Summation integrals for a Green function in a 3-D inhomogeneous anisotropic medium

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Outline

Introduction

Summation integral

Conclusions

Introduction

Wave modelling in complex anisotropic media

- ray method
- paraxial ray approximations, paraxial Gaussian beams (Gaussian packets)
- weighted summation of paraxial ray approximations or paraxial Gaussian beams (Gaussian packets)

Goal: algorithm for computation of a Green function based on the summation of Gaussian beams or two-parametric summation of Gaussian packets in Cartesian coordinates

Summation integral

$$G_{ij}(R,S,\omega) = \frac{\omega}{2\pi}\iint_{\mathcal{D}} G_{ij}^{\rm ray}(R_\gamma,S)[-\det\boldsymbol{\mathcal{N}}(R_\gamma)]^{1/2}\exp[{\rm i}\omega T(R,R_\gamma)]d\gamma_1 d\gamma_2$$

$\mathbf{G}(R,S,\omega)$ - Green function

ω - circular frequency

γ_1,γ_2 - ray parameters defined on $\mathcal{D}$, specifying ray Ω

R, S, R_γ - receiver, source and a point on Ω, in a vicinity of R

$\mathbf{G}^{\rm ray}(R_\gamma,S)$ - elementary ray-theory Green function

$[-\det\boldsymbol{\mathcal{N}}(R_\gamma)]^{1/2}$ - the weighting function; $\boldsymbol{\mathcal{N}}$ - 2×2 matrix

$T(R,R_\gamma)$ - paraxial travel time at receiver R

Summation integral

Elementary ray-theory Green function

$$G_{ij}^{\mathrm{ray}}(R_\gamma, S) = \frac{g_i(R_\gamma)g_j(S)}{4\pi[\rho(S)\rho(R_\gamma)C(S)C(R_\gamma)]^{1/2}} \frac{\exp[i\,T^G(R_\gamma,S)]}{\mathcal{L}(R_\gamma,S)}\,\mathcal{R}^C$$

$\mathbf{g}(S)$, $\mathbf{g}(R_\gamma)$ - polarization vectors at S and R_γ

$C(S)$, $C(R_\gamma)$, $\rho(S)$, $\rho(R_\gamma)$ - phase velocities and densities at S and R_γ

$\mathcal{L}(R_\gamma, S)$ ... the relative geometrical spreading

$T^G(R_\gamma, S)$ ... complete phase shift due to caustics

$\mathcal{R}^C$ ... complete reflection/transmission coefficient

Summation integral

Weighting function

$$\boldsymbol{\mathcal{N}}(R_\gamma) = -\mathbf{Q}^{(x)T}\mathbf{P}^{(x)} + \mathbf{Q}^{(x)T}\boldsymbol{\mathcal{E}}\mathbf{M}(R_\gamma)\boldsymbol{\mathcal{E}}^T\mathbf{Q}^{(x)}$$

$\mathbf{Q}^{(x)}(R_\gamma)$, $\mathbf{P}^{(x)}(R_\gamma)$ - 3×2 parts of $\hat{\mathbf{Q}}^{(x)}$, $\hat{\mathbf{P}}^{(x)}$

$\hat{\mathbf{Q}}^{(x)}(R_\gamma)$, $\hat{\mathbf{P}}^{(x)}(R_\gamma)$ - 3×3 paraxial matrices obtained from DRT

$\mathbf{M}(R_\gamma)$ - 2×2 matrix of Gaussian-beam parameters (given)

$\boldsymbol{\mathcal{E}}(R_\gamma)$ - 3×2 matrix $\boldsymbol{\mathcal{E}} = (\mathbf{e}_1, \mathbf{e}_2)$ (given)

$\mathbf{e}_I$ - unit vectors, $\mathbf{e}_1^T\mathbf{e}_2 = 0$, $\mathbf{e}_I^T\mathbf{p} = 0$, $\mathbf{p}$ - slowness vector

Summation integral

Travel time

$$T(R,R_\gamma) = T(R_\gamma) + \mathbf{x}^T(R,R_\gamma)\mathbf{p}(R_\gamma) + \tfrac{1}{2}\mathbf{x}^T(R,R_\gamma)\hat{\mathbf{M}}^{(x)}(R_\gamma)\mathbf{x}(R,R_\gamma)$$

$T(R_\gamma)$ - travel time at R_γ, $\qquad \mathbf{x}(R,R_\gamma) = \mathbf{x}(R) - \mathbf{x}(R_\gamma)$

$\hat{\mathbf{M}}^{(x)}(R_\gamma) = \boldsymbol{\mathcal{F}}\mathbf{M}\boldsymbol{\mathcal{F}}^T + \mathbf{p}\boldsymbol{\eta}^T + \boldsymbol{\eta}\mathbf{p}^T - \mathbf{p}(\boldsymbol{\mathcal{U}}^T\boldsymbol{\eta})\mathbf{p}^T$; $\quad \hat{\mathbf{M}}^{(x)}$ - 3×3 matrix

$\mathbf{M}(R_\gamma)$ - 2×2 - matrix of Gaussian-beam parameters (given)

$\boldsymbol{\mathcal{F}}(R_\gamma)$ - 3×2 matrix $\boldsymbol{\mathcal{F}} = (\mathbf{f}_1, \mathbf{f}_2)$ $\quad \mathbf{f}_1 = \mathcal{C}^{-1}(\mathbf{e}_2 \times \boldsymbol{\mathcal{U}})$, $\quad \mathbf{f}_2 = \mathcal{C}^{-1}(\boldsymbol{\mathcal{U}} \times \mathbf{e}_1)$

$\mathbf{p}$, $\boldsymbol{\eta}$, $\boldsymbol{\mathcal{U}}$ - slowness, eta, ray-velocity vector, $\mathbf{e}_I$ vectors given

Summation integral

Summation along a target surface Σ^R

Related to the two-parametric summation

of Gaussian packets (Klimeš, 2014)

Weighting function

$$\boldsymbol{\mathcal{N}}(R_\gamma) = -(\mathbf{Q}^{(x)})^T_{\Sigma^R}(\mathbf{P}^{(x)})_{\Sigma^R} + (\mathbf{Q}^{(x)})^T_{\Sigma^R}\mathbf{Z}(R_\gamma)\mathbf{F}(R_\gamma)\mathbf{Z}^T(R_\gamma)(\mathbf{Q}^{(x)})_{\Sigma^R}$$

Travel time

$$T(R, R_\gamma) = T(R_\gamma) + \mathbf{x}^T(R, R_\gamma)\mathbf{p}(R_\gamma) + \frac{1}{2}\mathbf{x}^T(R, R_\gamma)\mathbf{Z}(R_\gamma)\mathbf{F}(R_\gamma)\mathbf{Z}^T(R_\gamma)\mathbf{x}(R, R_\gamma)$$

Conclusions

- applicable to 3D inhomogeneous anisotropic media
  with curved structural interfaces

- applicable to separate P, S1 and S2 waves

- applicable to coupled S waves in weak anisotropy

- applicable to summation of paraxial ray approximations,
  including Maslov-Chapman integrals

- applicable to moment-tensor point sources

Conclusions

- DRT performed in Cartesian coordinates
- 3×2 parts of 3×3 paraxial matrices sufficient
- no need for two-point ray tracing
- removes or smoothes singularities of standard ray theory
- no need for computation of vector bases along ray Ω

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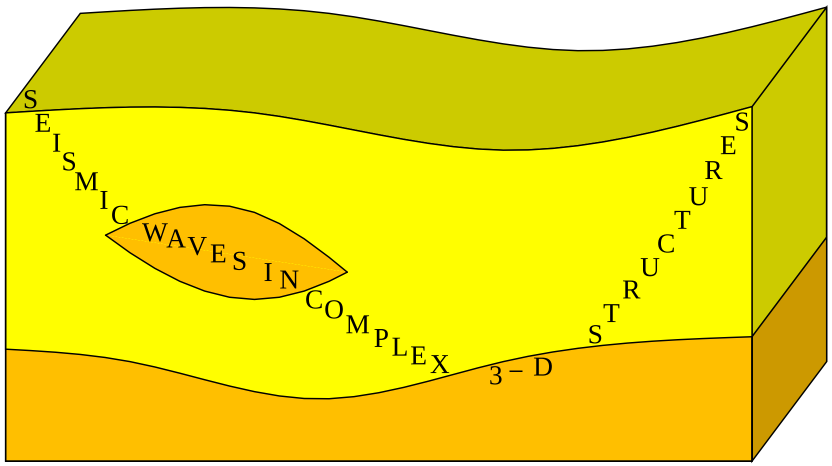

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