P-wave traveltime inversion in weakly anisotropic media: a preliminary study

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SW3D meeting

June 23-24, 2014

Outline

Introduction

Linear equations for WA (weak-anisotropy) parameters

Inversion scheme

Configuration and models

Tests

Uniqueness and stability

Conclusions

Future plans

Introduction

Long-term goal:

traveltime inversion

for 3D inhomogeneous media of arbitrary anisotropy

Present goal:

P-wave traveltime inversion

for 3D homogeneous media of arbitrary anisotropy

"Observed" data:

traveltimes generated by ANRAY program package

without or with an additional Gaussian noise

Calculated data:

traveltimes calculated from first-order approximations

of ray velocity

Linear equations for WA parameters

$$c^2(\mathbf{n}) = r^2/t^2$$

t - "observed" traveltime

r - source - receiver distance

$\mathbf{n}$ - unit vector in the source-receiver direction

$c(\mathbf{n})$ - first-order phase (ray) velocity

$c(\mathbf{n})$ depends on weak-anisotropy (WA) parameters

Linear equations for WA parameters

First-order expression for c^2

$$c^2(\mathbf{n}) = \alpha_0^2(1 + 2(\epsilon_x n_1^4 + \epsilon_y n_2^4 + \epsilon_z n_3^4 + \delta_x n_2^2 n_3^2 + \delta_y n_1^2 n_3^2 + \delta_z n_1^2 n_2^2)$$

$$+4[(\epsilon_{15} n_3 + \epsilon_{16} n_2) n_1^3 + (\epsilon_{24} n_3 + \epsilon_{26} n_1) n_2^3 + (\epsilon_{34} n_2 + \epsilon_{35} n_1) n_3^3$$

$$+(\chi_x n_1 + \chi_y n_2 + \chi_z n_3) n_1 n_2 n_3])$$

α_0 - P-wave reference velocity

ϵ_x, ϵ_y, ϵ_z, δ_x, δ_y, δ_z,

ϵ_{15}, ϵ_{16}, ϵ_{24}, ϵ_{26}, ϵ_{34}, ϵ_{35}, χ_x, χ_y, χ_z - 15 P-wave WA parameters

Linear equations for WA parameters

Alternative form of linear equations

linearized expression for c^2

$$c^{-1}(\mathbf{n}) = t/r$$

First-order expression for c^{-1}

$$c^{-1}(\mathbf{n}) = \alpha_0^{-1}(1 - (\epsilon_x n_1^4 + \epsilon_y n_2^4 + \epsilon_z n_3^4 + \delta_x n_2^2 n_3^2 + \delta_y n_1^2 n_3^2 + \delta_z n_1^2 n_2^2)$$

$$-2[(\epsilon_{16} n_2 + \epsilon_{15} n_3) n_1^3 + (\epsilon_{24} n_3 + \epsilon_{26} n_1) n_2^3 + (\epsilon_{35} n_1 + \epsilon_{34} n_2) n_3^3$$

$$+(\chi_x n_1 + \chi_y n_2 + \chi_z n_3) n_1 n_2 n_3])$$

Inversion scheme

$\mathbf{Gm} = \mathbf{d}$ $\quad$ $\mathbf{G}$ - $N \times M$ matrix

N - number of observations $\quad$ M - number of sought parameters (15)

Row of matrix $\mathbf{G}$:

$(n_1^4 \quad n_2^4 \quad n_3^4 \quad n_2^2 n_3^2 \quad n_1^2 n_3^2 \quad n_1^2 n_2^2 \quad 2n_1^3 n_3 \quad 2n_1^3 n_2 \quad 2n_2^3 n_3$
$2n_2^2 n_1 \quad 2n_3^3 n_2 \quad 2n_3^3 n_1 \quad 2n_1^2 n_2 n_3 \quad 2n_1 n_2^2 n_3 \quad 2n_1 n_2 n_3^2)$

Vector of model parameters $\mathbf{m}$:

$\mathbf{m} \equiv (\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15}, \epsilon_{16}, \epsilon_{24}, \epsilon_{26}, \epsilon_{34}, \epsilon_{35}, \chi_x, \chi_y, \chi_z)^T$

Data vector $\mathbf{d}$:

$\mathbf{d} \equiv (\frac{1}{2}(\frac{r_1^2}{\alpha_0^2 t_1^2} - 1),\ \frac{1}{2}(\frac{r_2^2}{\alpha_0^2 t_2^2} - 1),\ ....,\ \frac{1}{2}(\frac{r_N^2}{\alpha_0^2 t_N^2} - 1))^T$

Configuration and models

Models of orthorhombic media

Model	ϵ_x	ϵ_y	ϵ_z	δ_x	δ_y	δ_z
12%	0.193	0.211	0.128	0.277	0.237	0.342
25%	0.256	0.326	-0.001	0.075	-0.085	0.337

Parameters of the non-rotated models; $\alpha_0 = 2.44$ km/s

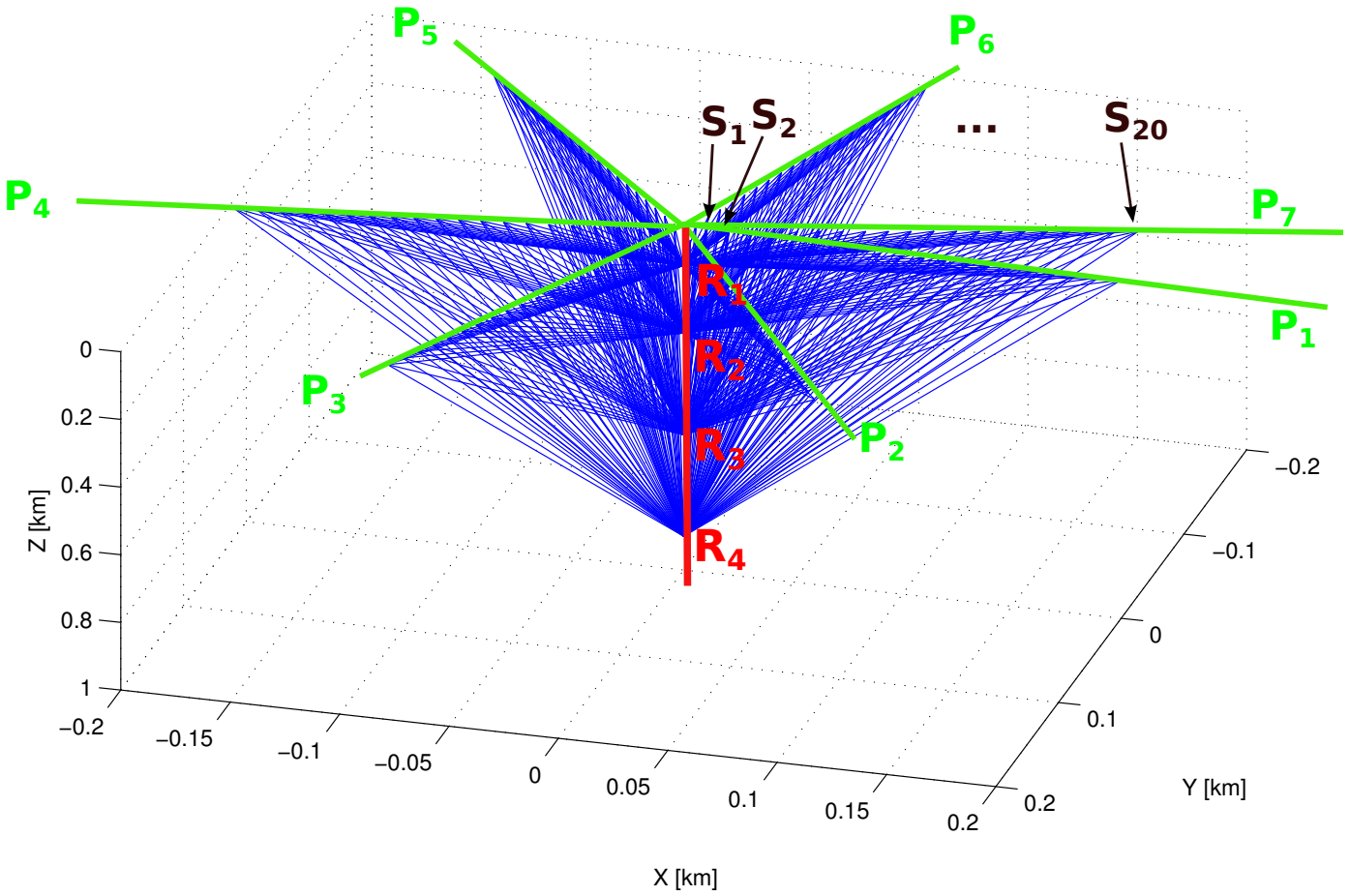
Model A1 - non-rotated model (12% anisotropy)

Model A2 - 45^0 around z-axis, 30^0 around new y-axis (12% anisotropy)

Model A3 - non-rotated model (25% anisotropy)

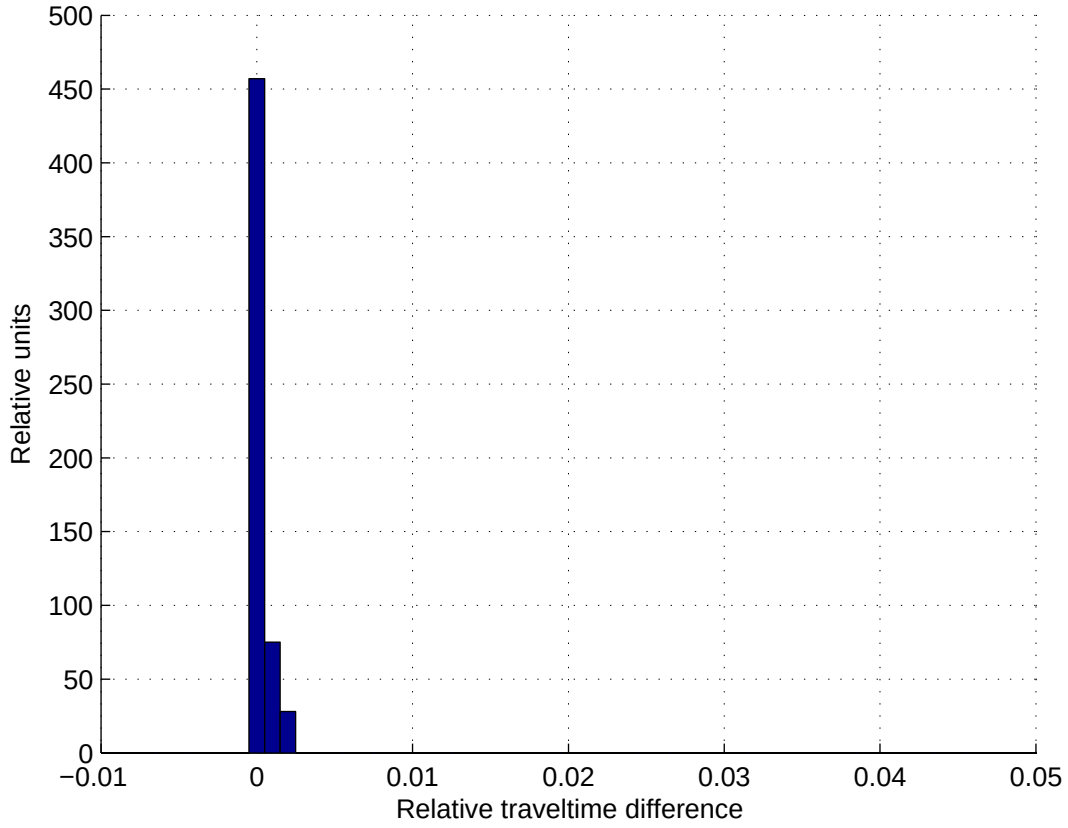
Model A4 - 30^0 around y-axis (25% anisotropy)

Configuration and models



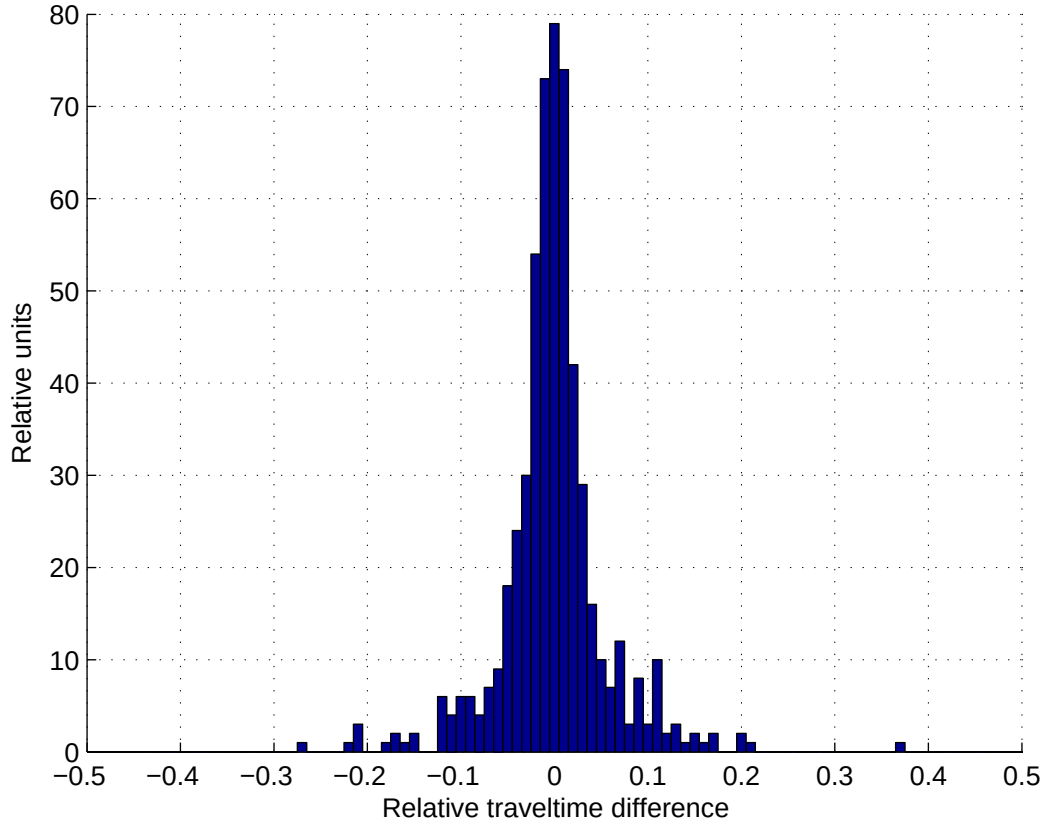
20 sources $\mathrm{S_i}$ on 7 surface profiles $\mathrm{P_i}$ and 4 receivers $\mathrm{R_i}$ in the borehole

Tests



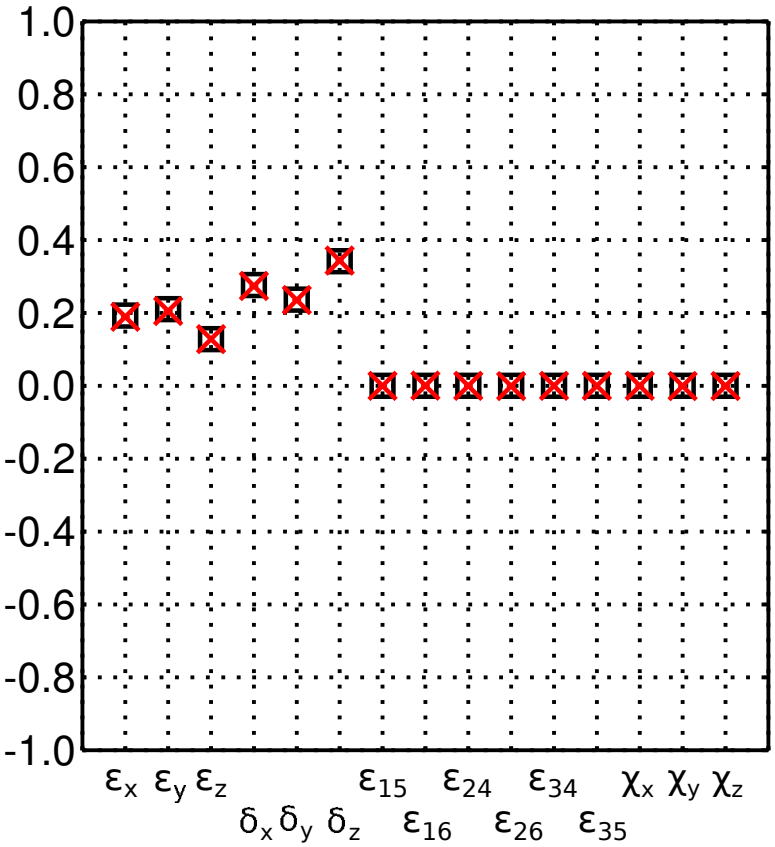
Relative traveltime mismodeling for model A1 - no noise

Tests



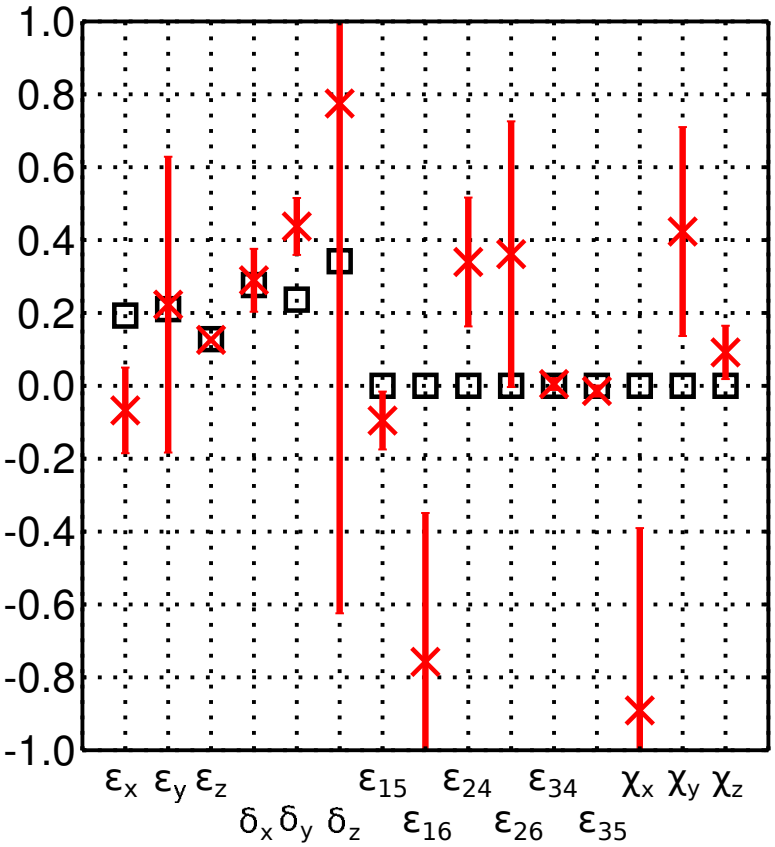
Relative traveltime mismodeling for model A1 - 5 ms random noise

Tests



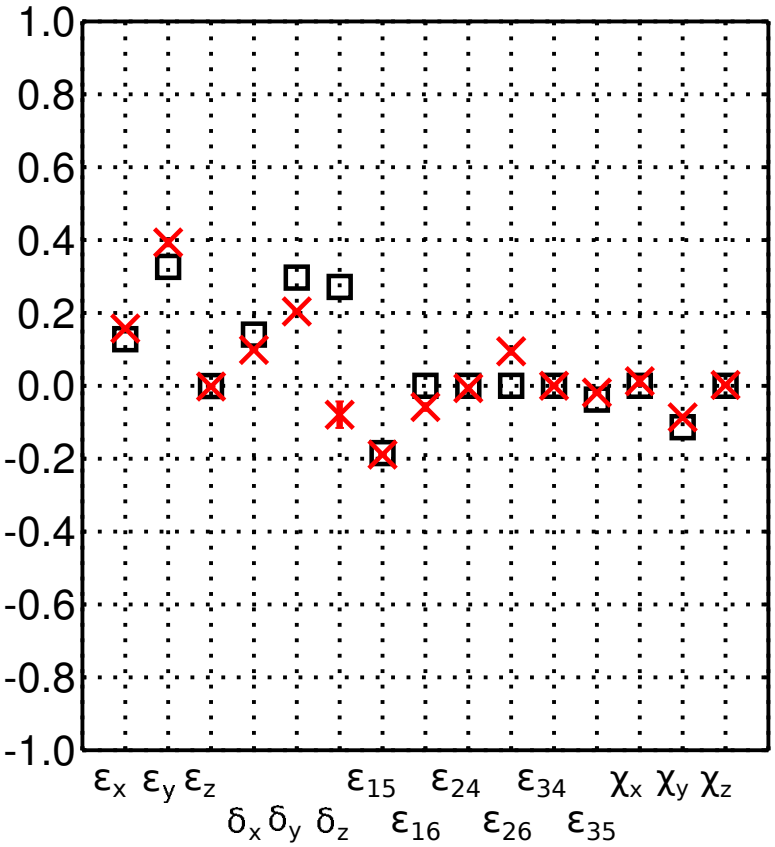
Model A1, no noise; black squares - exact, red crosses - inverted values, red bars - estimated errors of WA parameters

Tests



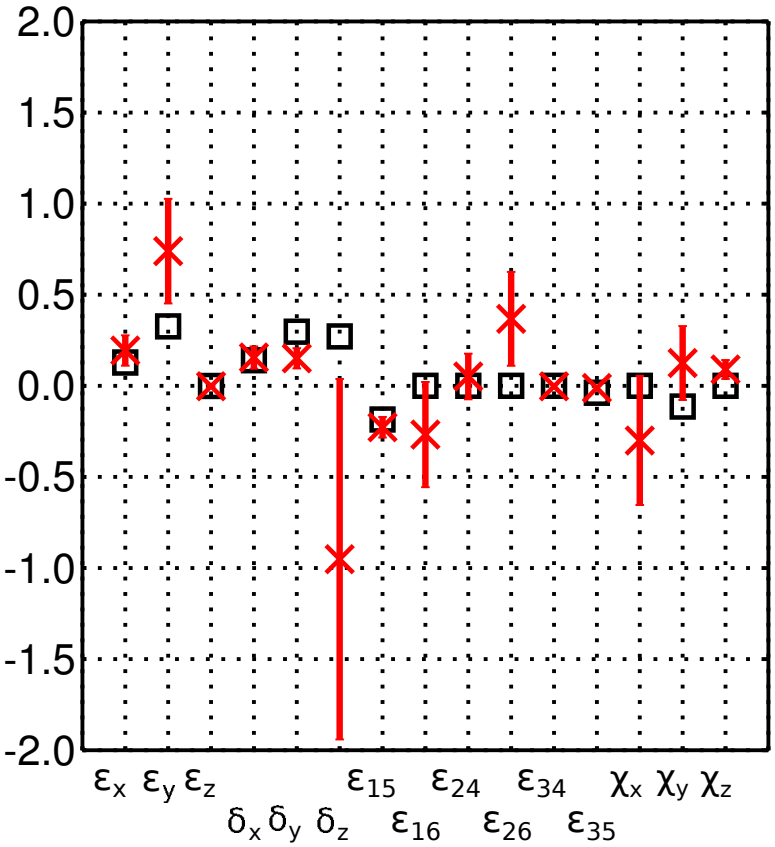
Model A1, 5 ms noise; black squares - exact, red crosses - inverted values, red bars - estimated errors of WA parameters

Tests



Model A4, no noise; black squares - exact, red crosses - inverted values, red bars - estimated errors of WA parameters

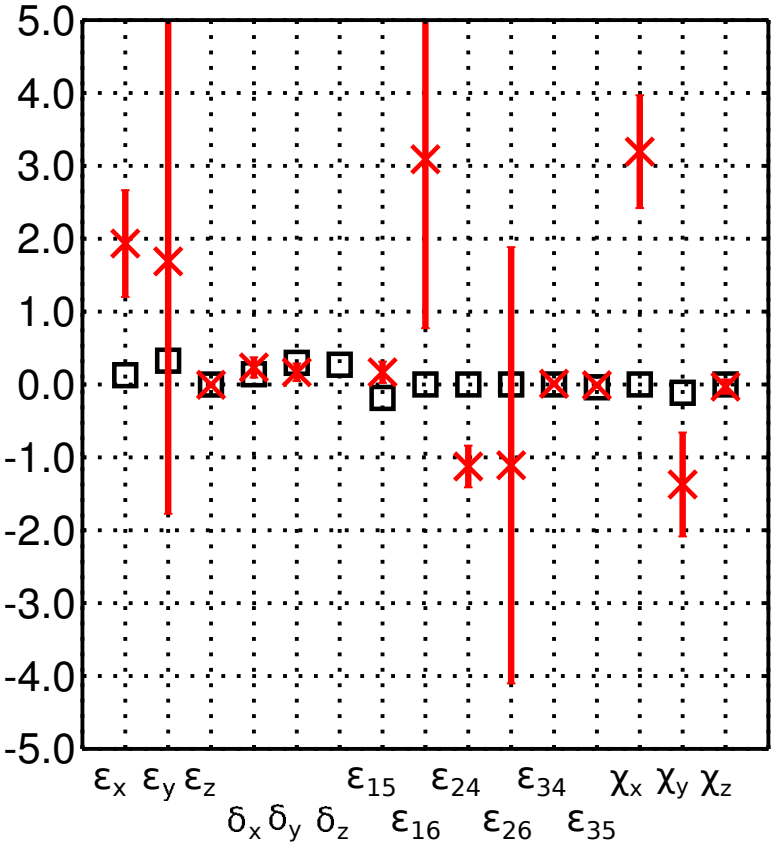
Tests



Model A4, 5 ms noise; black squares - exact, red crosses - inverted values, red bars - estimated errors of WA parameters

Tests

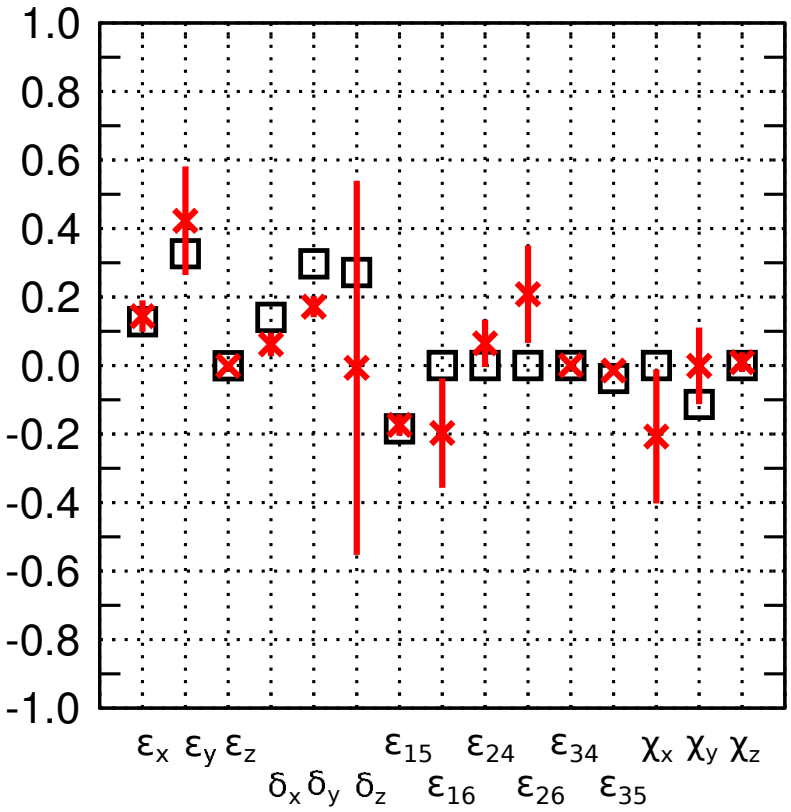
Receiver 0.1 km omitted

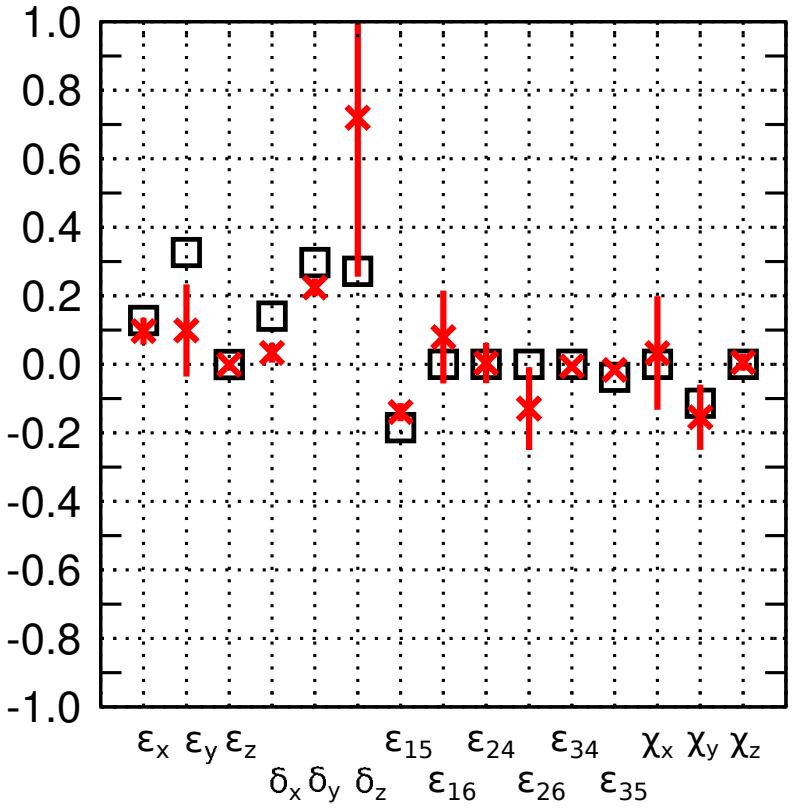


Model A4, 5 ms noise; black squares - exact, red crosses - inverted values, red bars - estimated errors of WA parameters

Tests

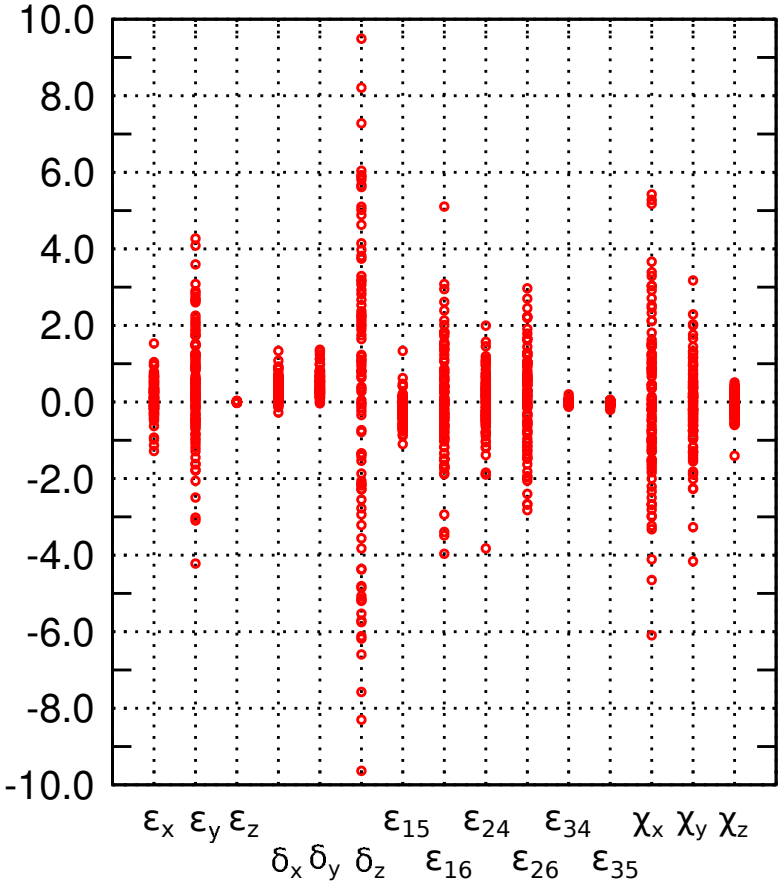
Use of expression for c^2(n) (left) and c^{-1}(n) (right), model A4, 3 ms noise, 1 realization

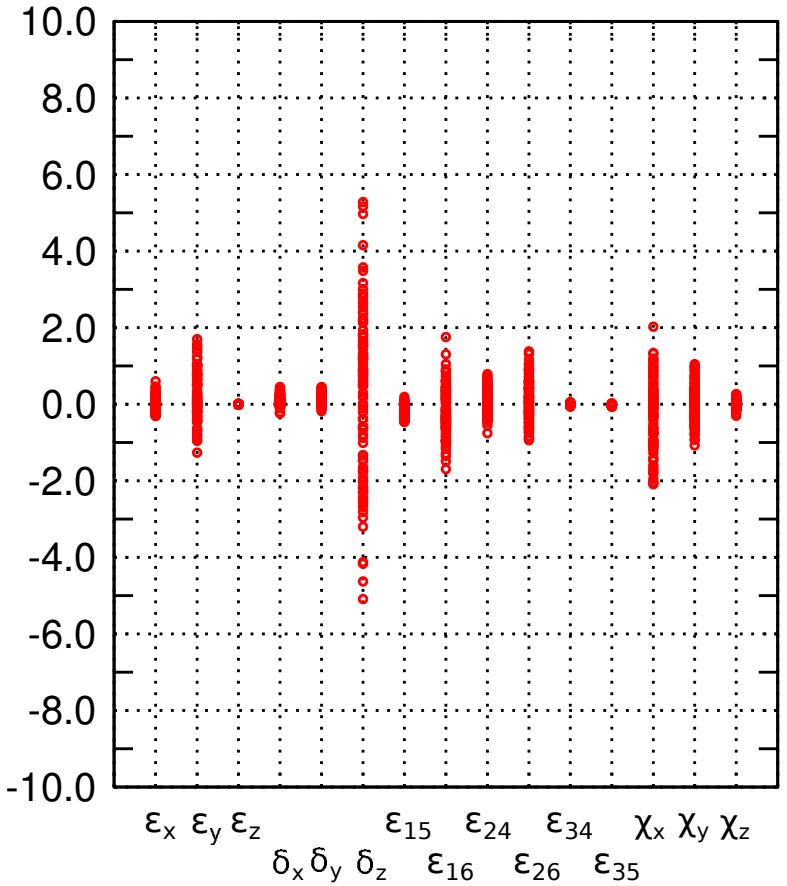




Tests

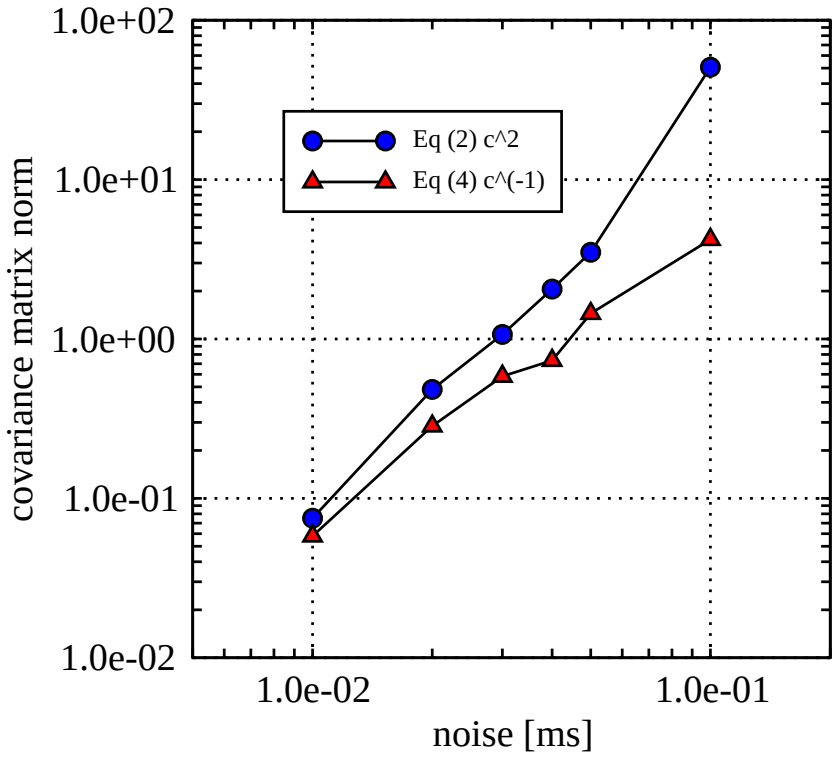
Use of expression for $c^2(\mathbf{n})$ (left) and $c^{-1}(\mathbf{n})$ (right), model A4, 10 ms noise, 100 realizations





Tests

Covariance matrix norms for model A4



100 realizations for 1,2,3,4,5 and 10 ms random Gaussian noise

Uniqueness and stability

Two fundamental aspects of linear inverse problems

Uniqueness $\times$ nonuniqueness - indications

- rank of $\mathbf{G}$ versus M (number of sought parameters)
- determinant of $\mathbf{G}^T\mathbf{G}$
- resolution matrix $\mathbf{G}^\dagger\mathbf{G}$

Stability $\times$ instability - indication

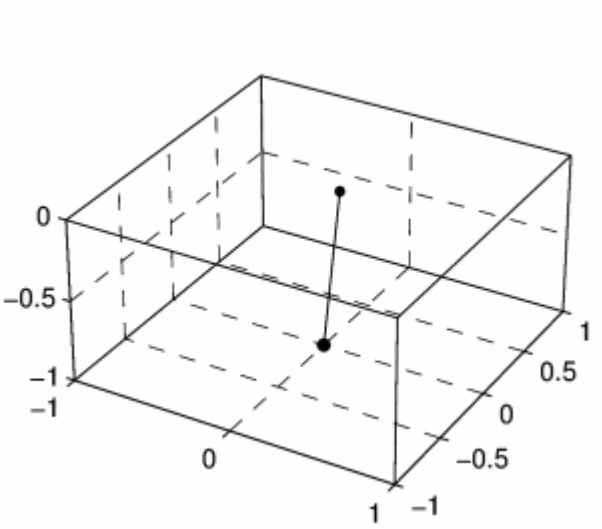
- condition number of $\mathbf{G}$: $s_{max}(\mathbf{G})/s_{min}(\mathbf{G})$

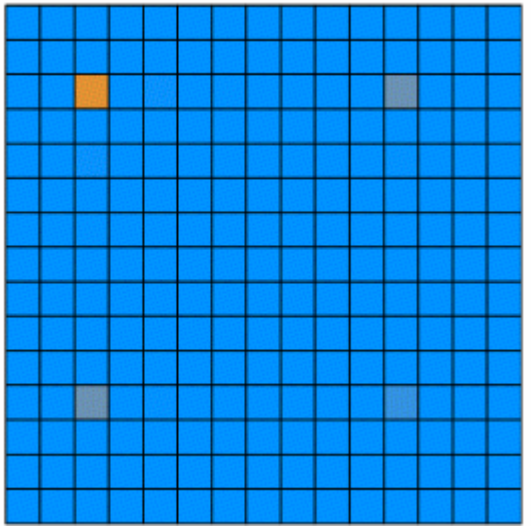
  $s_{max}(\mathbf{G})$, $s_{min}(\mathbf{G})$ - maximum, minimum singular value of $\mathbf{G}$

Uniqueness of the solution

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

Resolution matrix





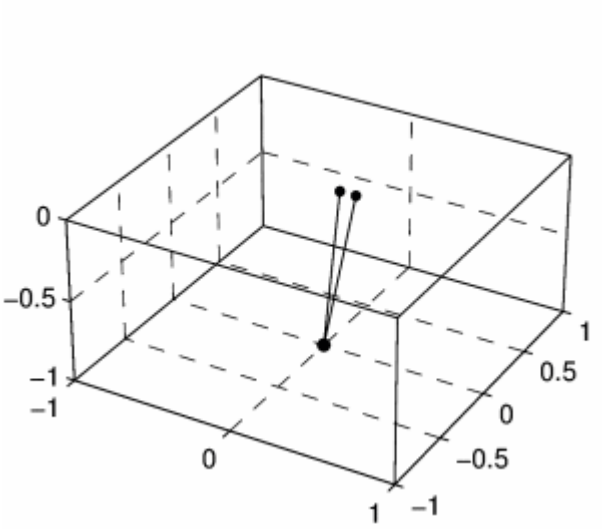
$$\boldsymbol{G} = \begin{bmatrix} 0.0001 & 0 & 0.9803 & 0 & 0.0196 & 0 & 0.0039 & 0 & 0 & 0 & 0 & 0.3921 & 0 & 0 & 0 \end{bmatrix}$$

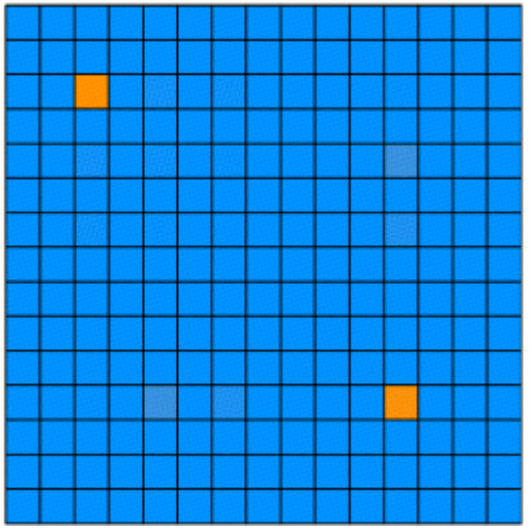
Measurements along a surface profile : 1 shot, rank = 1

Uniqueness of the solution

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

Resolution matrix





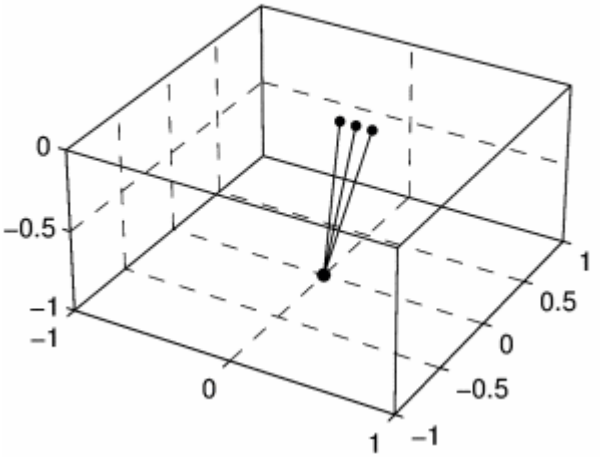
$$\boldsymbol{G} = \begin{bmatrix} 0.0001 & 0 & 0.9803 & 0 & 0.0196 & 0 & 0.0039 & 0 & 0 & 0 & 0 & 0.3921 & 0 & 0 & 0 \\ 0.0015 & 0 & 0.9246 & 0 & 0.0740 & 0 & 0.0296 & 0 & 0 & 0 & 0 & 0.7396 & 0 & 0 & 0 \end{bmatrix}$$

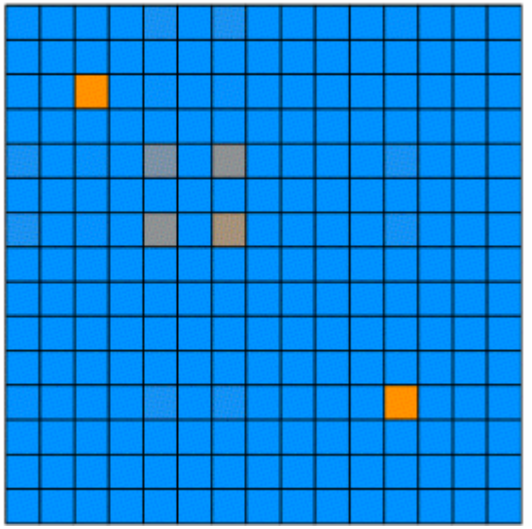
Measurements along a surface profile : 2 shots, rank = 2

Uniqueness of the solution

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

Resolution matrix





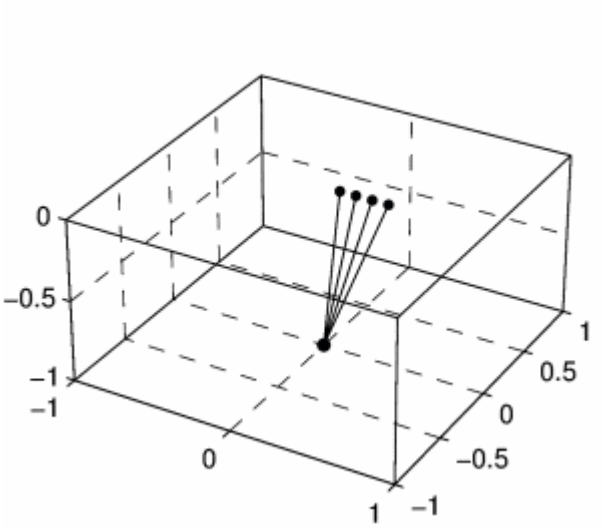
$$\boldsymbol{G} = \begin{bmatrix} 0.0001 & 0 & 0.9803 & 0 & 0.0196 & 0 & 0.0039 & 0 & 0 & 0 & 0 & 0.3921 & 0 & 0 & 0 \\ 0.0015 & 0 & 0.9246 & 0 & 0.0740 & 0 & 0.0296 & 0 & 0 & 0 & 0 & 0.7396 & 0 & 0 & 0 \\ 0.0068 & 0 & 0.8417 & 0 & 0.1515 & 0 & 0.0909 & 0 & 0 & 0 & 0 & 1.0100 & 0 & 0 & 0 \end{bmatrix}$$

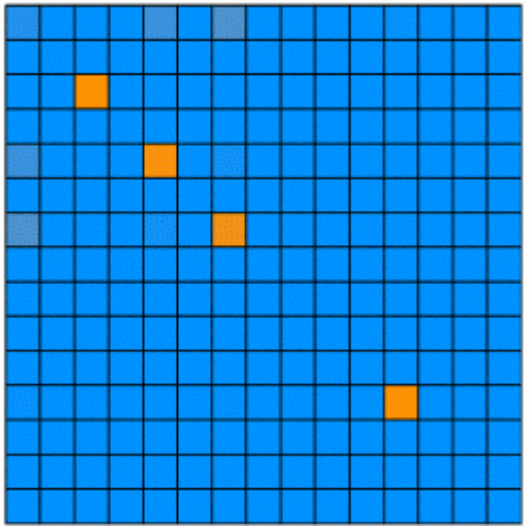
Measurements along a surface profile : 3 shots, rank = 3

Uniqueness of the solution

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

Resolution matrix



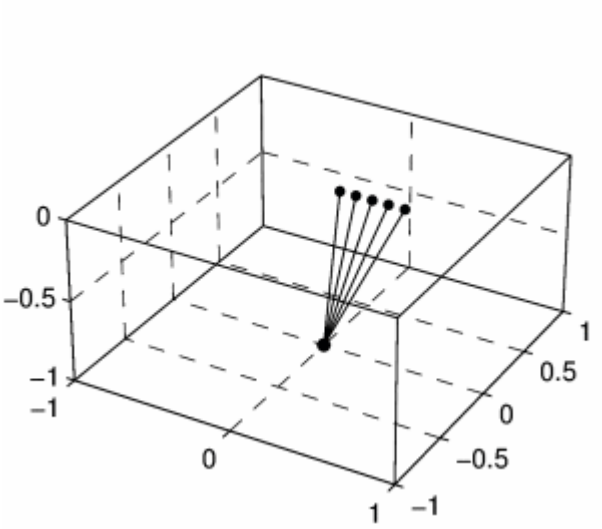


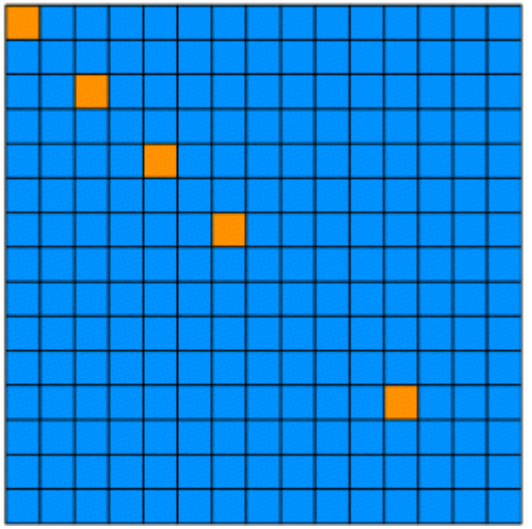
Measurements along a surface profile : 4 shots, rank = 4

Uniqueness of the solution

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

Resolution matrix



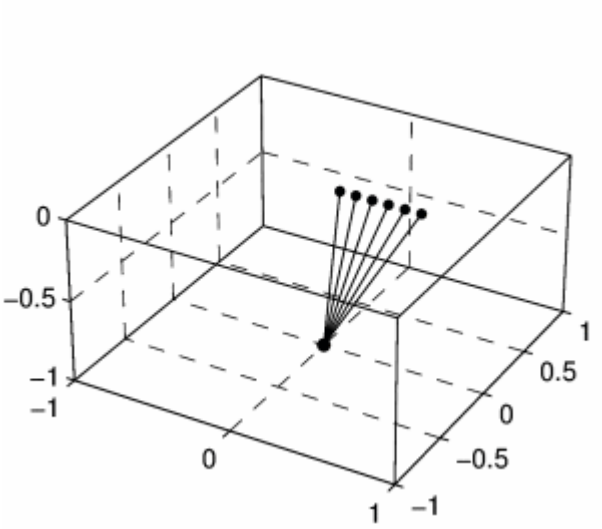


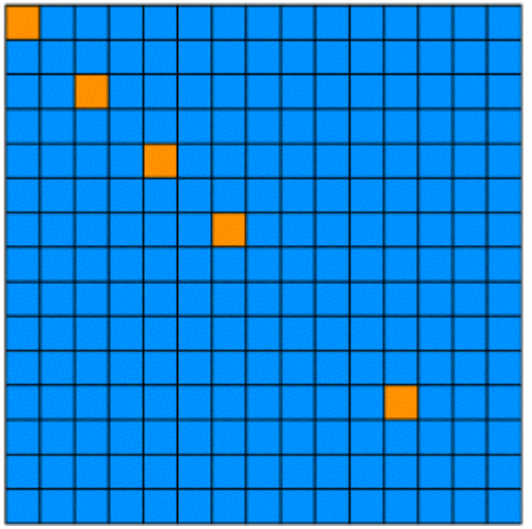
Measurements along a surface profile : 5 shots, rank = 5

Uniqueness of the solution

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

Resolution matrix



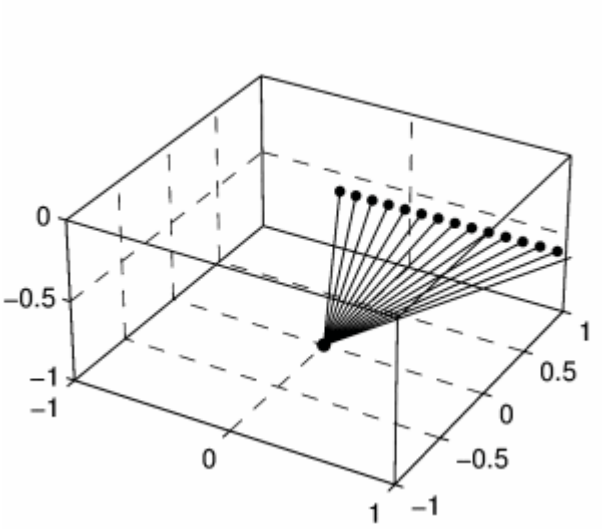


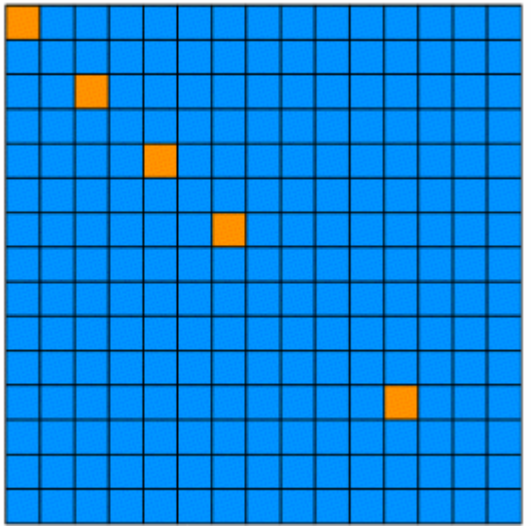
Measurements along a surface profile : 6 shots, rank = 5

Uniqueness of the solution

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

Resolution matrix



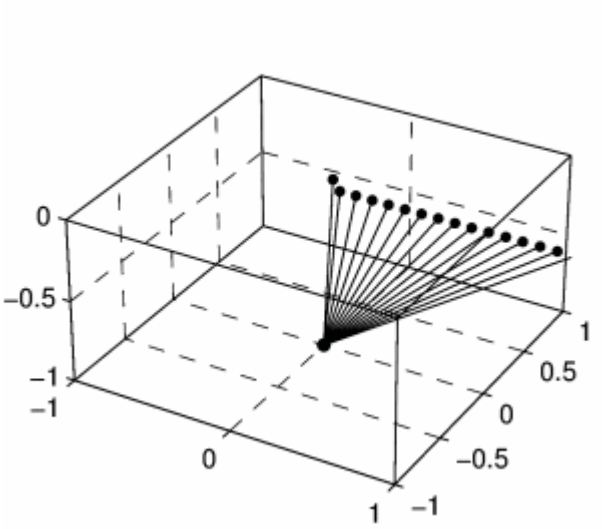


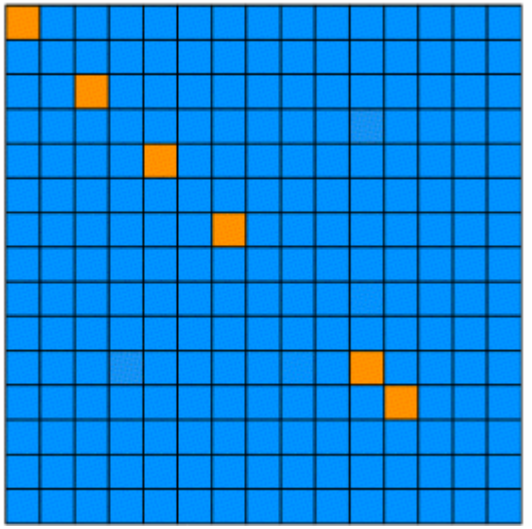
Measurements along a surface profile : 15 shots, rank = 5

Uniqueness of the solution

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

Resolution matrix



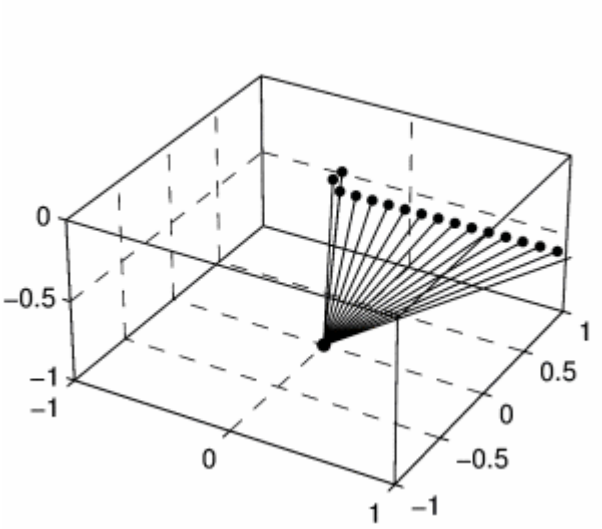


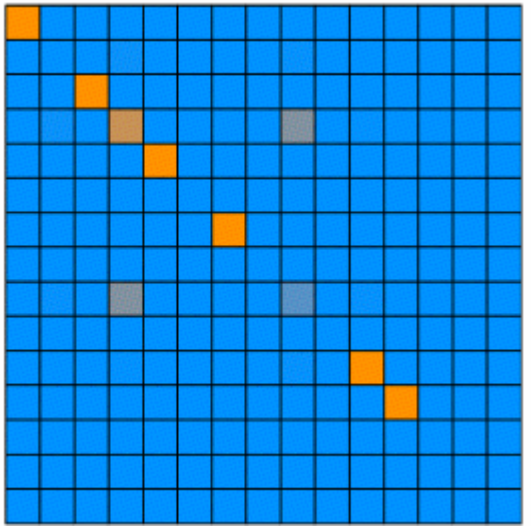
Measurements along 2 surface profiles : 15+1 shots, rank = 6

Uniqueness of the solution

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

Resolution matrix



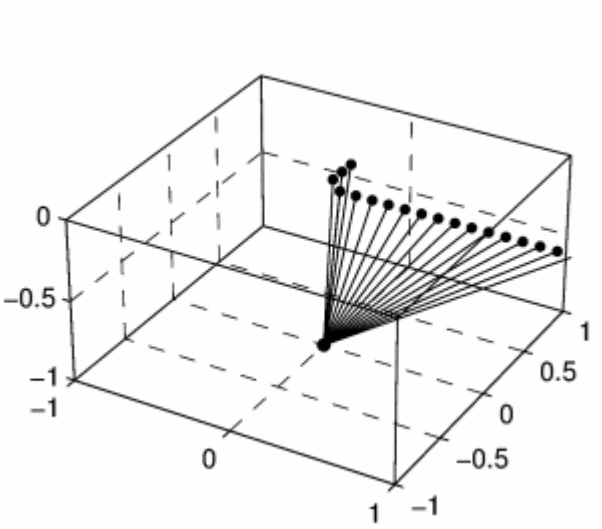


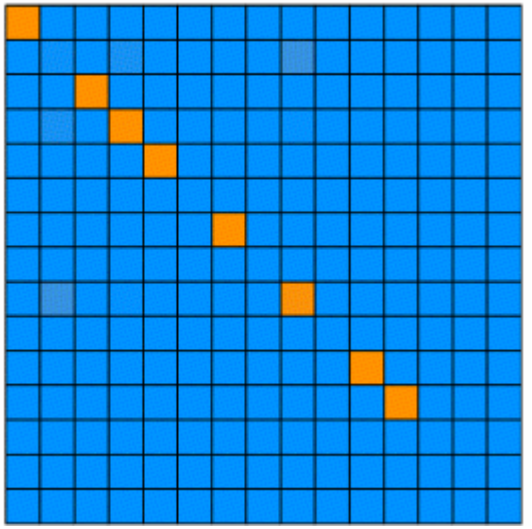
Measurements along 2 surface profiles : 15+2 shots, rank = 7

Uniqueness of the solution

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

Resolution matrix



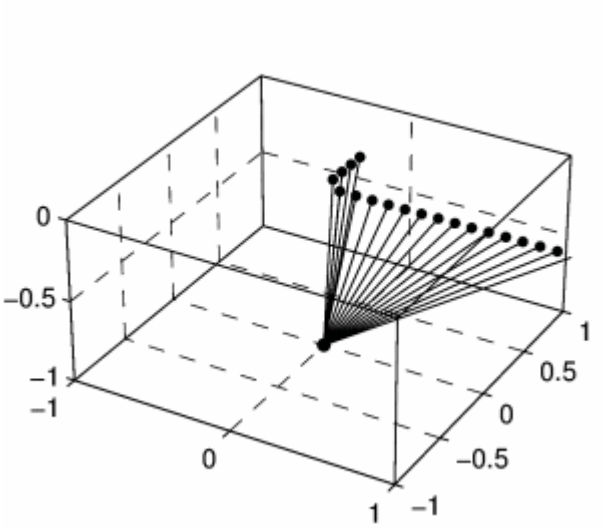


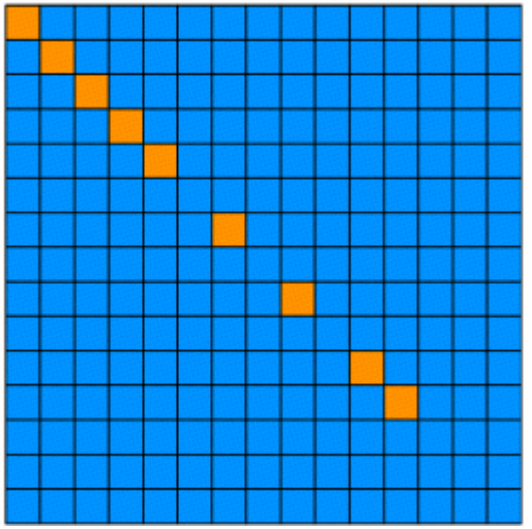
Measurements along 2 surface profiles : 15+3 shots, rank =8

Uniqueness of the solution

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

Resolution matrix



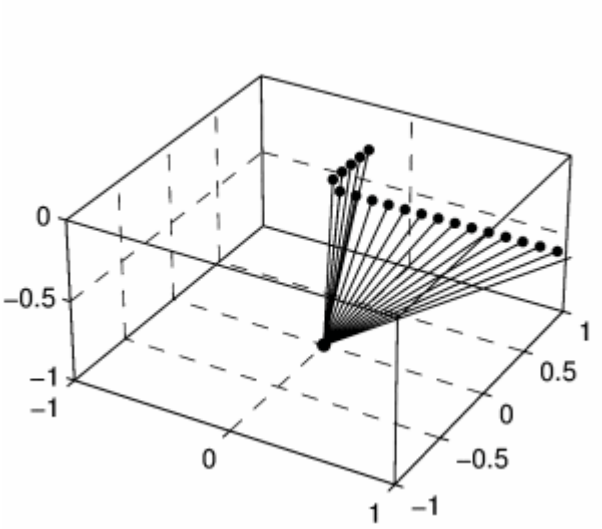


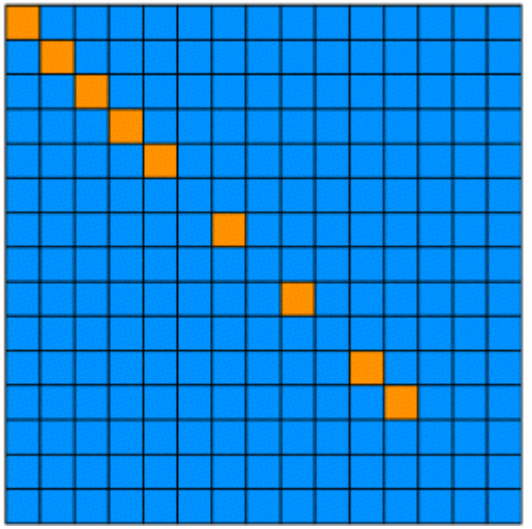
Measurements along 2 surface profiles : 15+4 shots, rank = 9

Uniqueness of the solution

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

Resolution matrix



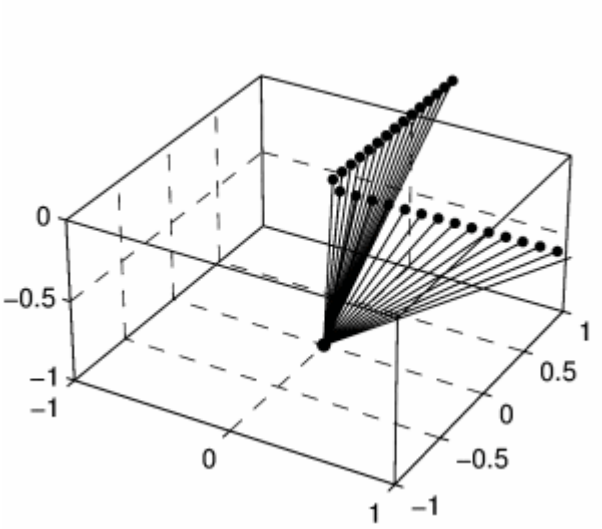


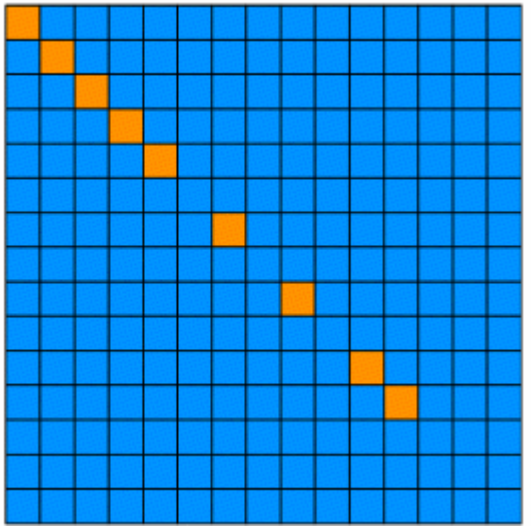
Measurements along 2 surface profiles : 15+5 shots, rank = 9

Uniqueness of the solution

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

Resolution matrix



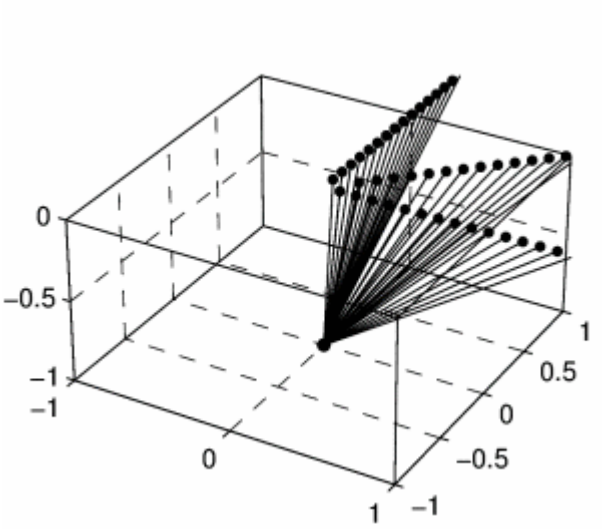


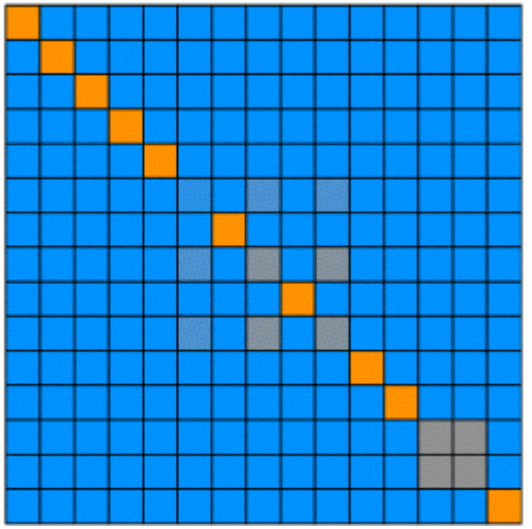
Measurements along 2 surface profiles : 15+15 shots, rank = 9

Uniqueness of the solution

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

Resolution matrix



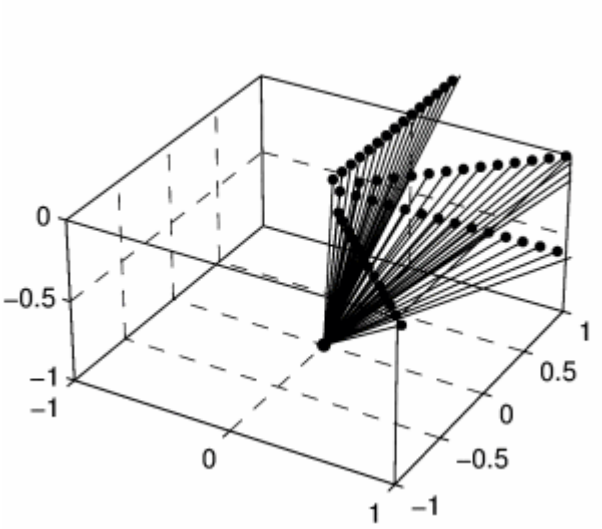


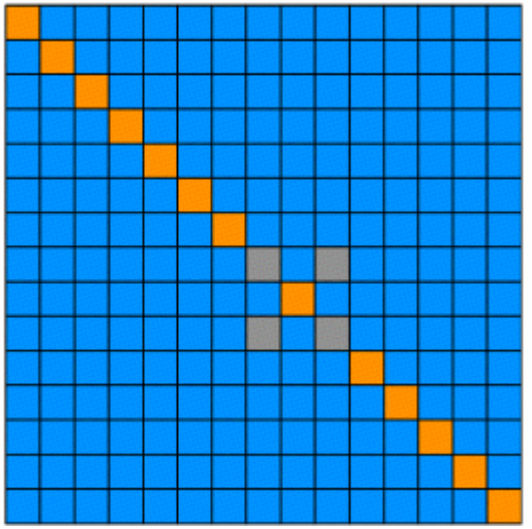
Measurements along 3 surface profiles : 15+15+15 shots, rank = 12

Uniqueness of the solution

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

Resolution matrix



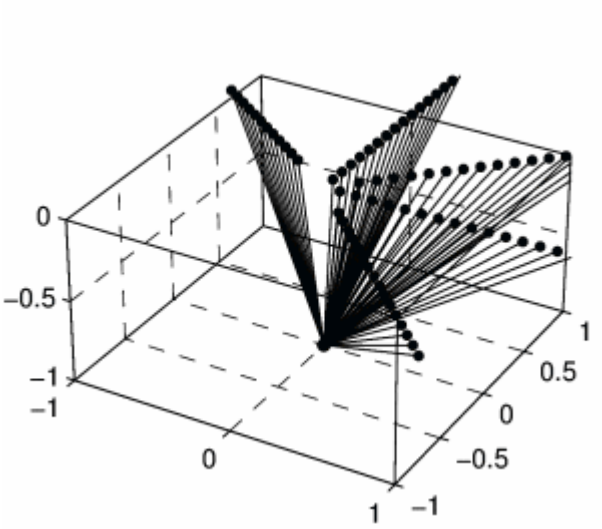


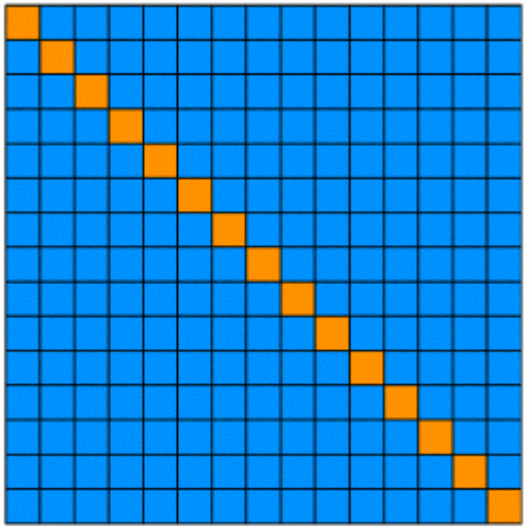
Measurements along 4 surface profiles : 15+15+15+15 shots, rank = 14

Uniqueness of the solution

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

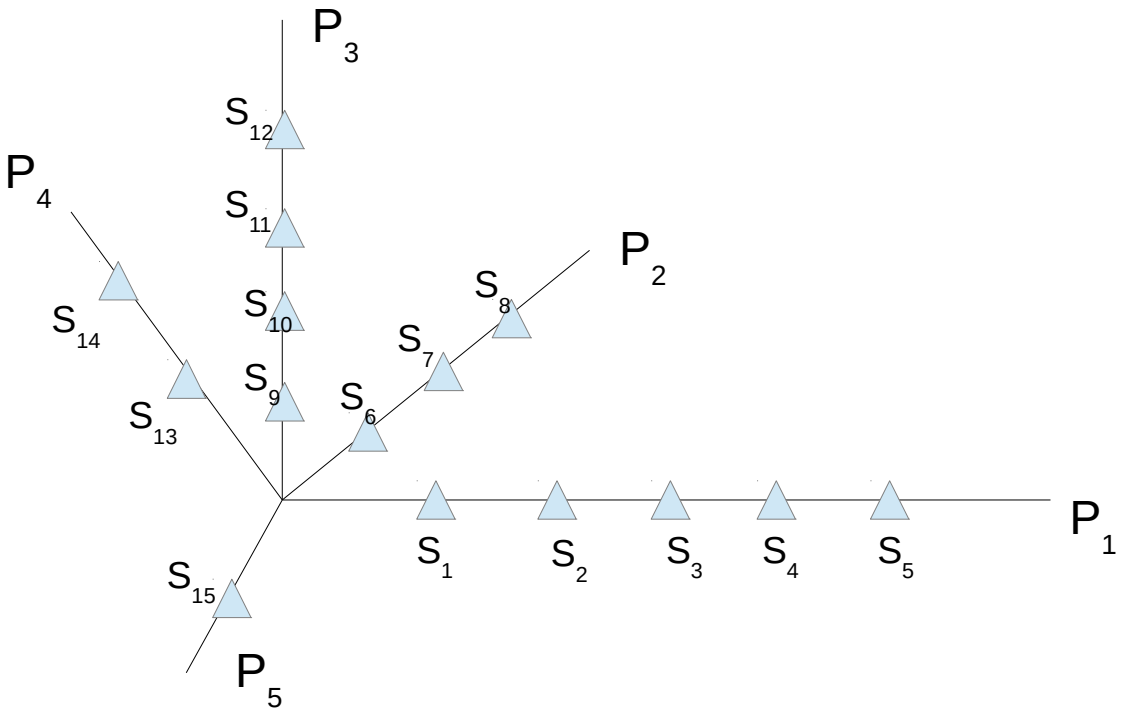
Resolution matrix





Measurements along 5 surface profiles : 15+15+15+15+15 shots, rank = 15

Economic variant of “profile” measurement

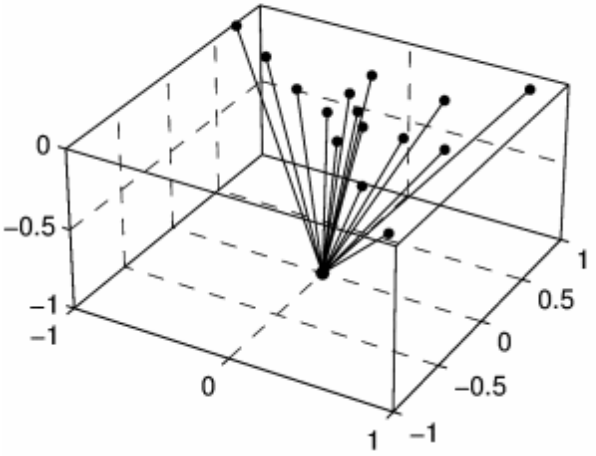


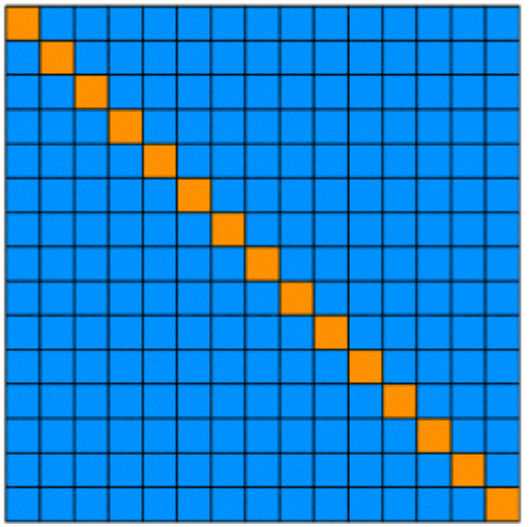
5 different azimuths, at most 5 shots/direction

Economic variant of “profile” measurement

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

Resolution matrix



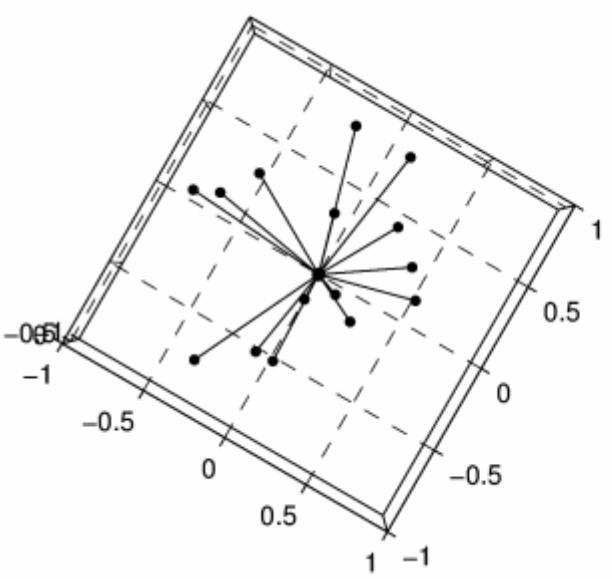


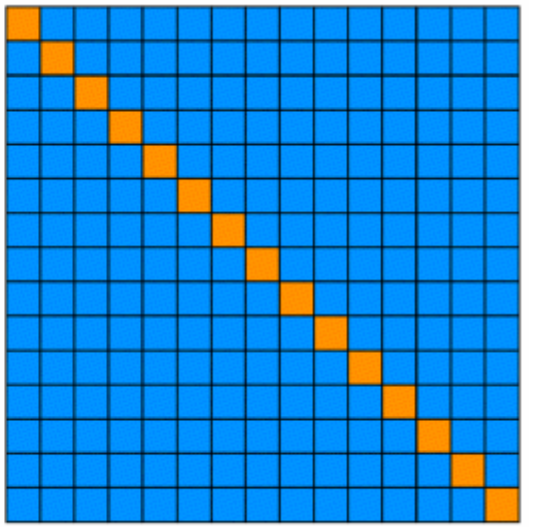
5 “profiles”, 3 shots/profile, **rank = 15**

Random distribution of surface shots

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

Resolution matrix



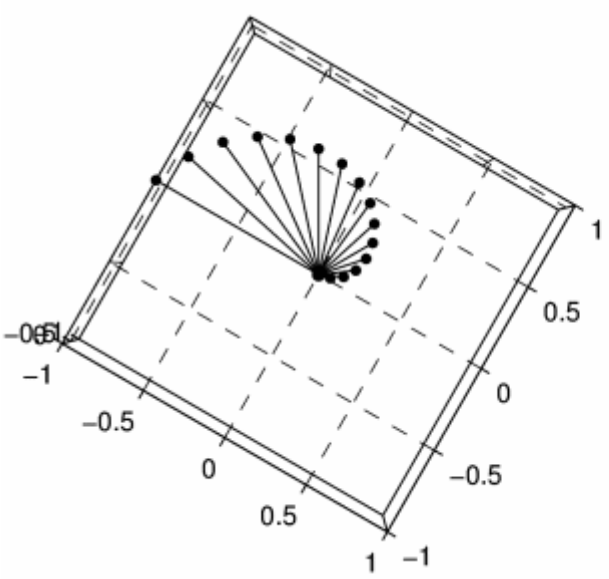


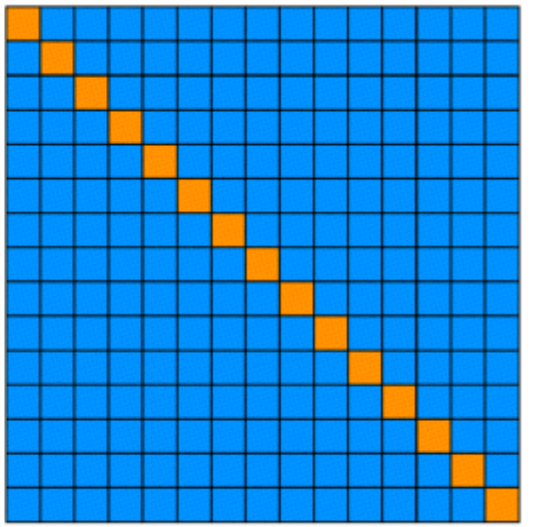
No profiles, **rank = 15**

Spiral-like distribution of surface shots

$$\left[\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15,} \epsilon_{16,} \epsilon_{24,} \epsilon_{26,} \epsilon_{34,} \epsilon_{35,} \chi_x, \chi_y, \chi_z\right]$$

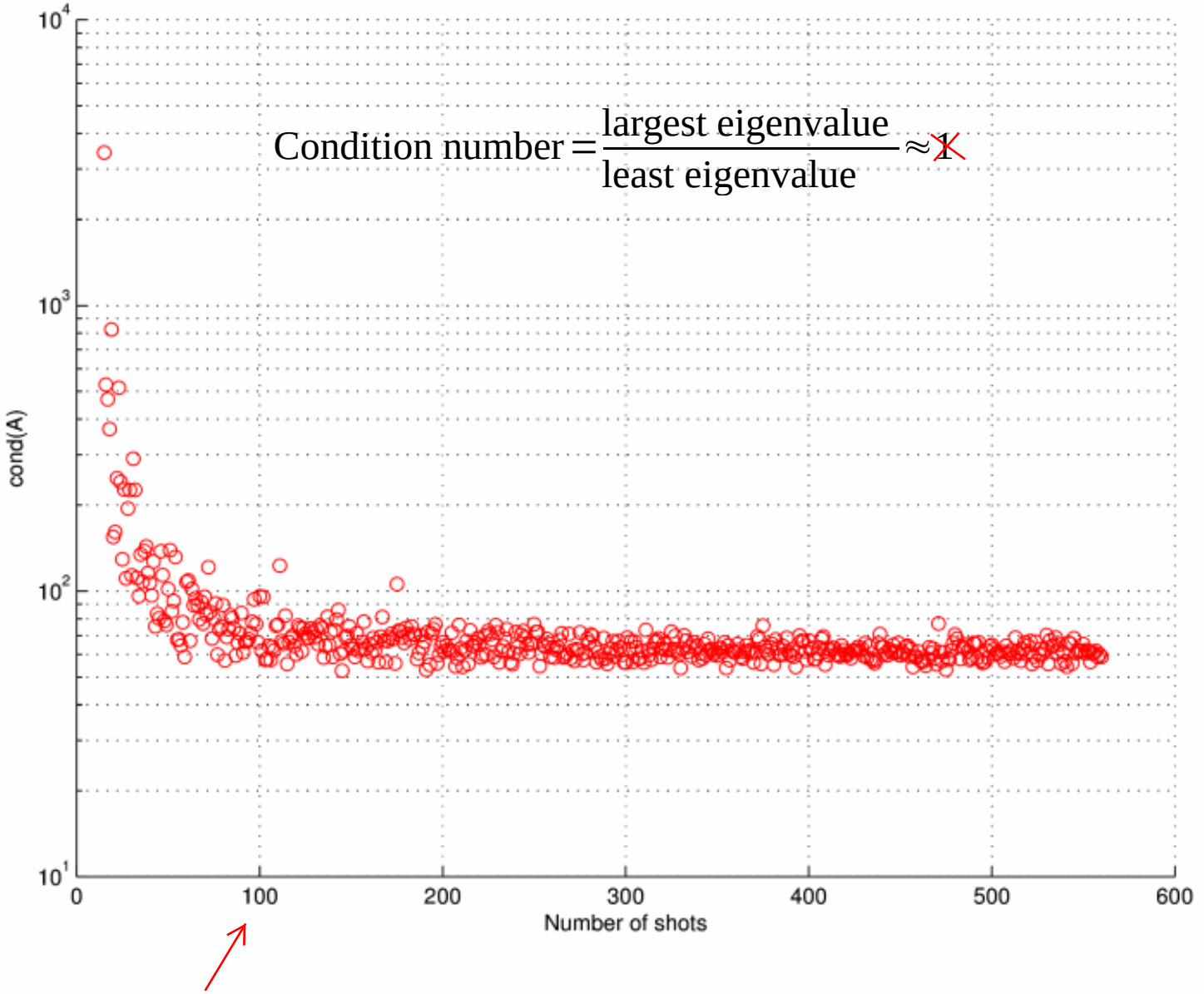
Resolution matrix





rank = 15

Stability of the solution



Conclusions

P-wave traveltime inversion - search for 15 P-wave WA parameters

Noise-free data: perfect fit for 12%, certain misfit for 26% anisotropy

Application of noise leads to instability

Stable WA parameters: ϵ_z, ϵ_{34}, ϵ_{35}, χ_z; unstable: δ_z

Expression for c^{-1} more suitable than expression for c^2

Future plans

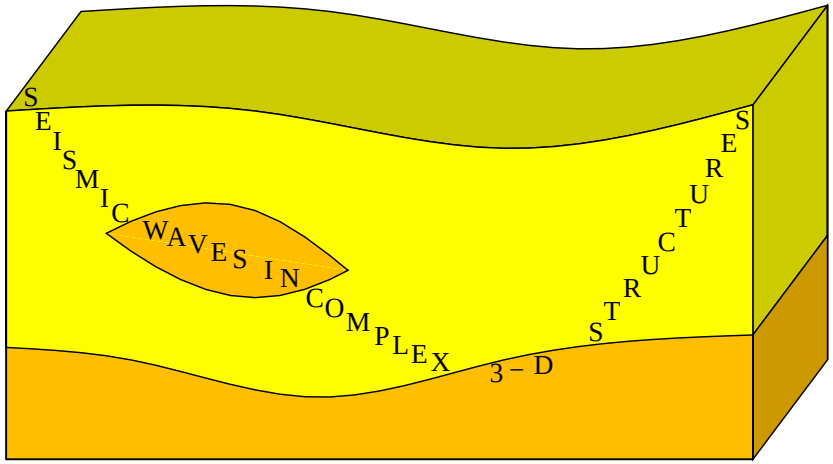
Continuing tests in homogeneous media

- study of resolution and stability
- use of more accurate expressions for the ray velocity
- estimate of type of anisotropic symmetry

Tests in inhomogeneous media

- study of resolution and stability
- use of an additional independent information
- use of S-wave traveltimes
- use of P- and S-wave polarizations

Acknowledgements



Research project P210/11/0117 of the Grant Agency of the CR