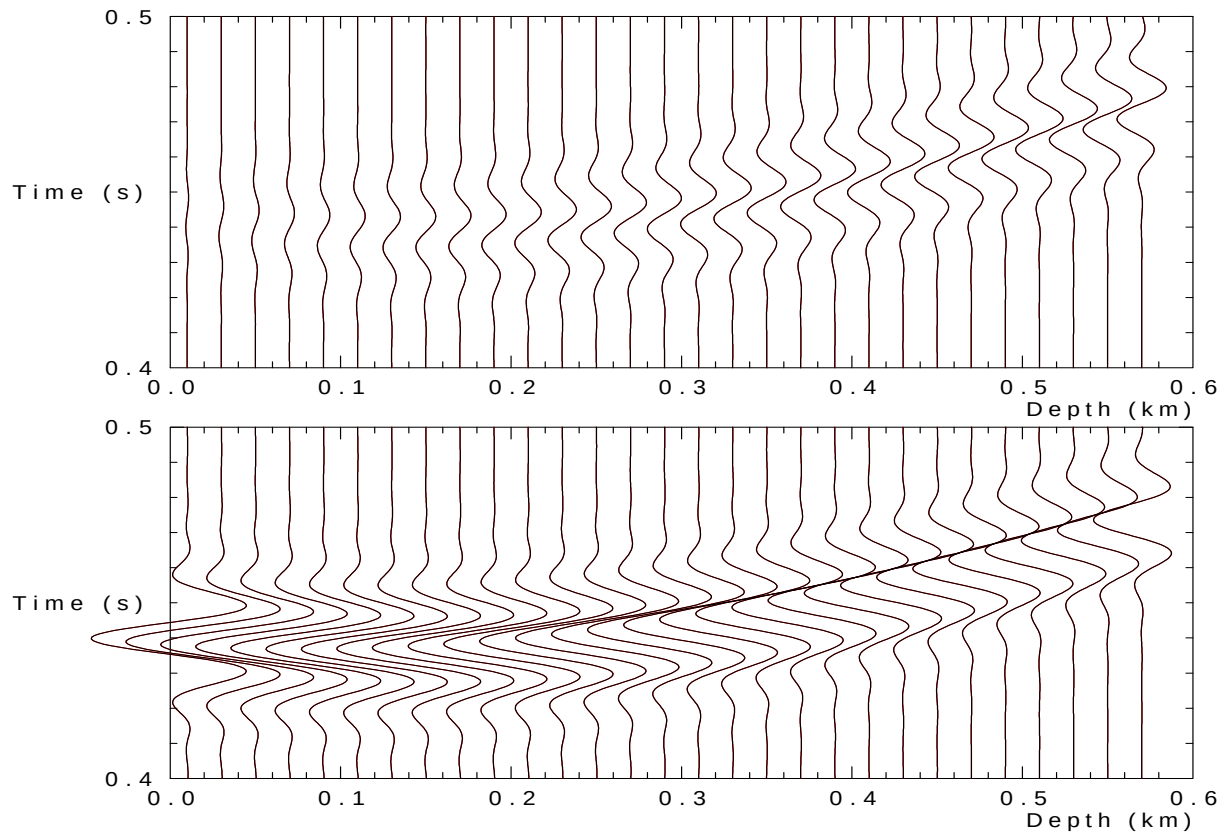


Numerical comparison of seismograms

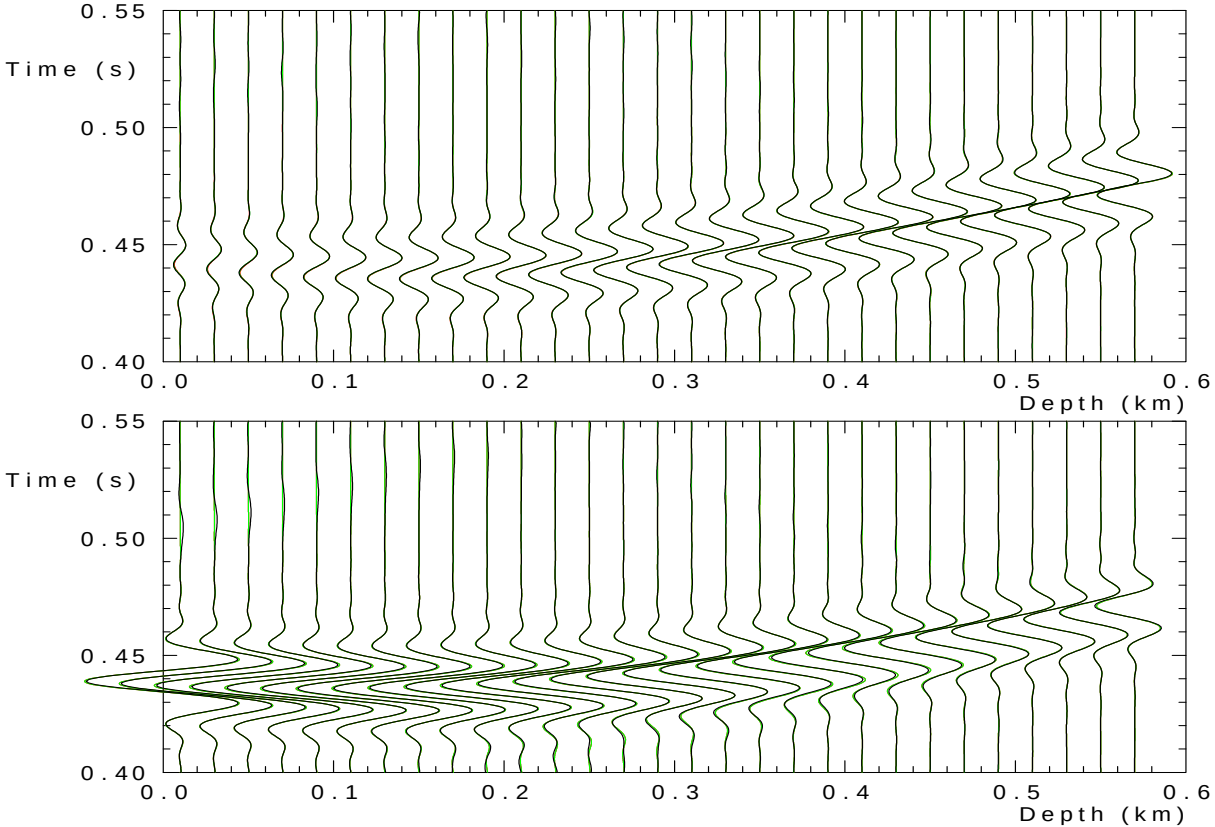
Single-frequency approximation.
Standard coupling ray theory.
Fourier pseudospectral method.

Source-receiver configuration:

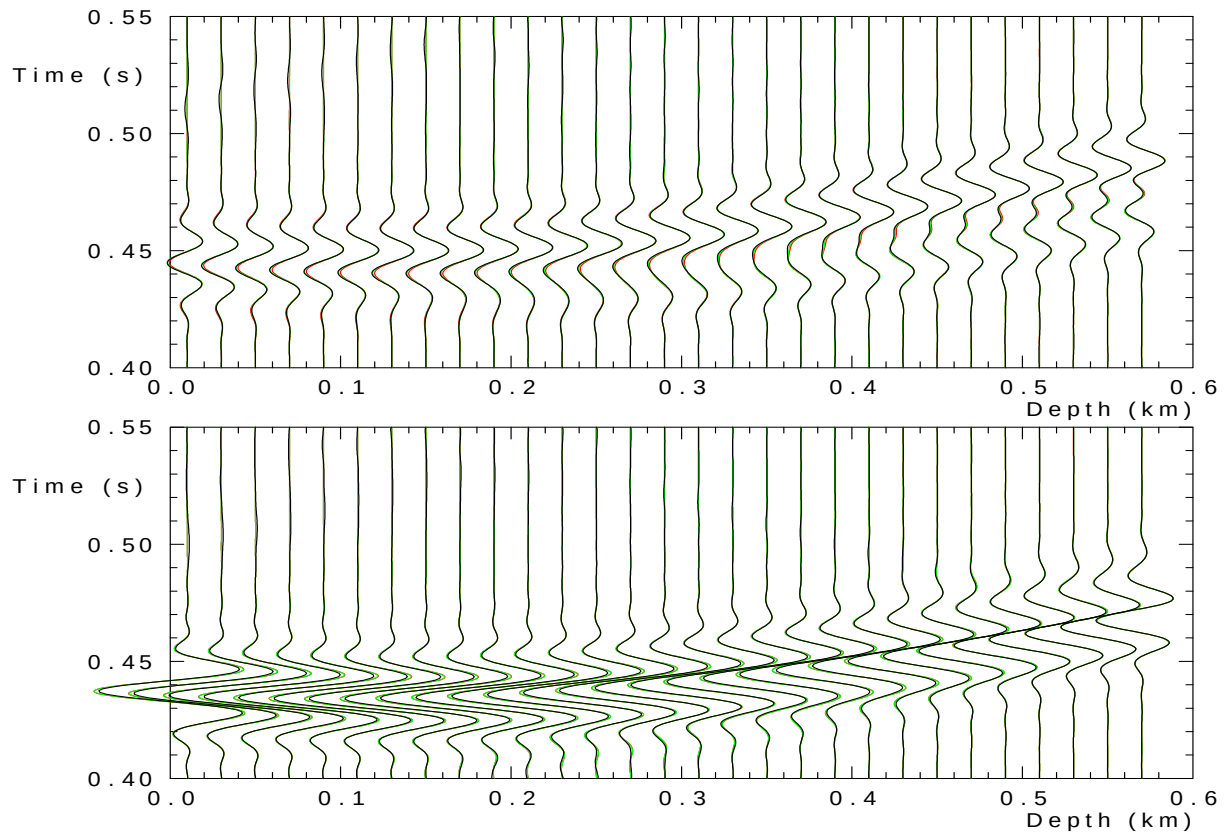




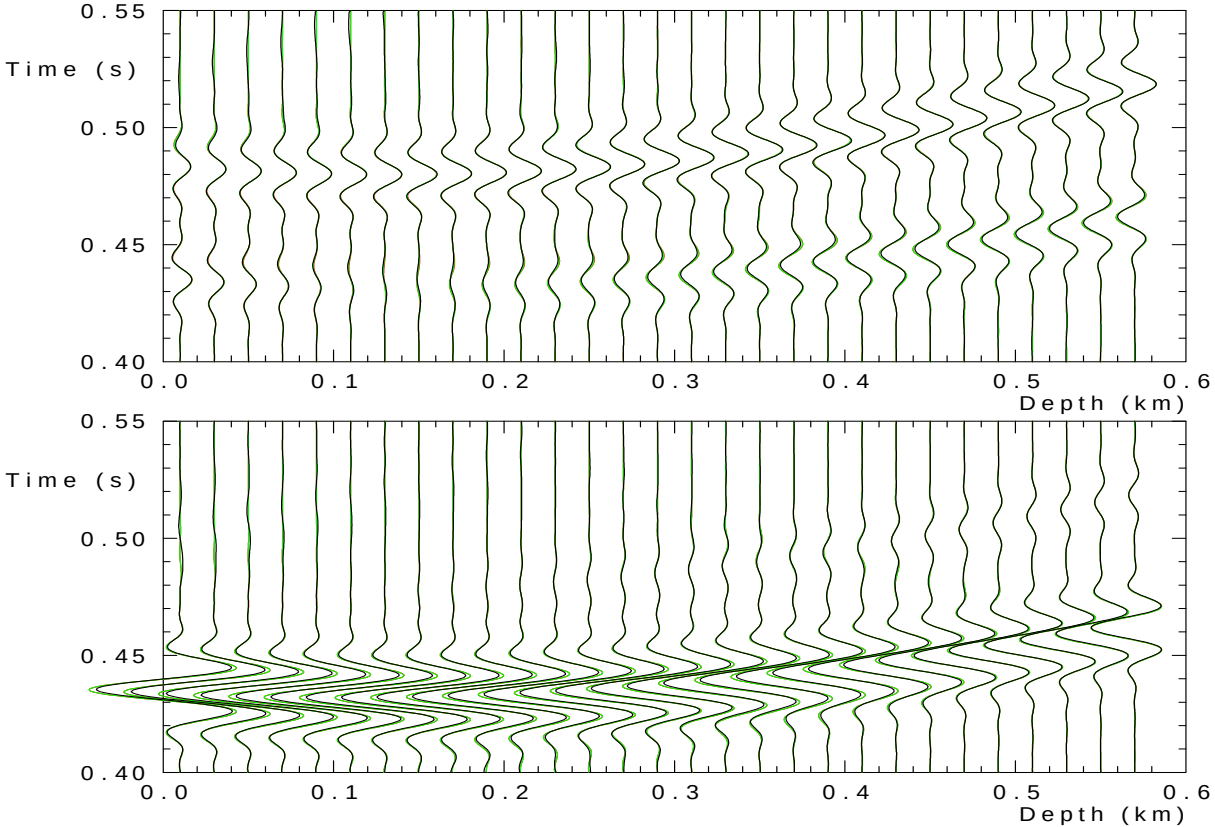
Velocity model QIH, transverse (top) and vertical (bottom) component. Single-frequency approximation, standard coupling ray theory.



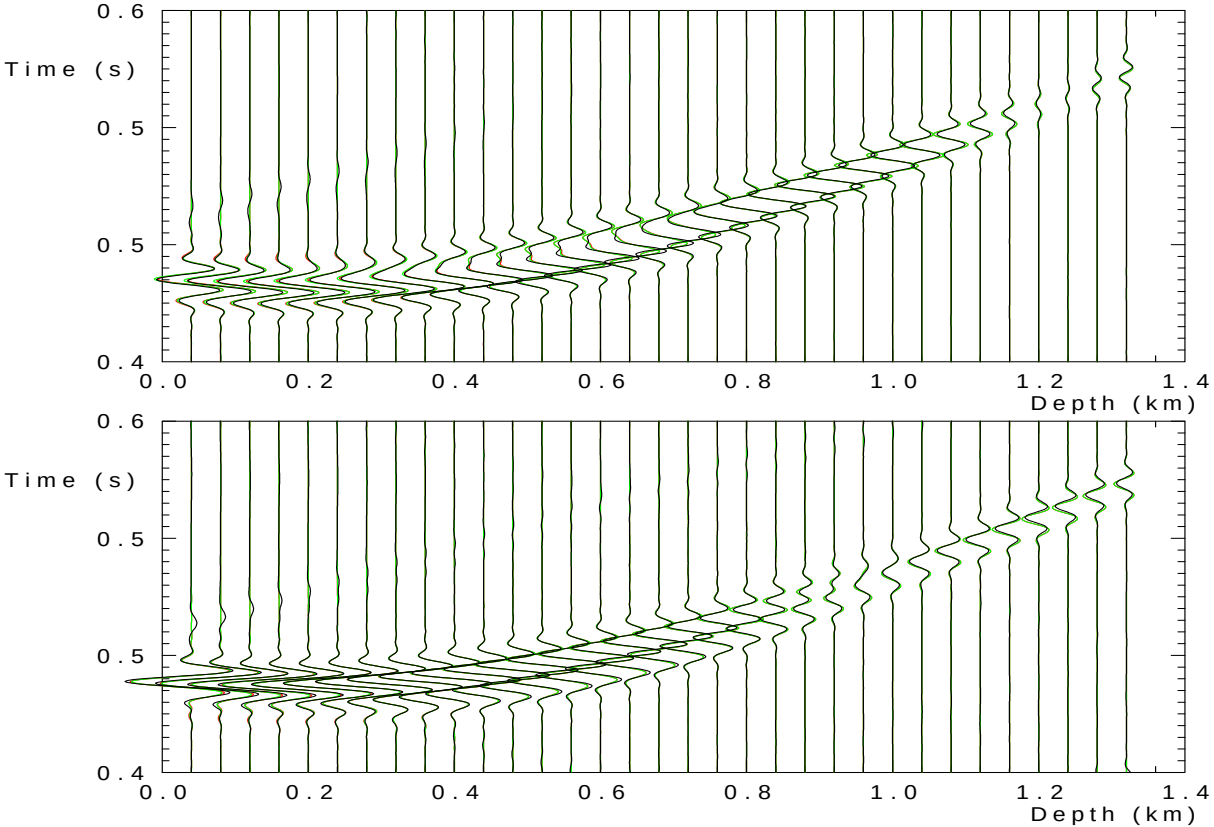
Velocity model QI, transverse (top) and vertical (bottom) component. Single-frequency, coupling ray theory, Fourier pseudospectral method.



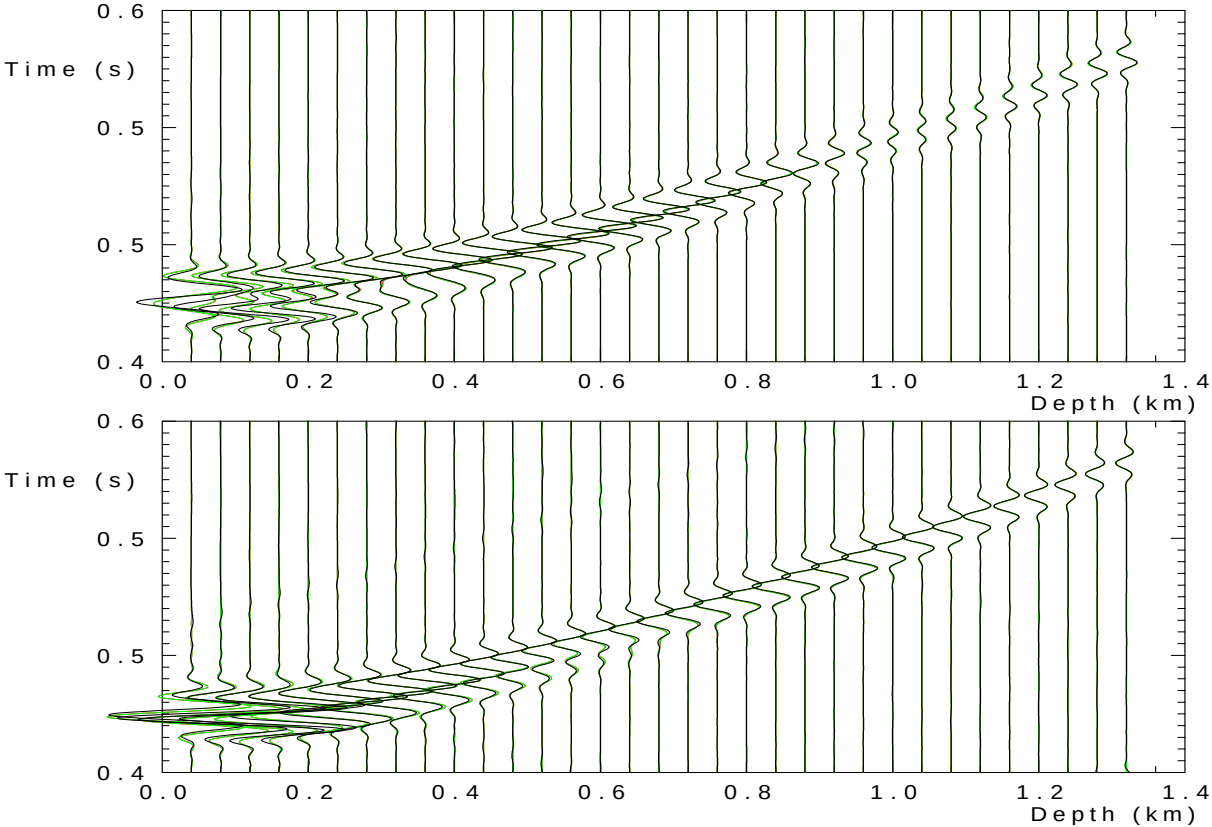
Velocity model QI2, transverse (top) and vertical (bottom) component. Single-frequency, coupling ray theory, Fourier pseudospectral method.



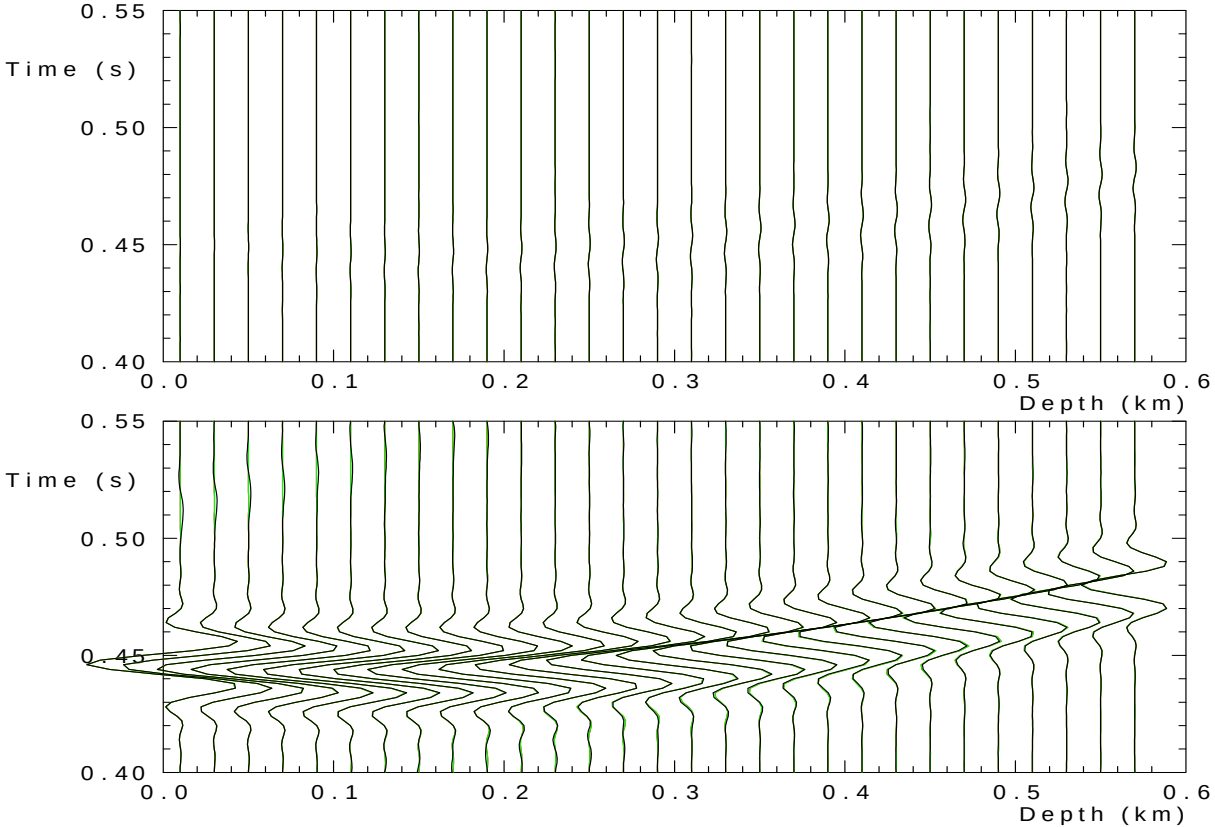
Velocity model QI4, transverse (top) and vertical (bottom) component. Single-frequency, coupling ray theory, Fourier pseudospectral method.



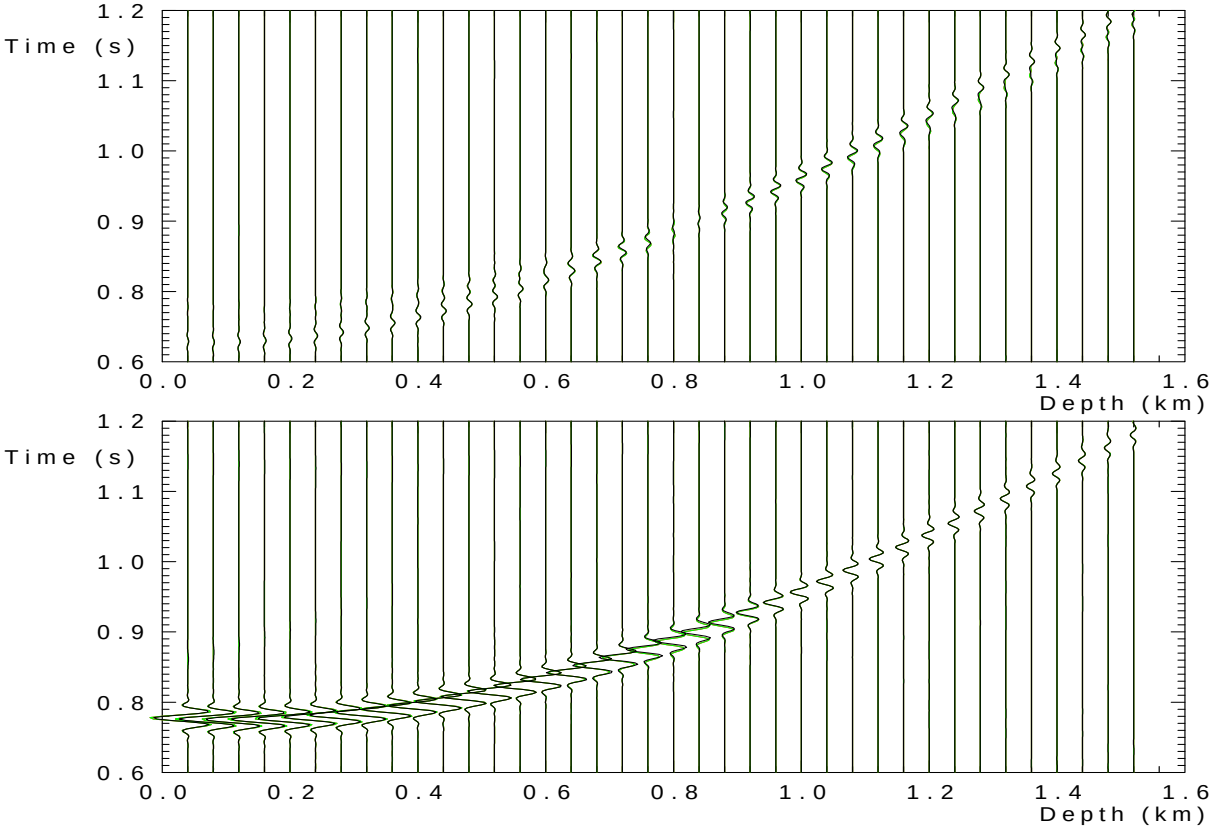
Velocity model SC1_I, transverse (top) and vertical (bottom) component.
Single-frequency, coupling ray theory, Fourier pseudospectral method.



Velocity model SC1_II, transverse (top) & vertical (bottom) component.
Single-frequency, coupling ray theory, Fourier pseudospectral method.



Velocity model KISS, transverse (top) and vertical (bottom) component. Single-frequency, coupling ray theory, Fourier pseudospectral method.



Velocity model ORT, transverse (top) and vertical (bottom) component.
Single-frequency, coupling ray theory, Fourier pseudospectral method.

Conclusions

The single-frequency approximation of the coupling ray theory allows us to process the coupling-ray-theory wavefield in the same way as the anisotropic-ray-theory wavefield.

The single-frequency approximation of the coupling ray theory can thus be included in wavefront tracing and in the interpolation within ray cells in anisotropic media.

This will enable common-source Kirchhoff prestack depth migration with coupling-ray-theory S waves.

We would like to further improve the accuracy of the coupling ray theory using the first-order perturbation of polarization vectors.

References:

Bulant, P. & Klimeš, L. (2002): Numerical algorithm of the coupling ray theory in weakly anisotropic media. *Pure appl. Geophys.*, **159**, 1419–1435, online at "http://sw3d.cz".

Coates, R.T. & Chapman, C.H. (1990): Quasi-shear wave coupling in weakly anisotropic 3-D media. *Geophys. J. int.*, **103**, 301–320.

Klimeš, L. (2007): Coupling ray series. In: *Seismic Waves in Complex 3-D Structures, Report 17*, pp. 99–109, Dep. Geophys., Charles Univ., Prague, online at "http://sw3d.cz".

Klimeš, L. & Bulant, P. (2012): Single-frequency approximation of the coupling ray theory. In: *Seismic Waves in Complex 3-D Structures, Report 22*, pp. 143–167, Dep. Geophys., Charles Univ., Prague.

Pšenčík, I., Farra, V. & Tessmer, E. (2011): Comparison of the FORT approximation of the coupling ray theory with the Fourier pseudospectral method. In: *Seismic Waves in Complex 3-D Structures, Report 21*, pp. 129–165, Dep. Geophys., Charles Univ., Prague, online at "http://sw3d.cz".

Acknowledgements

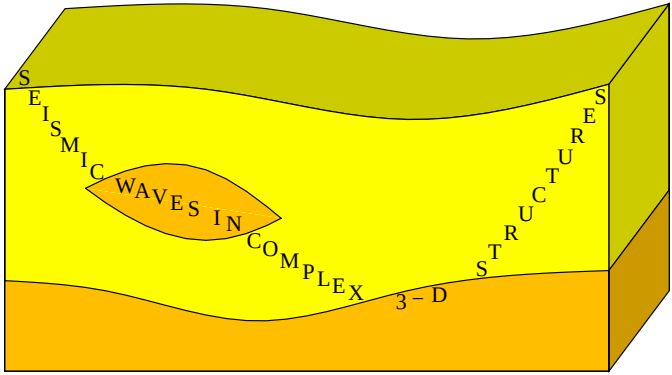
The research has been supported:

by the Grant Agency of the Czech Republic under contract P210/10/0736,

by the Ministry of Education of the Czech Republic within research project MSM0021620860,

by the European Commission under contract FP7-PEOPLE-IAPP-2009-230669 “Advanced Industrial Microseismic Monitoring”,

and by the consortium “Seismic Waves in Complex 3-D Structures”



http://sw3d.cz