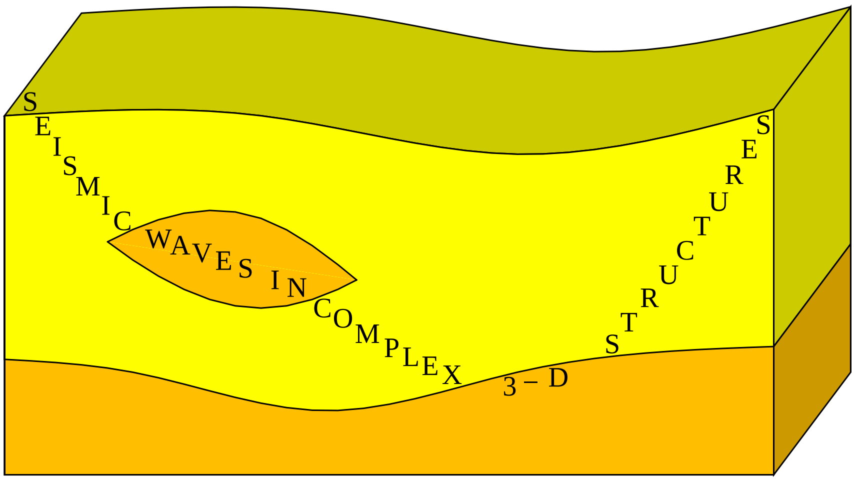
Interpolation of the coupling-ray-theory S-wave Green tensor within ray cells

Luděk Klimeš & Petr Bulant

Department of Geophysics
Faculty of Mathematics and Physics
Charles University in Prague



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Interpolation of the ray-theory Green tensor within ray cells

Interpolation of the ray-theory Green tensor within ray cells using the algorithm designed by Bulant & Klimeš (1999) has been proved especially efficient for calculating the ray-theory Green tensor at the nodes of dense 3-D grids.

The discretized ray-theory Green tensor can be used for various applications including the ray-based Born approximation, non-linear determination of seismic hypocentres, studies of seismic sources, or Kirchhoff prestack depth migration.

The current algorithms and code are applicable to both P and S waves in isotropic velocity models including velocity models with structural interfaces, and to P waves in smooth anisotropic velocity models.

This contribution is devoted to emerging challenges related to the interpolation of the S-wave ray-theory Green tensor in anisotropic velocity models. This interpolation is difficult because of polarization discontinuities.

Coupling ray theory for S waves (Coates & Chapman, 1990)

In weakly anisotropic media, the coupling ray theory for S waves is considerably more accurate than the anisotropic ray theory.

The coupling-ray-theory Green tensor is more continuous than the anisotropic-ray-theory Green tensor.

Unfortunately, the coupling-ray-theory Green tensor is frequency-dependent, and is calculated separately for each frequency. This frequency dependence represents the main obstacle for the interpolating the coupling-ray-theory Green tensor within ray cells.

Prevailing-frequency approximation of the coupling-ray-theory S-wave Green tensor

Klimeš & Bulant (2012) found the approximation of the coupling-ray-theory Green tensor in the vicinity of a given prevailing frequency by two prevailing-frequency coupling-ray-theory Green tensors corresponding to two elementary coupling-ray-theory S waves. Each prevailing-frequency coupling-ray-theory Green tensor is described by its coupling-ray-theory travel time and its complex-valued coupling-ray-theory amplitude tensor.

The prevailing-frequency approximation enables us to interpolate the coupling-ray-theory S-wave Green tensor within ray cells using the algorithm designed by Bulant & Klimeš (1999).

Continuity of the coupling-ray-theory S-wave Green tensor along rays

Both prevailing-frequency coupling-ray-theory Green tensors are calculated along a single anisotropic common S-wave reference ray. The anisotropic common reference rays then determine the ray cells for interpolation.

At each point of each reference ray, we have two prevailing-frequency coupling-ray-theory Green tensors.

We double each reference ray and match the pair of the prevailing-frequency coupling-ray-theory Green tensors with the pair of new rays so that each Green tensor is continuous along the corresponding new ray.

Length correction of the reference slowness vector

In order to increase the accuracy of the interpolation of travel time within ray cells, Bulant & Klimeš (1999) use the slowness vectors at the vertices of ray cells in addition to travel times themselves.

Unfortunately, we do not know the slowness vectors corresponding to the coupling-ray-theory travel times. We thus keep the directions of the reference slowness vectors used to calculate anisotropic common reference rays, and only correct the lengths of the reference slowness vectors.

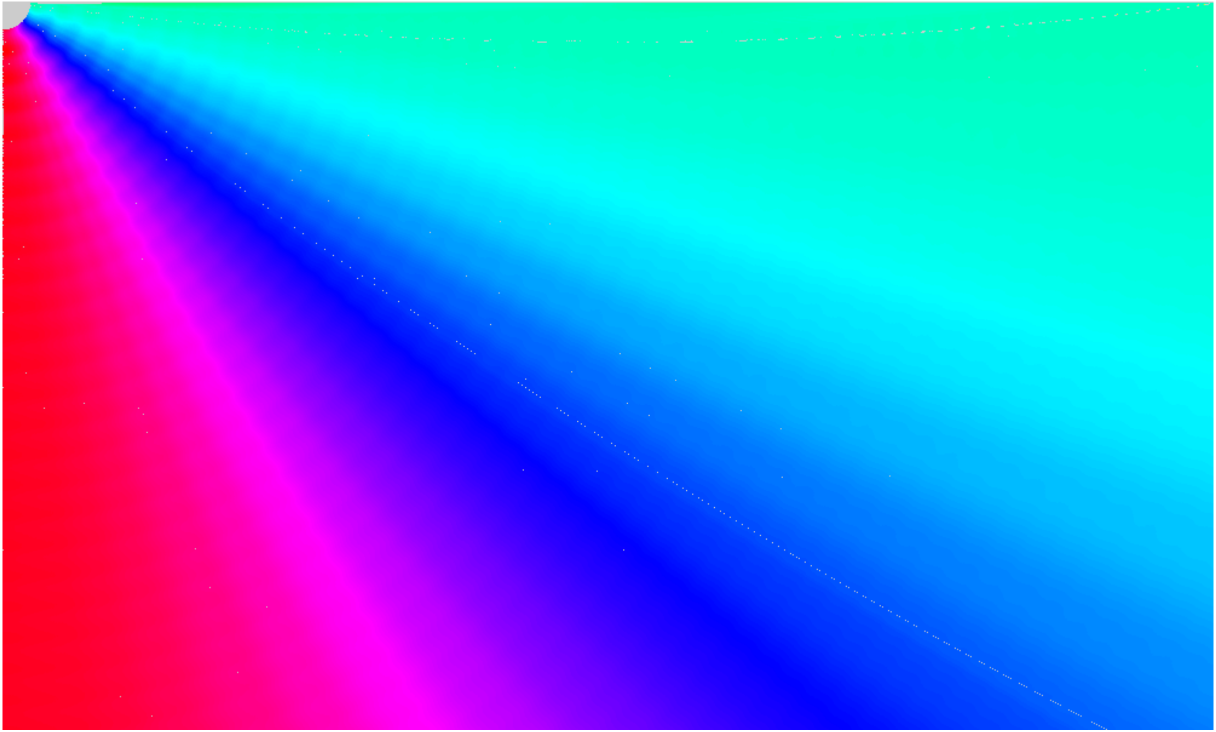
We shall see in the numerical examples that this idea is not good. The direction of the slowness vector is as important as its length.

Continuity of the coupling-ray-theory S-wave Green tensor within ray tubes

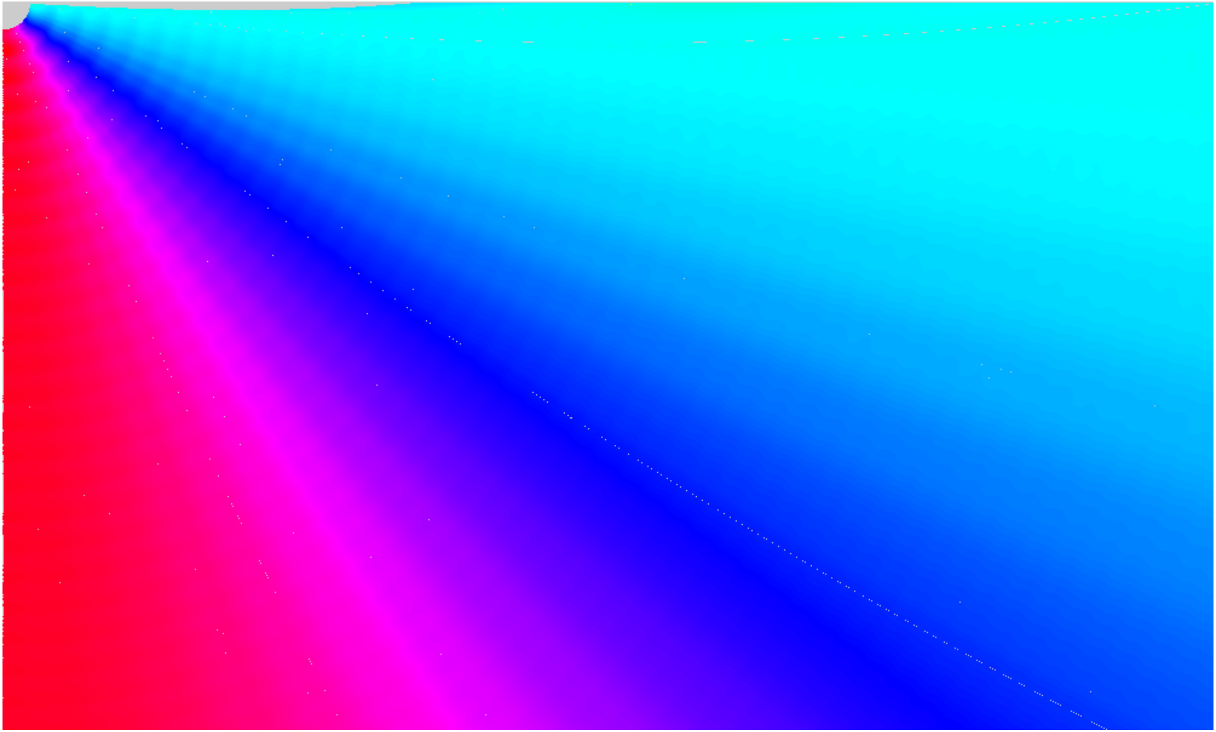
In place of each reference ray, we have two new rays corresponding to two prevailing-frequency coupling-ray-theory Green tensors. The Green tensor is continuous along the corresponding ray. As the result, each of three edges of each old ray tube is represented by two rays instead of one ray. We need to double each old ray tube and match the three pairs of edge rays with the pair of new ray tubes so that the Green tensor is continuous within either of the two new ray tubes.

The currently designed algorithm is obviously not optimal. The study of continuity of the prevailing-frequency coupling-ray-theory Green tensor within ray tubes represents the most challenging task.

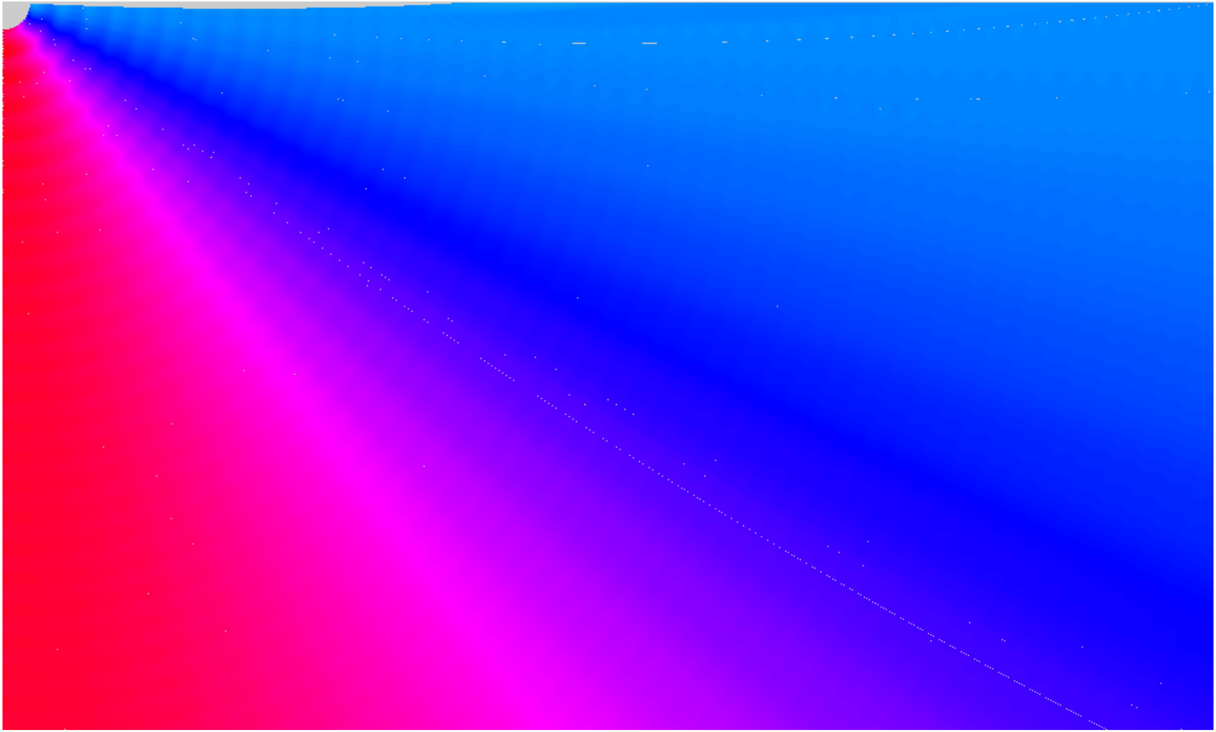
Numerical examples



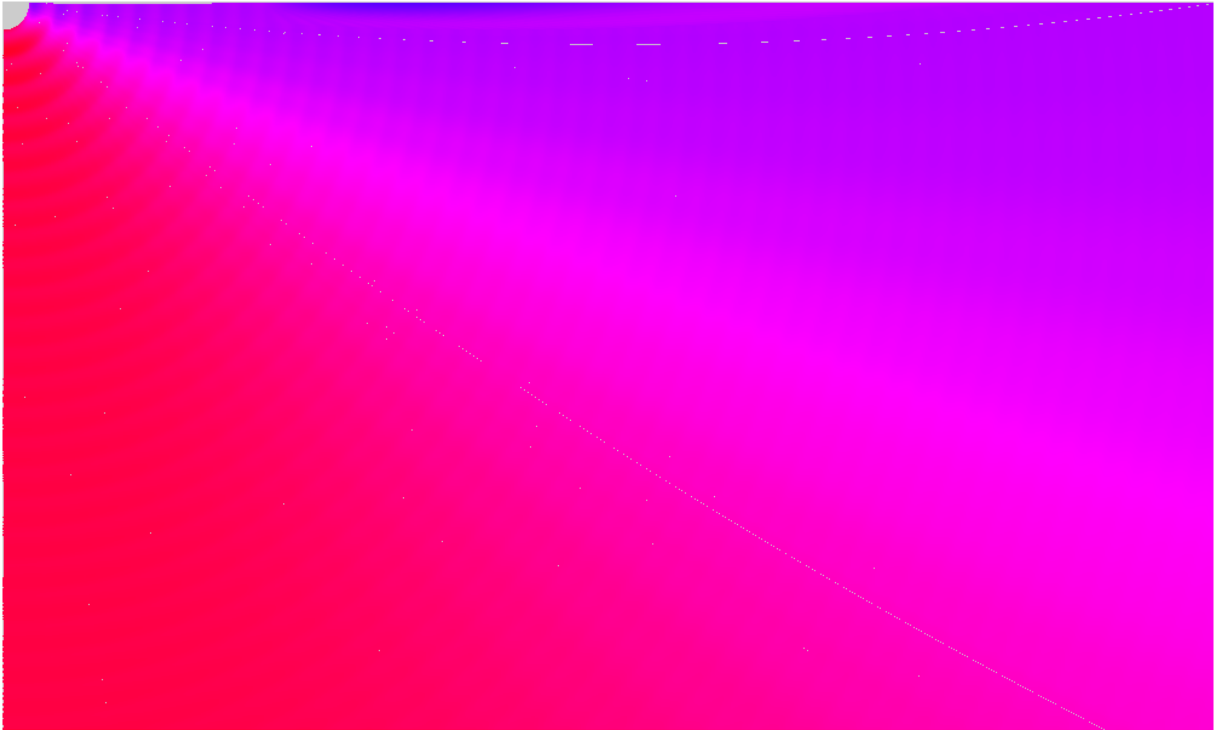
Relative coupling-ray-theory S-wave travel-time difference $|D/\overline{\tau}|$ in velocity model QIH.
Colour scale: **0.00%**, **0.175%**, **0.350%**, **0.525%**, **0.700%**, **0.875%**.



Relative coupling-ray-theory S-wave travel-time difference $|D/\overline{\tau}|$ in velocity model QI.
Colour scale: **0.00%**, **0.35%**, **0.70%**, **1.05%**, **1.40%**, **1.75%**.

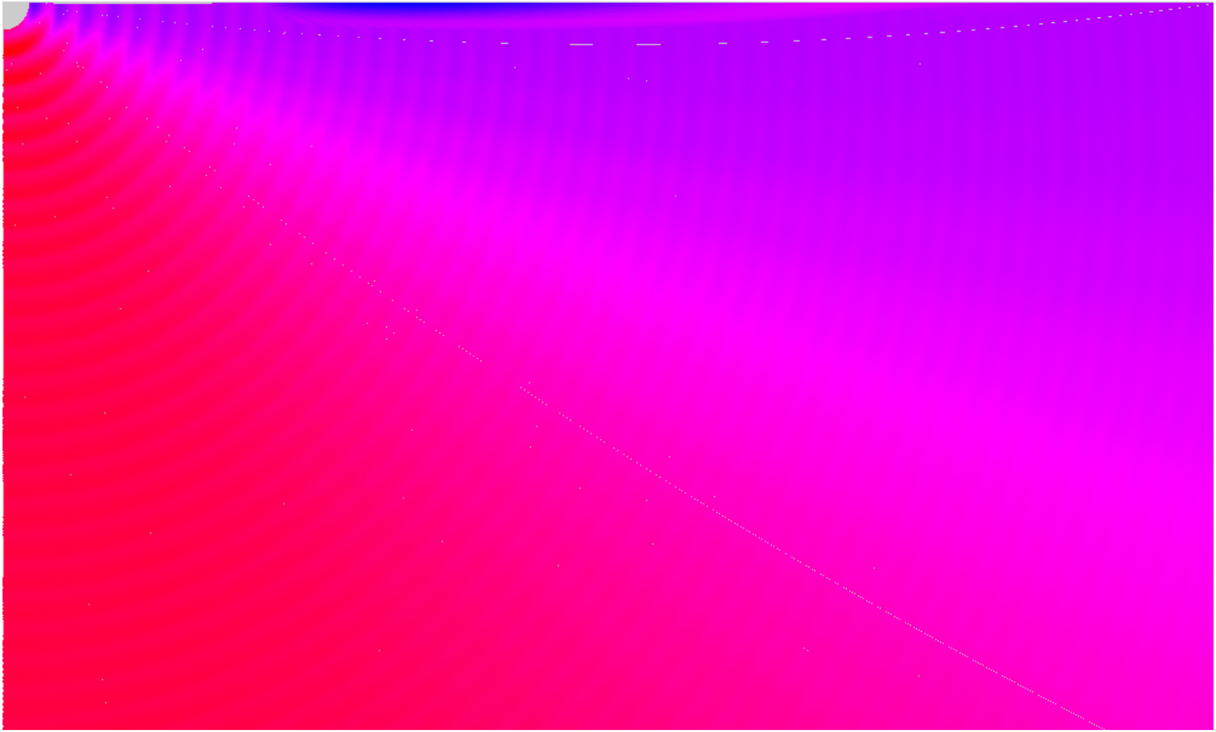


Relative coupling-ray-theory S-wave travel-time difference $|D/\overline{\tau}|$ in velocity model QI2.
Colour scale: **0.00%**, **0.7%**, **1.4%**, **2.1%**, **2.8%**, **3.5%**.

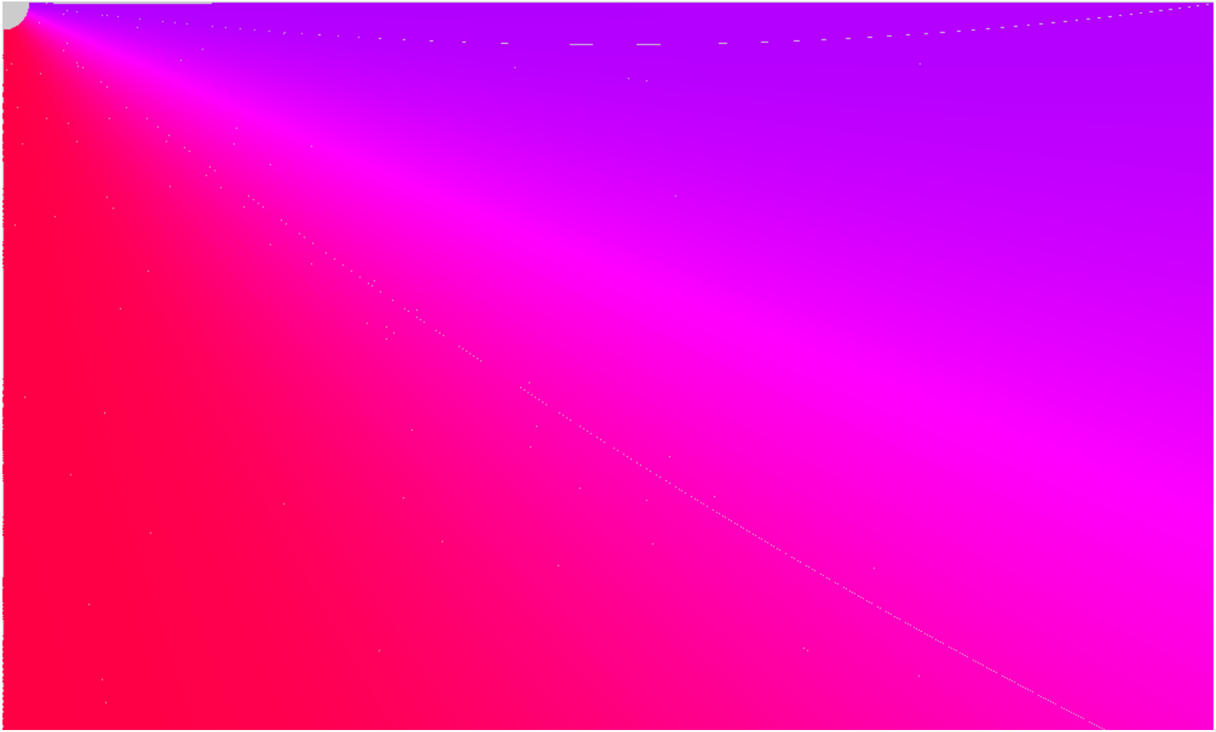


Relative coupling-ray-theory S-wave travel-time difference $|D/\overline{\tau}|$
in velocity model QI4.
Colour scale: **0.00%**, **1.4%**, **2.8%**, **4.2%**, **5.6%**, **7.0%**.

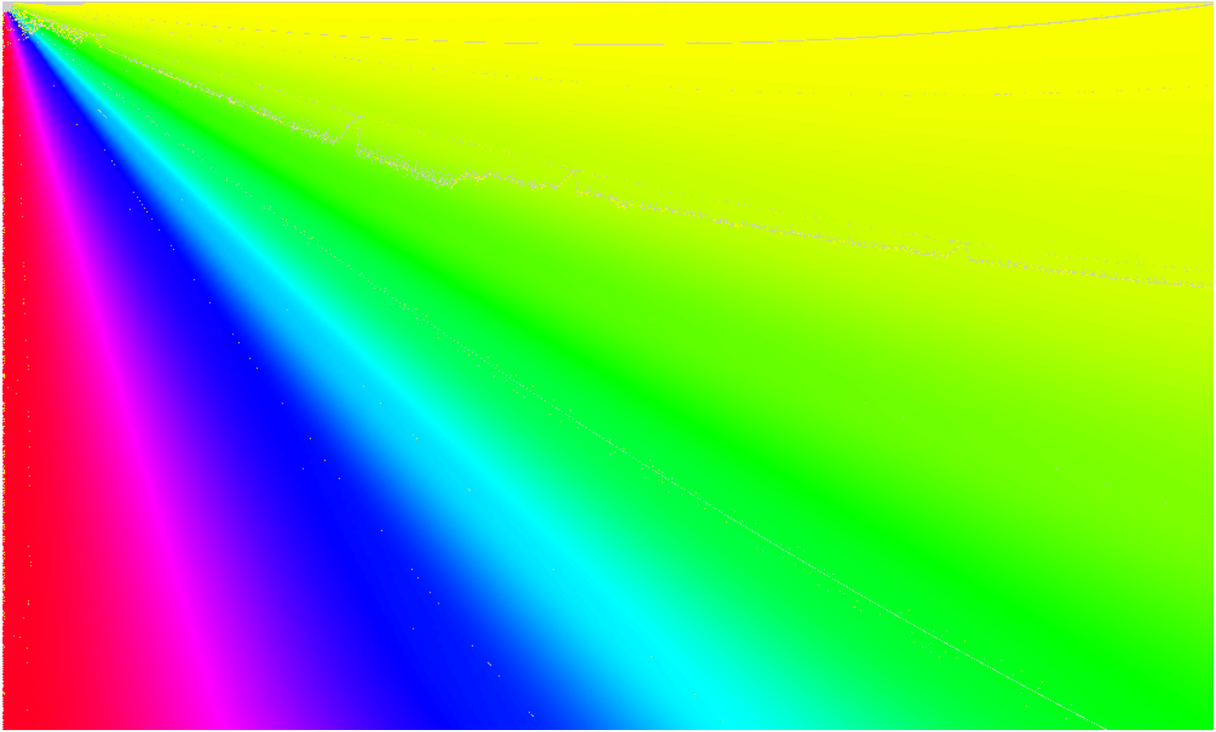
We can observe noticeable spurious oscillations of the travel-time difference due to the incorrect direction of the slowness vector used during the tricubic interpolation of travel time within ray cells.



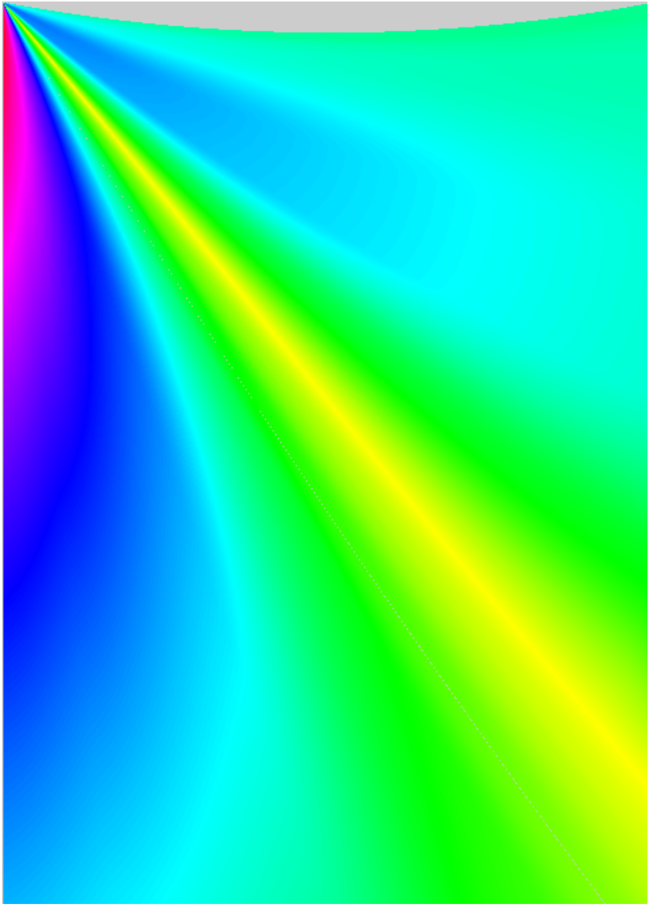
Relative coupling-ray-theory S-wave travel-time difference $|D/\overline{\tau}|$ in velocity model QI4. No length correction of the reference slowness vectors: the oscillations are considerably more pronounced.
Colour scale: 0.00%, 1.4%, 2.8%, 4.2%, 5.6%, 7.0%.



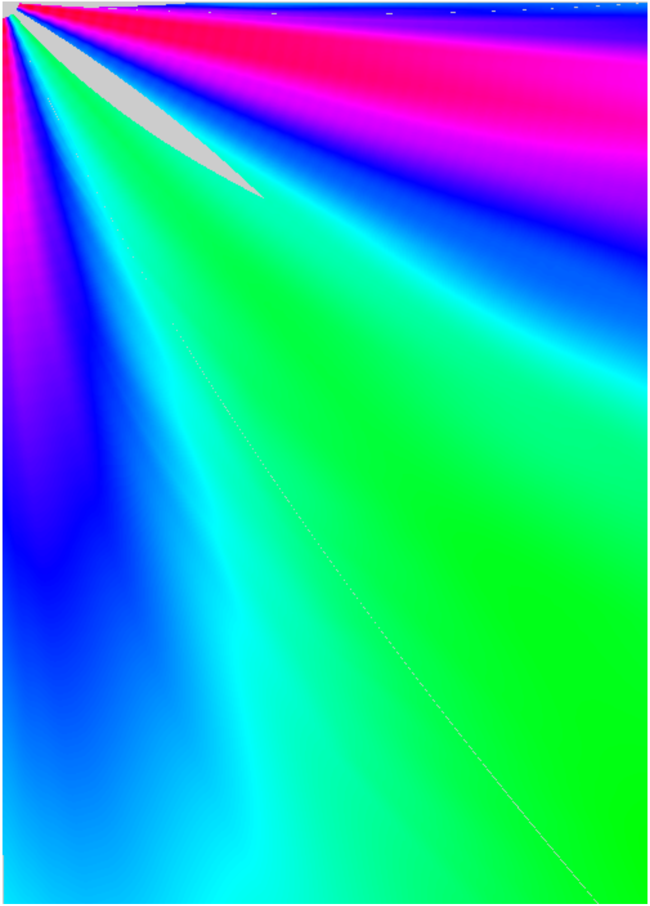
Relative coupling-ray-theory S-wave travel-time difference $|D/\overline{\tau}|$ in velocity model QI4. Tricubic interpolation of the reference travel time with trilinear interpolation of the travel-time correction.
Colour scale: 0.00%, 1.4%, 2.8%, 4.2%, 5.6%, 7.0%.



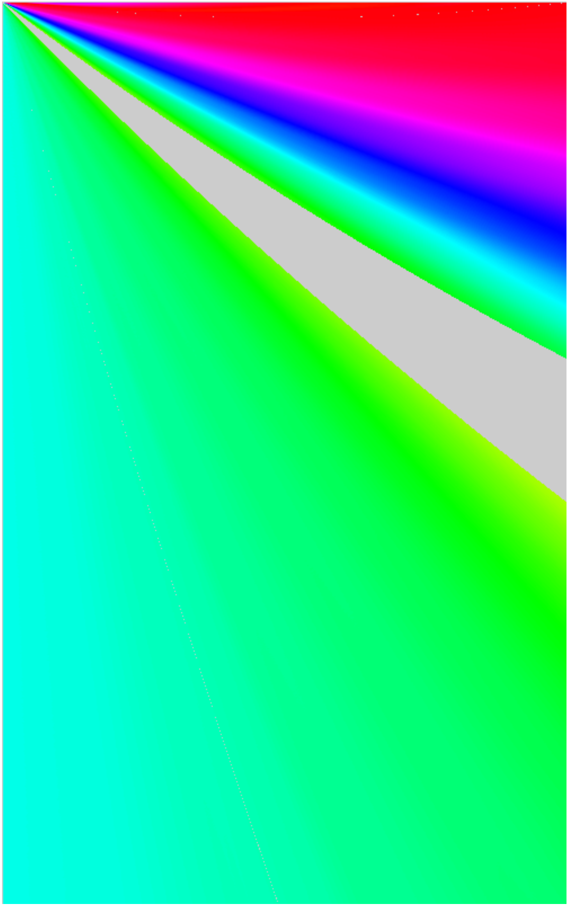
Relative coupling-ray-theory S-wave travel-time difference $|D/\overline{\tau}|$ in velocity model KISS.
Colour scale: **0.00%**, **0.35%**, **0.70%**, **1.05%**, **1.40%**, **1.75%**.



Relative coupling-ray-theory S-wave travel-time difference $|D/\overline{\tau}|$ in velocity model SC1_I.
Colour scale: **0.00%**, **1.1%**, **2.2%**, **3.3%**, **4.4%**, **5.5%**.



Relative coupling-ray-theory S-wave travel-time difference $|D/\overline{\tau}|$ in velocity model SC1_II. The diagonal shadow zone is probably caused by too narrow horizontal extent of the angular domain of the initial slowness vector. We have not been able to calculate the coupling-ray-theory S-wave Green tensor outside this narrow angular domain.
Colour scale: **0.00%**, **0.6%**, **1.2%**, **1.8%**, **2.4%**, **3.0%**.



Relative coupling-ray-theory S-wave travel-time difference $|D/\overline{\tau}|$ in velocity model ORT. The diagonal shadow zone corresponds to the ray tubes which cannot be split into the pairs of tubes with continuous prevailing-frequency coupling-ray-theory Green tensors. For infinitesimally short rays, the coupling-ray-theory polarization vectors approach the anisotropic-ray-theory polarization vectors which are discontinuous at a conical singularity. That is probably why the ray tubes in the vicinity of the conical singularity cannot be split into the pairs of tubes with continuous prevailing-frequency coupling-ray-theory Green tensors.
Colour scale: **0.00%**, **0.7%**, **1.4%**, **2.1%**, **2.8%**, **3.5%**.

Conclusions

The most challenging task is the study of continuity of the prevailing-frequency coupling-ray-theory Green tensor within ray tubes.

The erroneously oscillating interpolation of the coupling-ray-theory travel time can be corrected in two ways:
(a) interpolate the reference travel time by the tricubic interpolation within ray cells and separately interpolate the travel-time correction by the trilinear interpolation within ray cells;
(b) find the equations and the algorithm of calculating the correct slowness vector corresponding to the coupling-ray-theory travel time.

References

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Acknowledgements

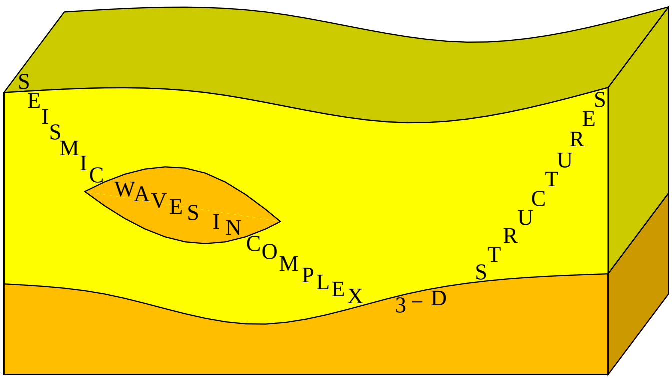
The research has been supported:

by the Grant Agency of the Czech Republic under contract P210/10/0736,

by the Ministry of Education of the Czech Republic within research project MSM0021620860,

by the European Commission under contract FP7-PEOPLE-IAPP-2009-230669 "Advanced Industrial Microseismic Monitoring",

and by the consortium "Seismic Waves in Complex 3-D Structures"



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