# P-wave VSP traveltime inversion in weakly and moderately anisotropic media

Bohuslav Růžek and Ivan Pšenčík

Institute of Geophysics, Acad. Sci. Praha, Czech Republic

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# Outline

Introduction

Equations for weak-anisotropy parameters

Tests

Conclusions

Future plans

# Introduction

stiffness tensor:

- $3 \times 3 \times 3 \times 3$ density-normalized stiffness tensor elements $a_{ijkl}$

Voigt notation:

- $6 \times 6$ matrix of density-normalized elastic moduli $A_{\alpha\beta}$

$$\begin{pmatrix} A11 & A12 & A13 & A14 & A15 & A16 \\ & A22 & A23 & A24 & A25 & A26 \\ & & A33 & A34 & A35 & A36 \\ & & & A44 & A45 & A46 \\ & & & & A55 & A56 \\ & & & & & A66 \end{pmatrix}$$

*Thomsen's (1986)* notation for VTI:

- 3 parameters $\epsilon$, $\delta$, $\gamma$, 2 vertical velocities $V_P$, $V_S$

# Introduction

Weak-anisotropy (WA) parameters: 21 WA parameters

$$\epsilon_x = \frac{A_{11}-\alpha_0^2}{2\alpha_0^2}, \quad \epsilon_y = \frac{A_{22}-\alpha_0^2}{2\alpha_0^2}, \quad \epsilon_z = \frac{A_{33}-\alpha_0^2}{2\alpha_0^2},$$

$$\delta_x = \frac{A_{23}+2A_{44}-\alpha_0^2}{\alpha_0^2}, \quad \delta_y = \frac{A_{13}+2A_{55}-\alpha_0^2}{\alpha_0^2}, \quad \delta_z = \frac{A_{12}+2A_{66}-\alpha_0^2}{\alpha_0^2},$$

$$\epsilon_{15} = \frac{A_{15}}{\alpha_0^2}, \quad \epsilon_{16} = \frac{A_{16}}{\alpha_0^2}, \quad \epsilon_{24} = \frac{A_{24}}{\alpha_0^2}, \quad \epsilon_{26} = \frac{A_{26}}{\alpha_0^2}, \quad \epsilon_{34} = \frac{A_{34}}{\alpha_0^2}, \quad \epsilon_{35} = \frac{A_{35}}{\alpha_0^2},$$

$$\chi_x = \frac{A_{14}+2A_{56}}{\alpha_0^2}, \quad \chi_y = \frac{A_{25}+2A_{46}}{\alpha_0^2}, \quad \chi_z = \frac{A_{36}+2A_{45}}{\alpha_0^2}.$$

$$\gamma_x = \frac{A_{44}-\beta_0^2}{2\beta_0^2}, \quad \gamma_y = \frac{A_{55}-\beta_0^2}{2\beta_0^2}, \quad \gamma_z = \frac{A_{66}-\beta_0^2}{2\beta_0^2}, \quad \epsilon_{46} = \frac{A_{46}}{\alpha_0^2}, \quad \epsilon_{56} = \frac{A_{56}}{\alpha_0^2}, \quad \epsilon_{45} = \frac{A_{45}}{\beta_0^2}$$

$\alpha_0$, $\beta_0$ - P- and S-wave reference velocities

# Introduction

Weak-anisotropy (WA) parameters: 21 WA parameters

- closely related to *Thomsen's (1986)* parameters
- an alternative to $A_{\alpha\beta}$
- linear relation of WA to $A_{\alpha\beta}$ parameters
- applicable to anisotropy of any type and strength
- all 21 WA parameters dimensionless and of comparable size
- describe exactly any wave attribute

# Introduction

Weak-anisotropy (WA) parameters: 21 WA parameters

- unique for all anisotropy symmetries
- natural combinations of $A_{\alpha\beta}$ taken into account
- freedom in the choice of reference velocity
- simple transformation from one coordinate system to another
- first-order P-wave attributes depend on only 15 WA parameters

# Introduction

Parameters of an **orthorhombic** medium in natural coordinates:

$$\begin{pmatrix} 9. & 3.6 & 2.25 & 0 & 0 & 0 \\ & 9.84 & 2.4 & 0 & 0 & 0 \\ & & 5.9375 & 0 & 0 & 0 \\ & & & 2. & 0 & 0 \\ & & & & 1.6 & 0 \\ & & & & & 2.182 \end{pmatrix}$$

Parameters of an **orthorhombic** medium - tilted ($\varphi = 10°$, $\theta = 40°$):
$\varphi$ - azimuth angle, $\theta$ - polar angle

$$\begin{pmatrix} 6.79139 & 3.16151 & 3.21889 & -0.12896 & -0.90788 & -0.10623 \\ & 9.68967 & 2.90586 & -0.13455 & -0.58655 & -0.42122 \\ & & 6.22391 & -0.10096 & -0.57255 & 0.05697 \\ & & & 2.09039 & 0.08614 & -0.09290 \\ & & & & 2.56375 & -0.04192 \\ & & & & & 2.16412 \end{pmatrix}$$

$\Rightarrow$ reason for considering general, triclinic symmetry

# Introduction

Two ways of the formulation of the inverse problem:

- inversion of the complete set of 15 WA parameters
  - successive search for possible symmetry elements
  (Helbig, 2013; Klimeš, 2016)

- inversion of the reduced (9 - MONO, 6 - ORT, 3 - TI)
  sets of WA parameters plus 2 (TI) or 3 Euler angles

# Equations for weak-anisotropy parameters

$$v^2(\epsilon_i, \mathbf{N}) = r^2/t^2$$

$t$ - “observed” traveltime

$r$ - source - receiver distance

$\mathbf{N}$ - ray vector (unit vector in the source-receiver direction)

$v^2(\epsilon_i, \mathbf{N})$ - first-order approximation of the square of ray velocity

$\epsilon_i$ - WA parameters

# Equations for weak-anisotropy parameters

**First-order expression for $v^2(\epsilon_i, \mathbf{N})$**

$$\alpha_0^2(1 + 2(\epsilon_x N_1^4 + \epsilon_y N_2^4 + \epsilon_z N_3^4 + \delta_x N_2^2 N_3^2 + \delta_y N_1^2 N_3^2 + \delta_z N_1^2 N_2^2)$$

$$+4[(\epsilon_{15} N_3 + \epsilon_{16} N_2) N_1^3 + (\epsilon_{24} N_3 + \epsilon_{26} N_1) N_2^3 + (\epsilon_{34} N_2 + \epsilon_{35} N_1) N_3^3$$

$$+(\chi_x N_1 + \chi_y n_2 + \chi_z N_3) N_1 N_2 N_3]) = v^2(\epsilon_i, \mathbf{N}) = r^2/t^2$$

$\alpha_0$ - P-wave isotropic reference velocity

$v^2(\epsilon_i, \mathbf{N})$ **independent** of $\alpha_0$!

# Equations for weak-anisotropy parameters

$$v^{-1}(\epsilon_i, \mathbf{N}) = t/r$$

$t$ - “observed” traveltime

$r$ - source - receiver distance

$\mathbf{N}$ - ray vector (unit vector in the source-receiver direction)

$v(\epsilon_i, \mathbf{N})$ - first-order approximation of ray velocity

$\epsilon_i$ - WA parameters

# Equations for weak-anisotropy parameters

**First-order expression for $v^{-1}(\epsilon_i, \mathbf{N})$**

$$\alpha_0^{-1}(1 - (\epsilon_x N_1^4 + \epsilon_y N_2^4 + \epsilon_z N_3^4 + \delta_x N_2^2 N_3^2 + \delta_y N_1^2 N_3^2 + \delta_z N_1^2 N_2^2)$$

$$-2[(\epsilon_{15} N_3 + \epsilon_{16} N_2) N_1^3 + (\epsilon_{24} N_3 + \epsilon_{26} N_1) N_2^3 + (\epsilon_{34} N_2 + \epsilon_{35} N_1) N_3^3$$

$$+(\chi_x N_1 + \chi_y n_2 + \chi_z N_3) N_1 N_2 N_3]) = v^{-1}(\epsilon_i, \mathbf{N}) = t/r$$

$\alpha_0$ - P-wave isotropic reference velocity

$v^{-1}(\epsilon_i, \mathbf{N})$ **depends** on the choice of $\alpha_0$!

# Equations for weak-anisotropy parameters

$v^2(\epsilon_i, \mathbf{N})$ $\quad \mathbf{Gm} = \mathbf{d}$ $\quad$ $\mathbf{G}$ - $N \times M$ matrix

$N$ - number of observations $\quad$ $M$ - number of sought parameters (15)

Row of matrix $\mathbf{G}$:

$$(N_1^4 \quad N_2^4 \quad N_3^4 \quad N_2^2 N_3^2 \quad N_1^2 N_3^2 \quad N_1^2 N_2^2 \quad 2N_1^3 N_3 \quad 2N_1^3 N_2 \quad 2N_2^3 N_3$$
$$2N_2^2 N_1 \quad 2N_3^3 N_2 \quad 2N_3^3 N_1 \quad 2N_1^2 N_2 N_3 \quad 2N_1 N_2^2 N_3 \quad 2N_1 N_2 N_3^2)$$

Vector of model parameters $\mathbf{m}$:

$$\mathbf{m} \equiv (\epsilon_x, \epsilon_y, \epsilon_z, \delta_x, \delta_y, \delta_z, \epsilon_{15}, \epsilon_{16}, \epsilon_{24}, \epsilon_{26}, \epsilon_{34}, \epsilon_{35}, \chi_x, \chi_y, \chi_z)^T$$

Data vector $\mathbf{d}$:

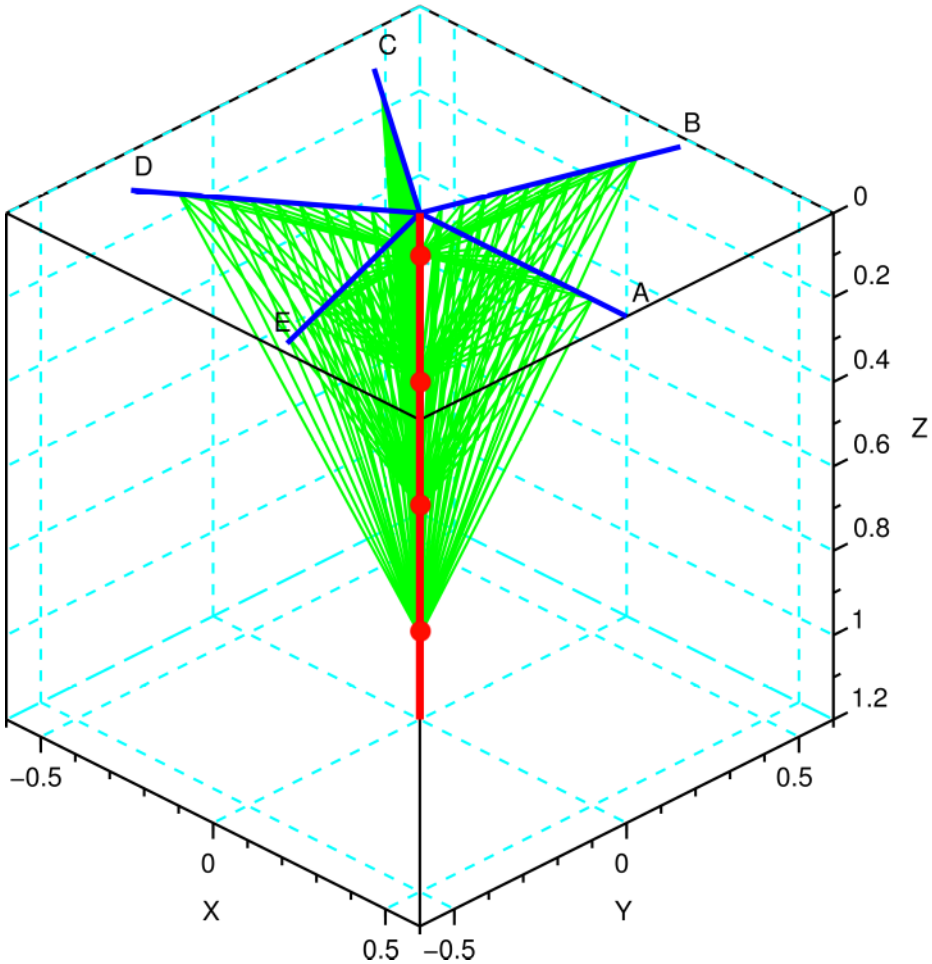
$$\mathbf{d} \equiv (\tfrac{1}{2}(\tfrac{r_1^2}{\alpha_0^2 t_1^2} - 1),\ \tfrac{1}{2}(\tfrac{r_2^2}{\alpha_0^2 t_2^2} - 1),\ ....,\ \tfrac{1}{2}(\tfrac{r_N^2}{\alpha_0^2 t_N^2} - 1))^T$$

# Numerical Tests

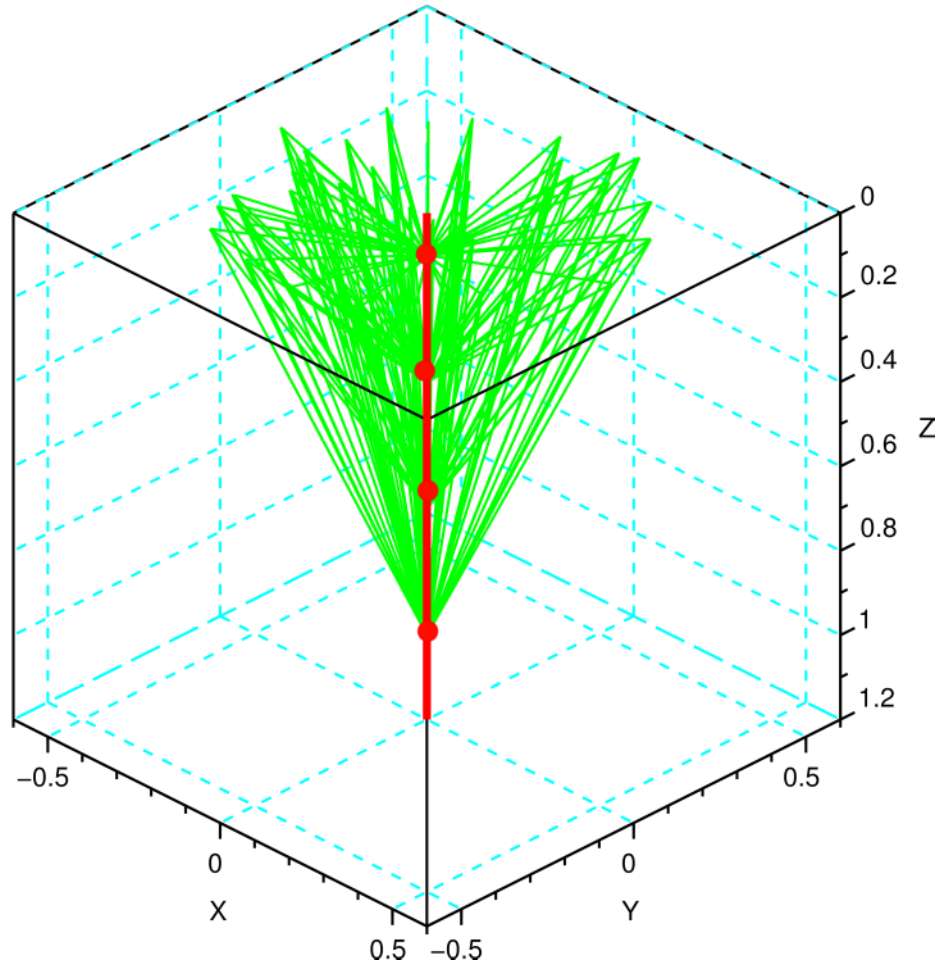
- Measurement setups
- Velocity models
- Inverted traveltimes
- Inversion methodology
- List of experiments
- Inversion results

# Measurement setups

Profile distribution of surface sources




Random distribution of surface sources

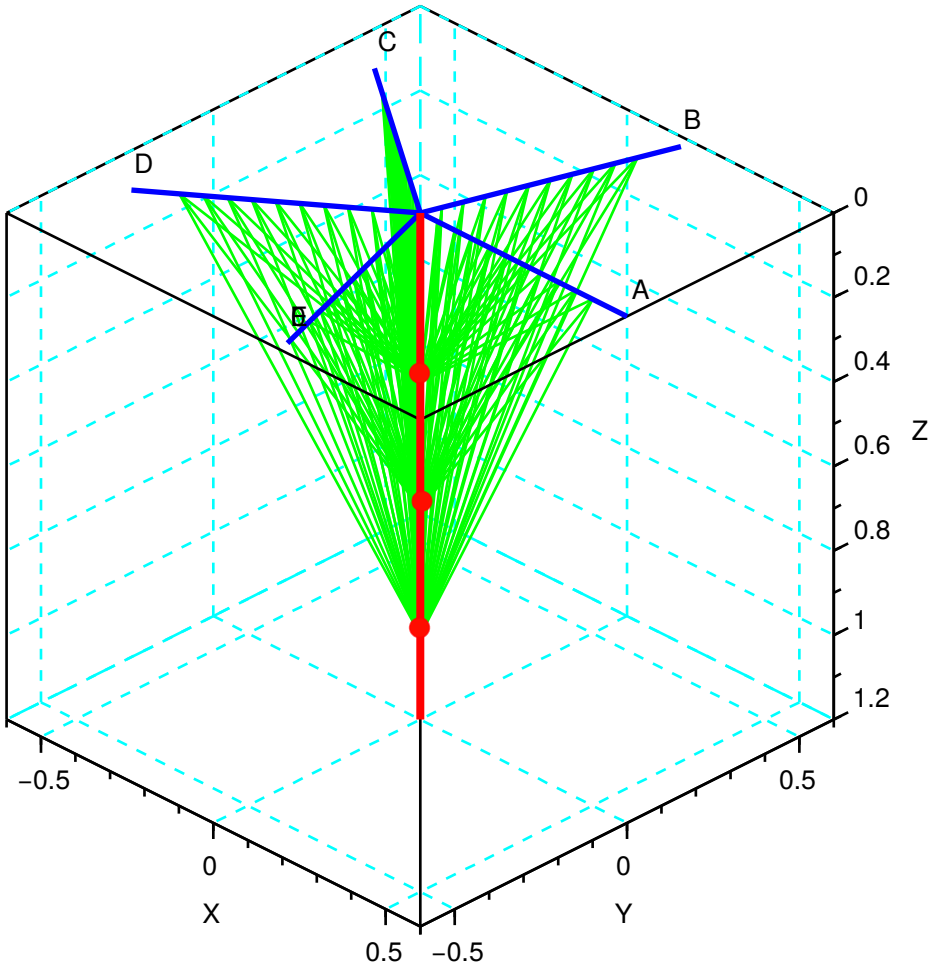



5 profiles, 10 sources/profile, 4 receivers

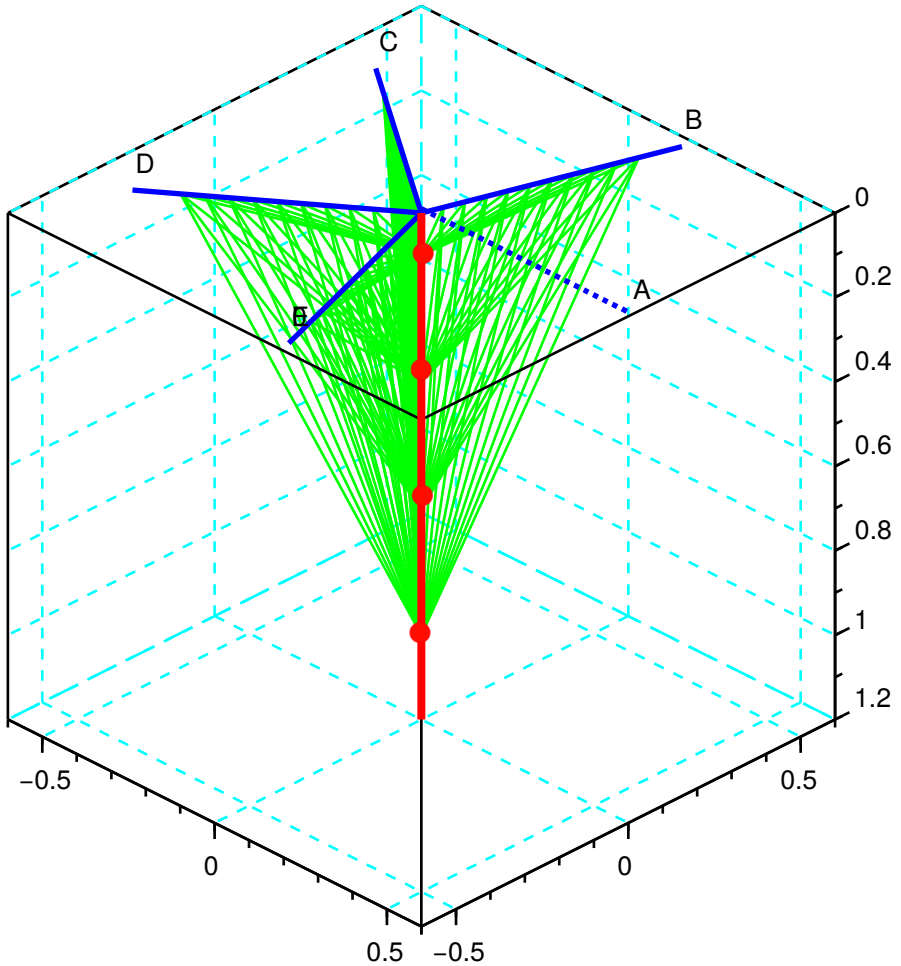
no profiles, 50 sources, 4 receivers

# Measurement setups

Profile distribution of surface sources



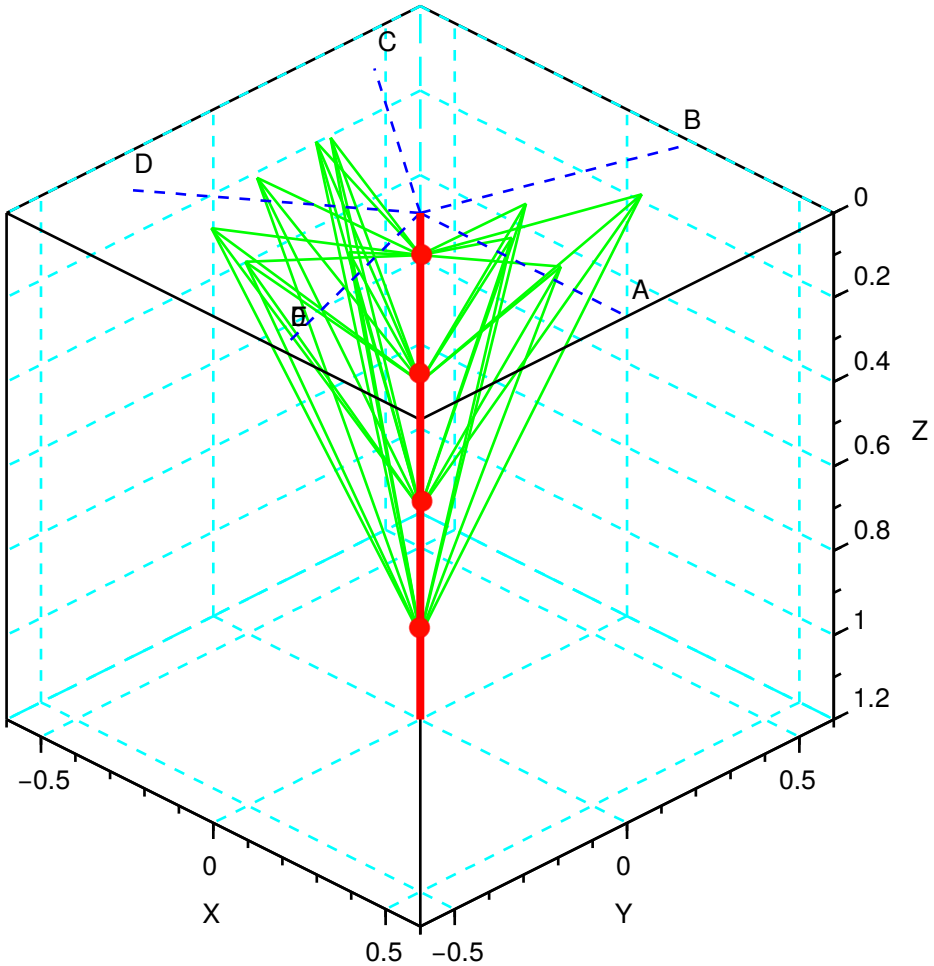

5 → 4 receivers



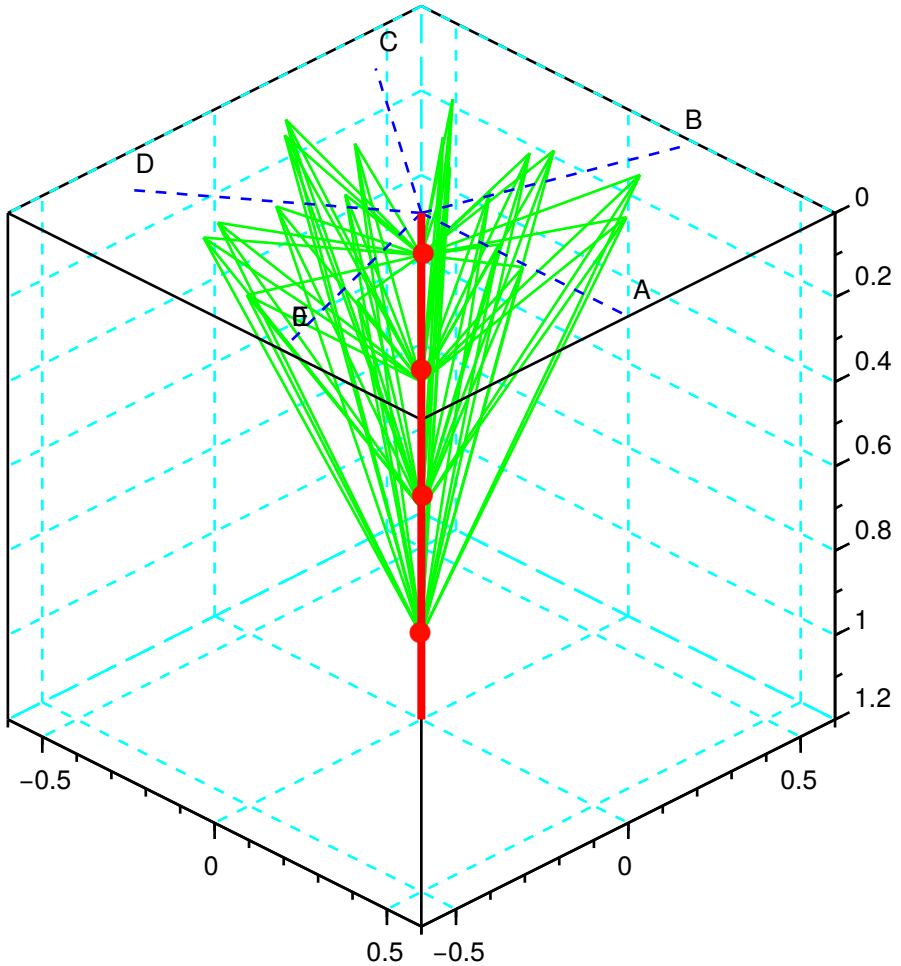

5 → 4 profiles

# Measurement setups

Random distribution of surface sources




50 → 10 sources

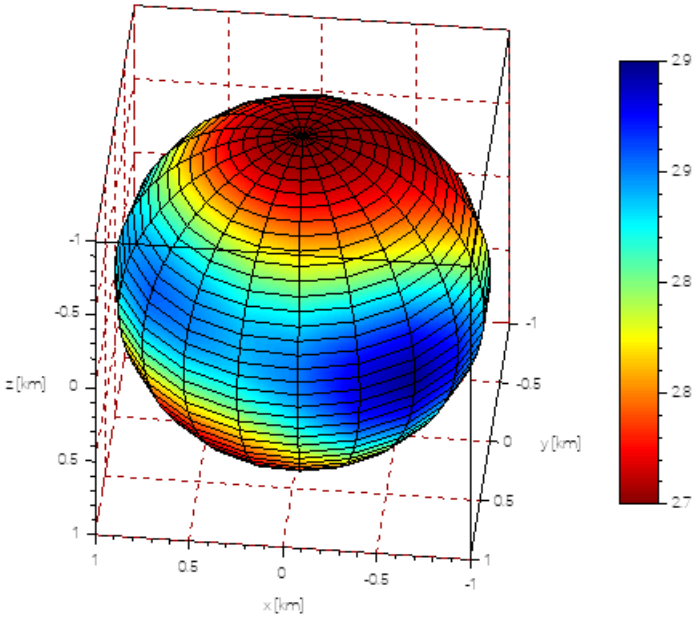



50 → 20 sources

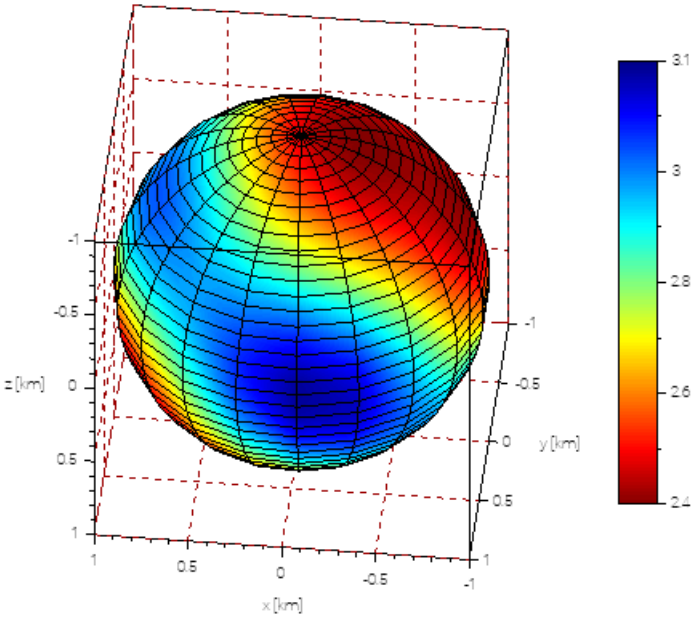
# Velocity models

Tested anisotropic orthorhombic velocity models of inclined symmetry planes

a) “Weak”, anisotropy 6%

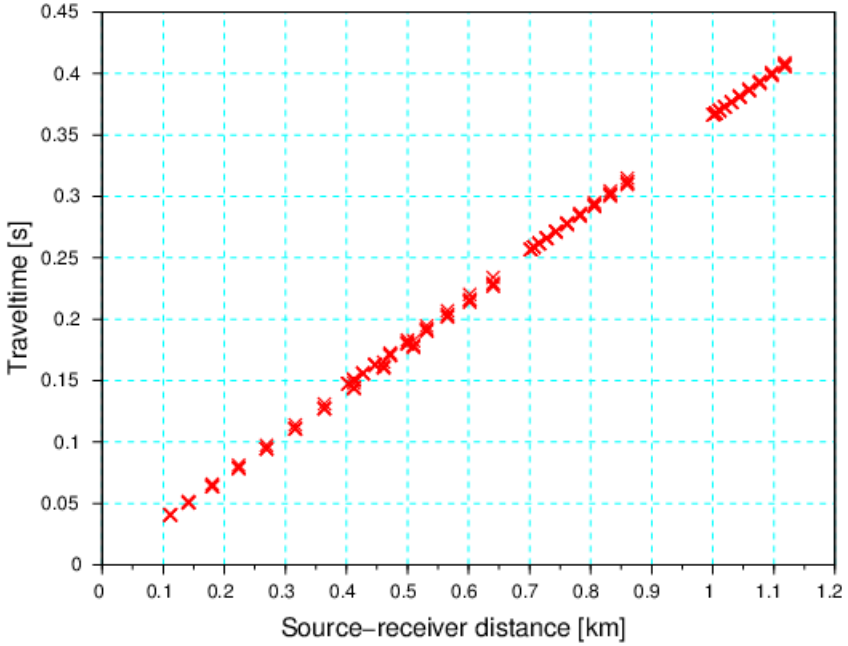


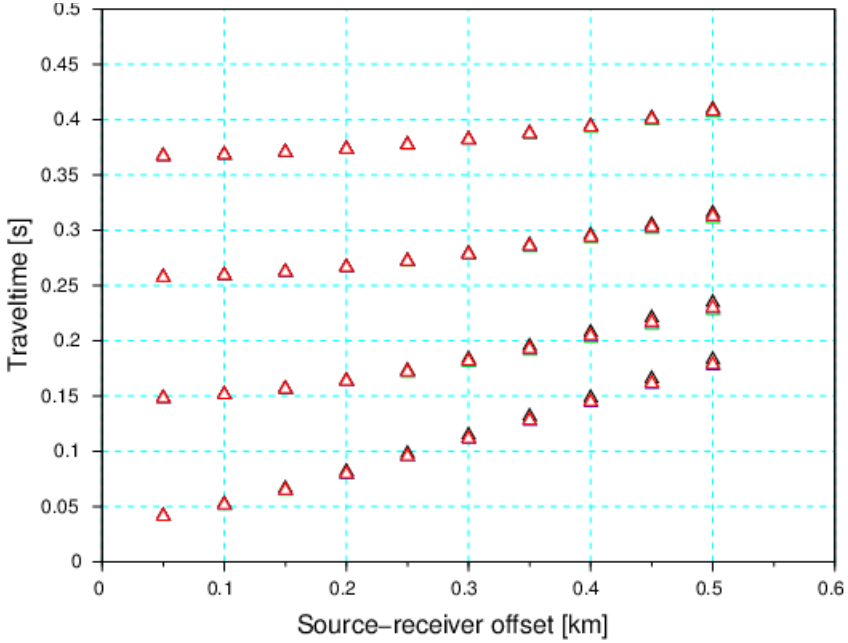

b) “Strong”, anisotropy 25%



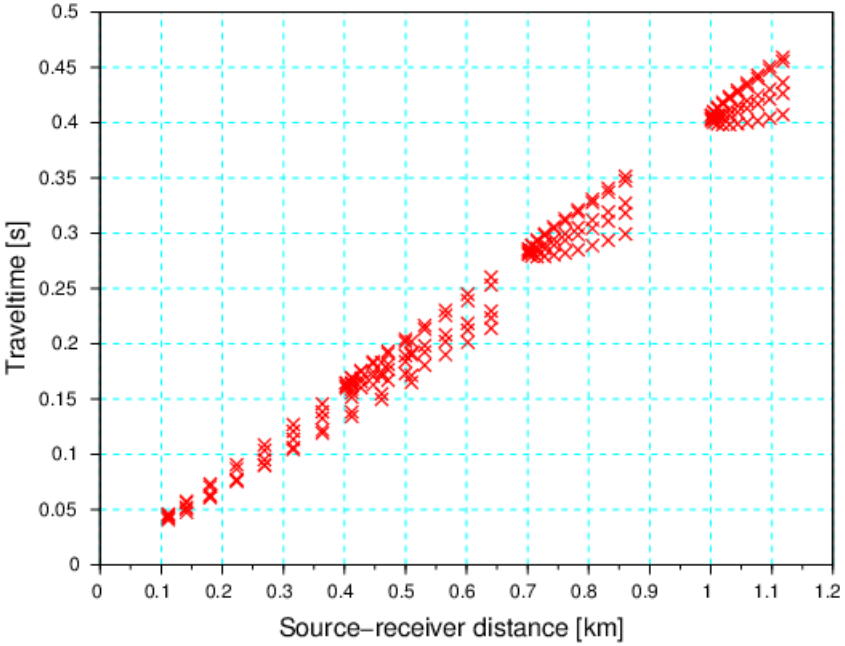
# Traveltimes (profile setup)

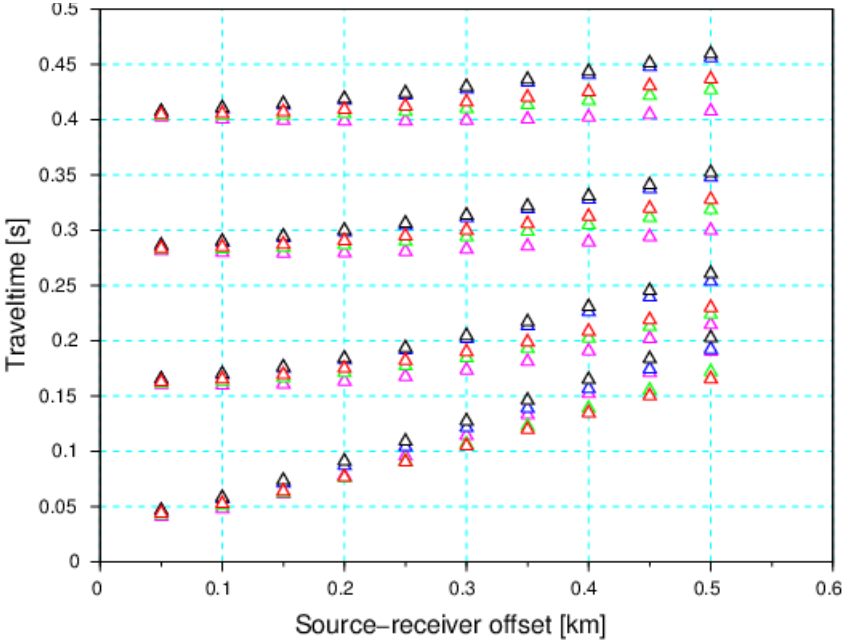
## model “Weak”



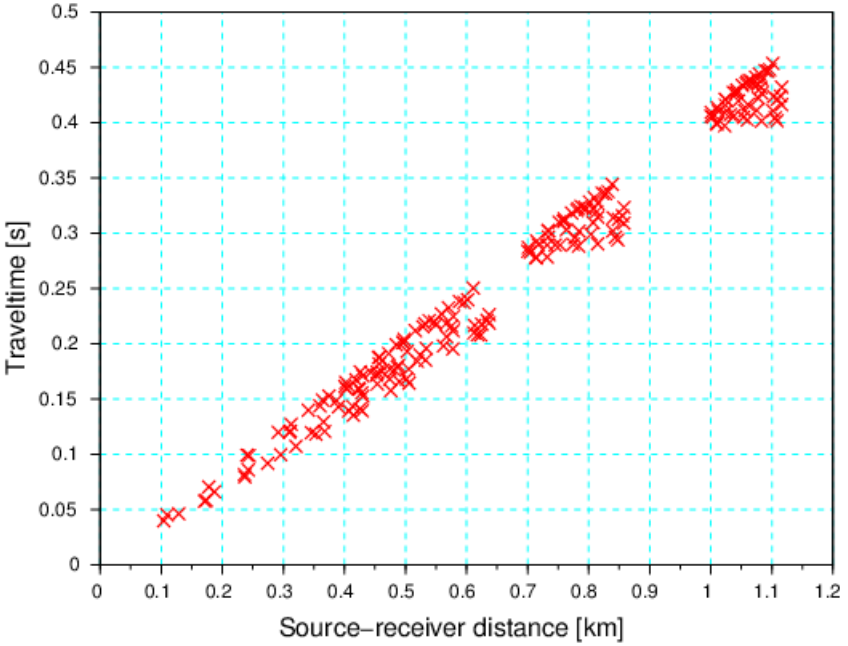

## model “Strong”

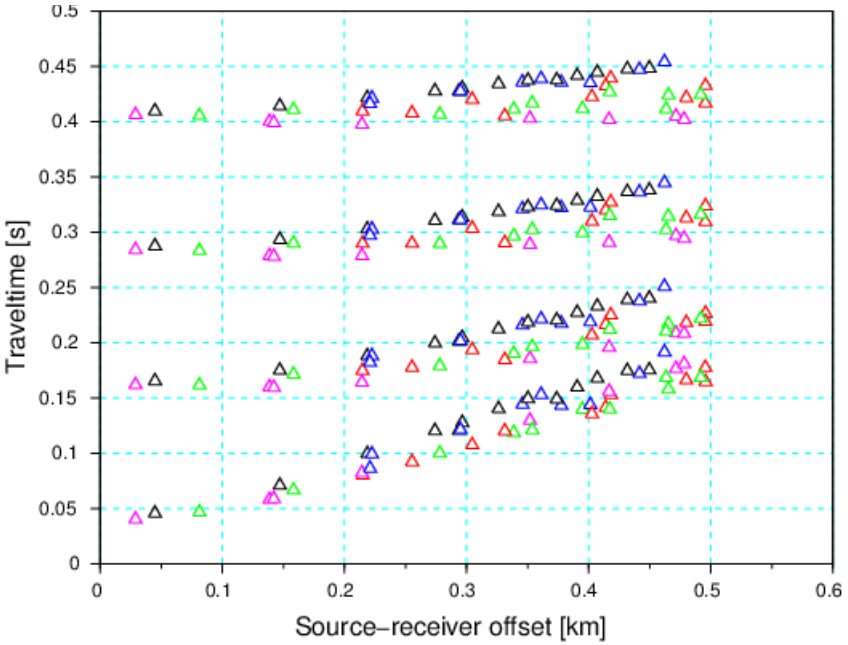


# Traveltimes (random setup)

all traveltimes calculated by
- ANRAY software or,
- solving Christoffel equation



# Inversion methodology

Taking advantages of linear algebra:

Solution search for by pseudoinverse

$$\boldsymbol{G}\boldsymbol{m}=\boldsymbol{d}$$
$$\boldsymbol{m}=\boldsymbol{G}^{\dagger}(\boldsymbol{d}+\boldsymbol{err})$$
$$dim(\boldsymbol{d})=200, dim(\boldsymbol{m})=15$$

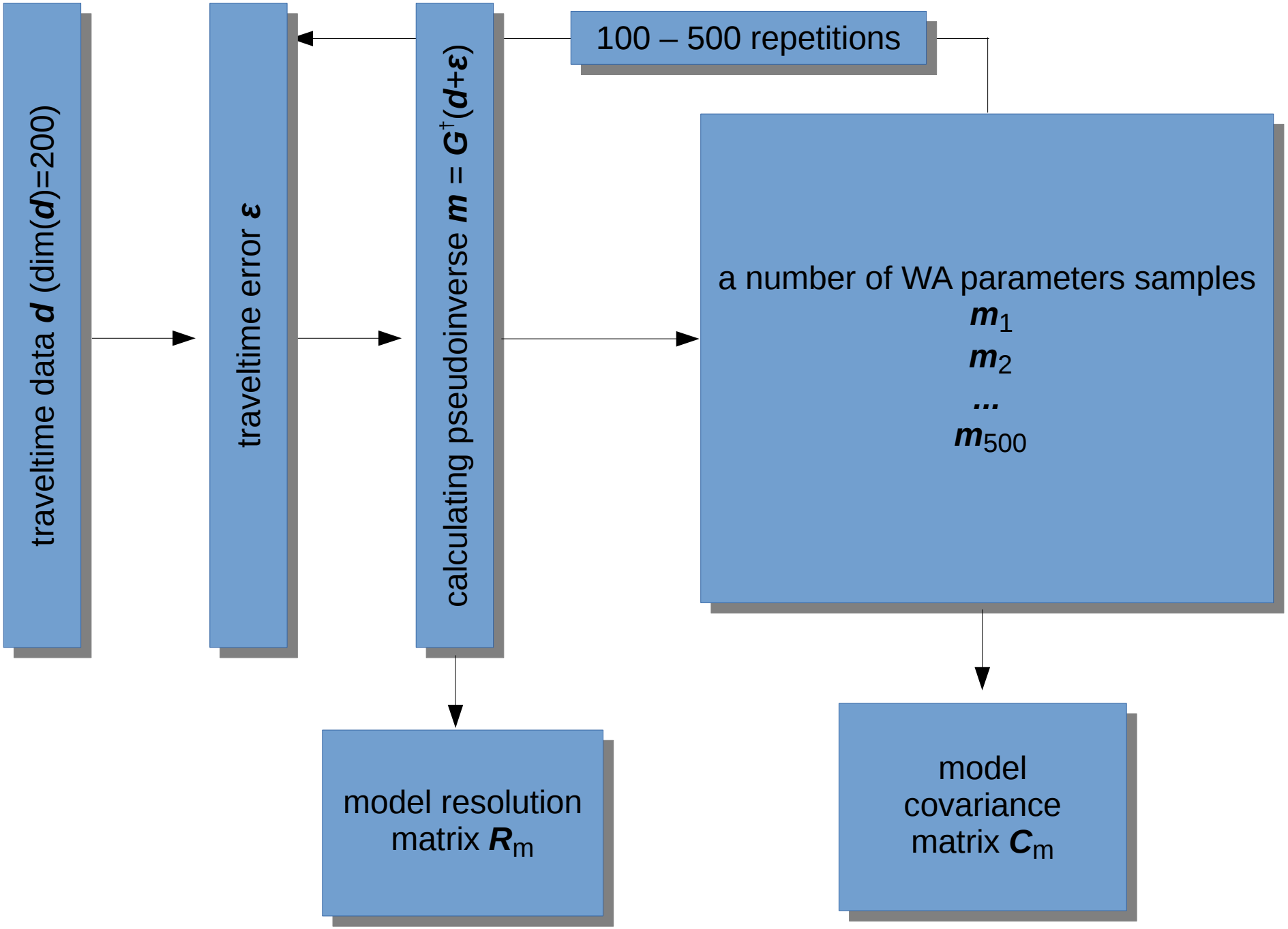
Resolution (***R***) matrix

$$\boldsymbol{G}\,\boldsymbol{m}^{true}=\boldsymbol{d}^{true}$$
$$\boldsymbol{m}^{est}=\boldsymbol{G}^{\dagger}\,\boldsymbol{d}^{true}=\boldsymbol{G}^{\dagger}\,\boldsymbol{G}\,\boldsymbol{m}^{true}$$
$$\boldsymbol{R}=\boldsymbol{G}^{\dagger}\,\boldsymbol{G}$$
$$\boldsymbol{m}^{est}=\boldsymbol{R}\,\boldsymbol{m}^{true}$$
$$\boldsymbol{R}\rightarrow\boldsymbol{I}$$

Covariance (***C***$_{\mathbf{m}}$, ***C***$_{\mathbf{d}}$) matrices

$$\boldsymbol{G}\,\boldsymbol{m}=\boldsymbol{d}$$
$$\boldsymbol{G}\,\boldsymbol{C}_m\,\boldsymbol{G}^T=\boldsymbol{C}_d$$
$$\boldsymbol{H}\,\boldsymbol{C}_d\,\boldsymbol{H}^T=\boldsymbol{C}_m, \boldsymbol{H}=\boldsymbol{G}^{\dagger}$$

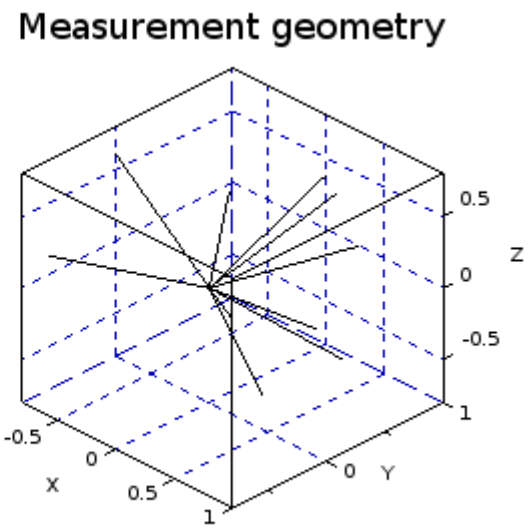
# Inversion methodology

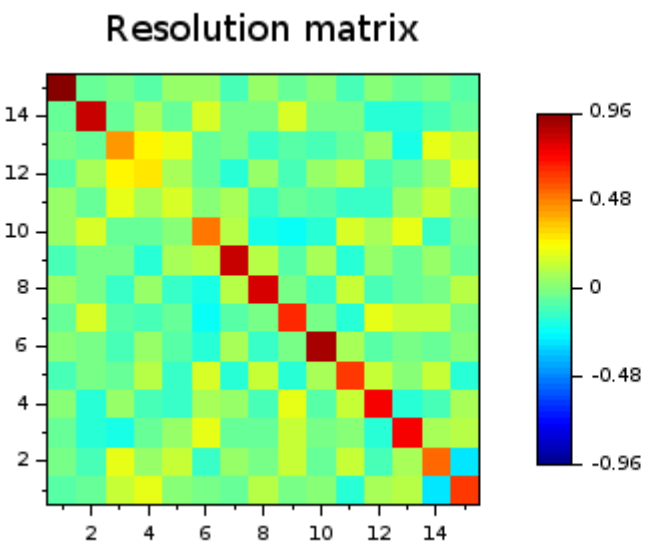


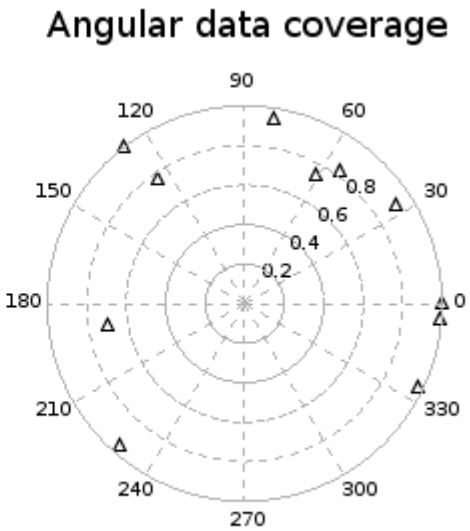
# Discussion of the resolution matrix - search for minimum configuration

Rank = 10
Nray = 10

10 randomly oriented rays



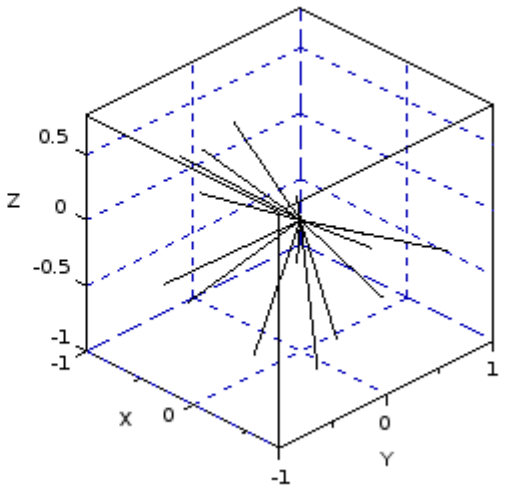



# Discussion of the resolution matrix - search for minimum configuration

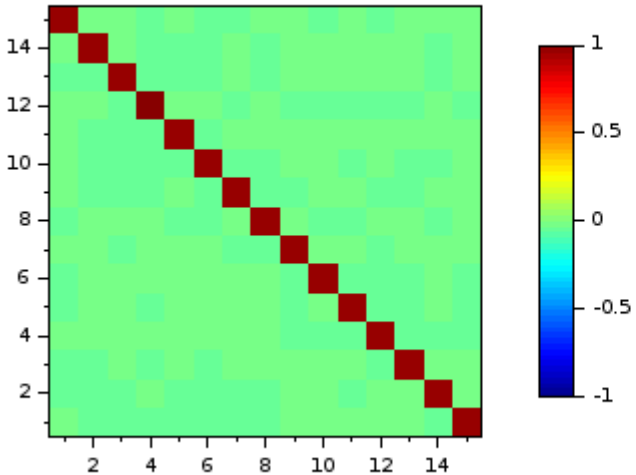
Rank = 15
Nray = 15

<u>15 randomly oriented rays</u>

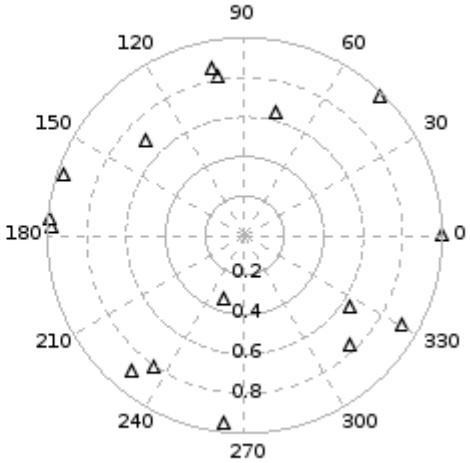
Measurement geometry




Resolution matrix



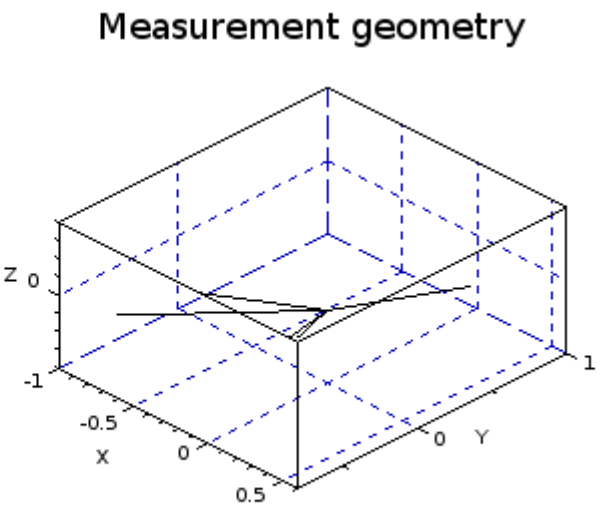

Angular data coverage

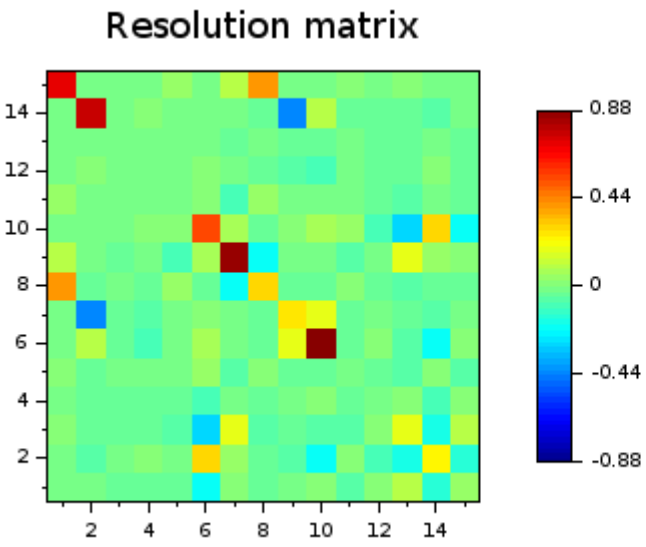


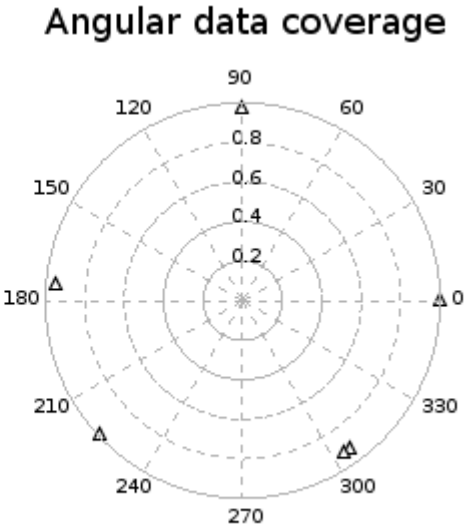
# Discussion of the resolution matrix - search for minimum configuration

Rank = 5
Nray = 5

5 rays in 1 plane



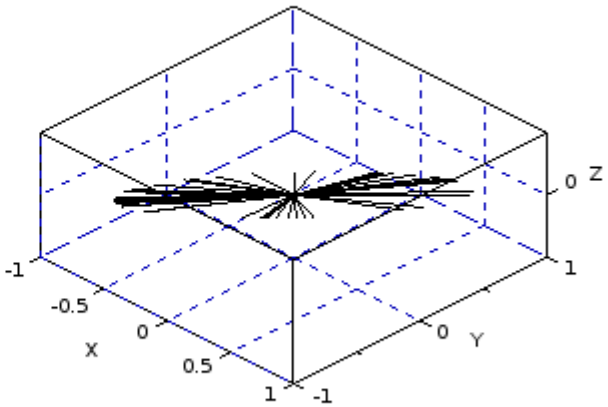



# Discussion of the resolution matrix - search for minimum configuration

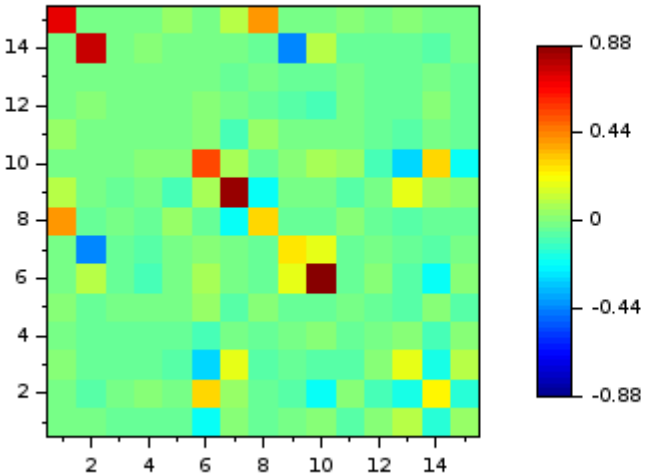
Rank = 5
Nray = 50

50 rays in 1 plane

Measurement geometry




Resolution matrix



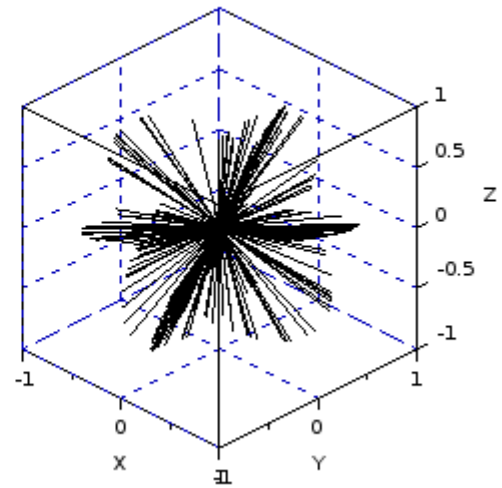

Angular data coverage

# Discussion of the resolution matrix - search for minimum configuration

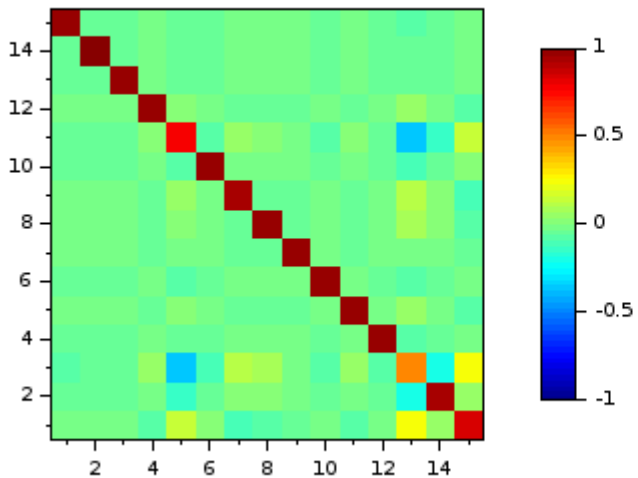
Rank = 14
Nray = 200

50+50+50+50 rays in 4 planes

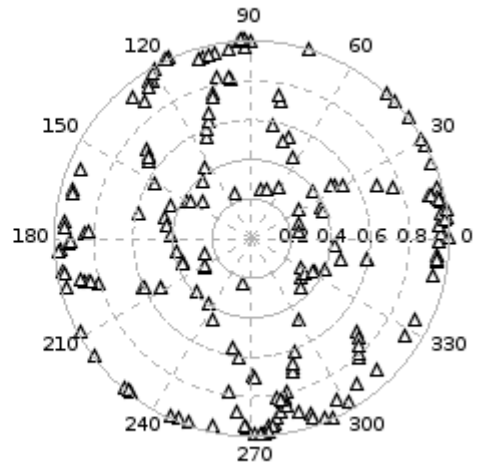
Measurement geometry




Resolution matrix




Angular data coverage

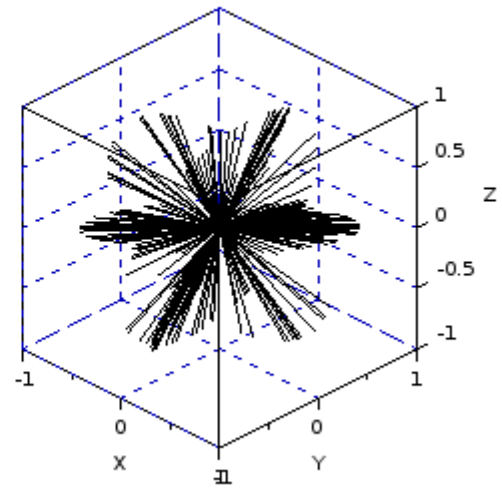


# Discussion of the resolution matrix - search for minimum configuration

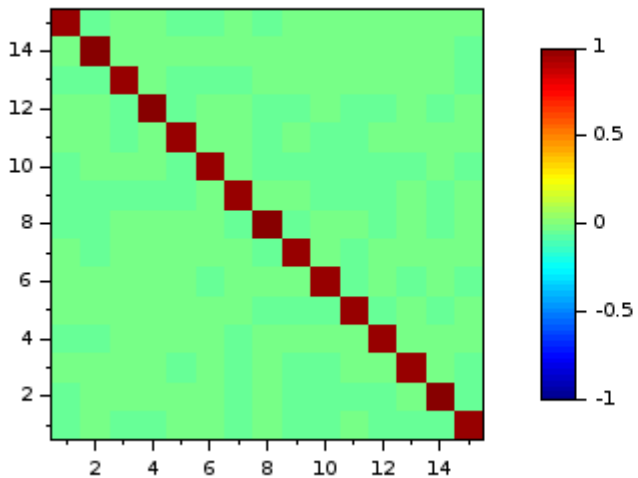
Rank = 15
Nray = 250

<u>5*50 rays in 5 planes</u>

Measurement geometry




Resolution matrix



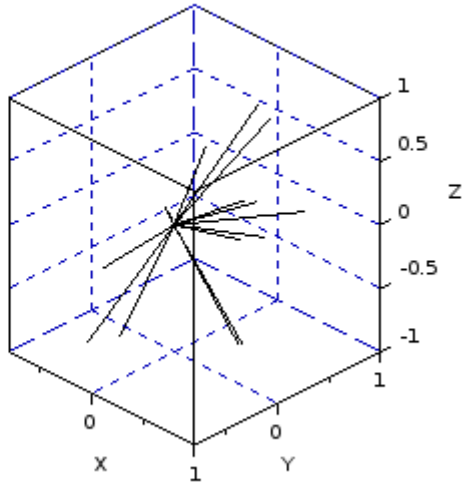

Angular data coverage

# Discussion of the resolution matrix - search for minimum configuration

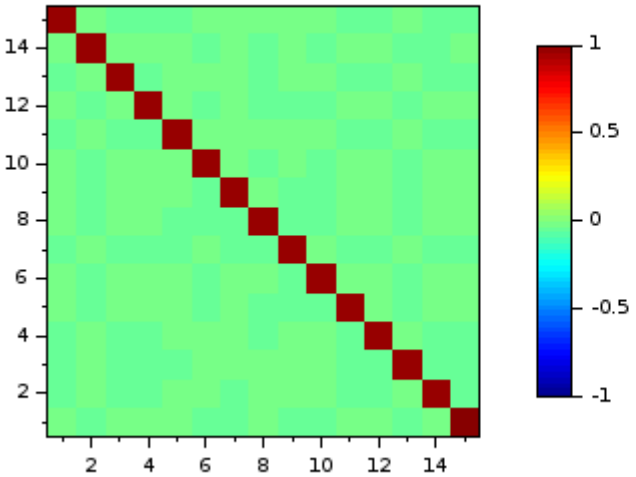
Rank = 15
Nray = 15

<u>5*3 rays in 5 planes</u>

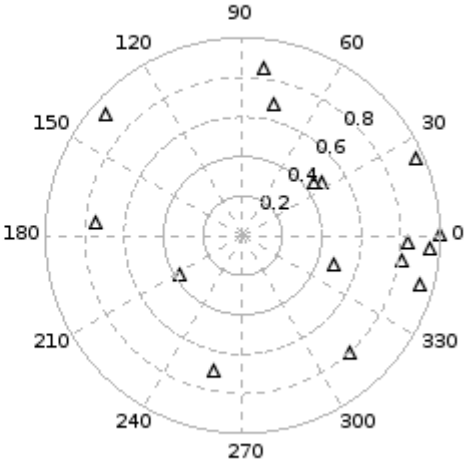
Measurement geometry




Resolution matrix




Angular data coverage



# Discussion of the resolution matrix - search for minimum configuration

- Minimum number of rays = 15
- More than 5 rays in a plane cause linear dependence

<u>Sources distributed along profiles</u>

- 15 rays
- 5 planes configurations
  - 3+3+3+3+3
  - 5+4+3+2+1
  - ~~6+3+3+2+1~~

<u>Randomly distributed sources</u>

- 15 randomly oriented rays

# Discussion of the model covariance matrix - testing the inversion accuracy

$$\boldsymbol{d} = \boldsymbol{G}\,\boldsymbol{m}$$

$$\boldsymbol{d} \pm \boldsymbol{C}_{\boldsymbol{d}}, \boldsymbol{m} \pm \boldsymbol{C}_{\boldsymbol{m}}$$

$$\boldsymbol{C}_{\boldsymbol{d}} = \boldsymbol{G}\,\boldsymbol{C}_{\boldsymbol{m}}\,\boldsymbol{G}^{\boldsymbol{T}}$$

$$\boldsymbol{H} = \boldsymbol{G}^{\dagger}$$

$$\boldsymbol{C}_{\boldsymbol{m}} = \boldsymbol{H}\,\boldsymbol{C}_{\boldsymbol{d}}\,\boldsymbol{H}^{\boldsymbol{T}}$$

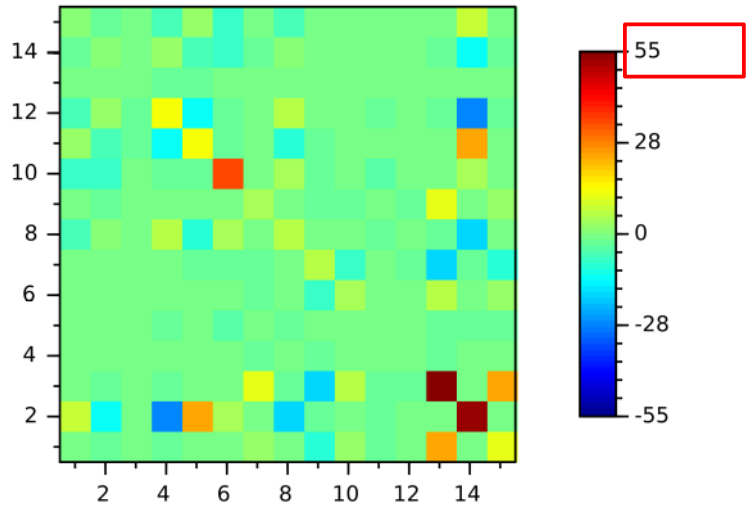
$$\boldsymbol{C}_{\boldsymbol{d}} = \boldsymbol{I} \rightarrow \boldsymbol{C}_{\boldsymbol{m}} = \boldsymbol{H}\,\boldsymbol{H}^{\boldsymbol{T}}$$

# Discussion of the model covariance matrix - testing the inversion accuracy

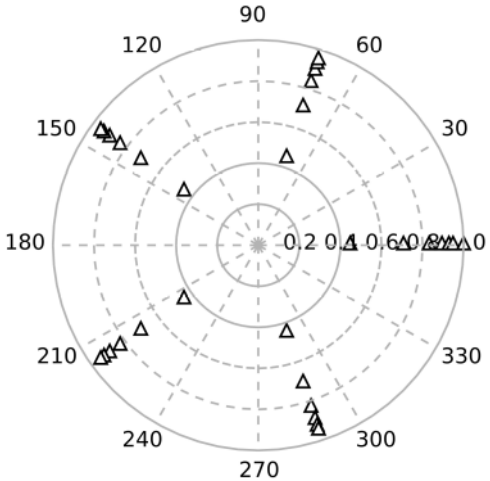
## 30 rays / 5 profiles

$$m = \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \delta_x \\ \delta_y \\ \delta_z \\ \epsilon_{16} \\ \epsilon_{15} \\ \epsilon_{24} \\ \epsilon_{26} \\ \epsilon_{35} \\ \epsilon_{34} \\ \chi_x \\ \chi_y \\ \chi_z \end{bmatrix}$$

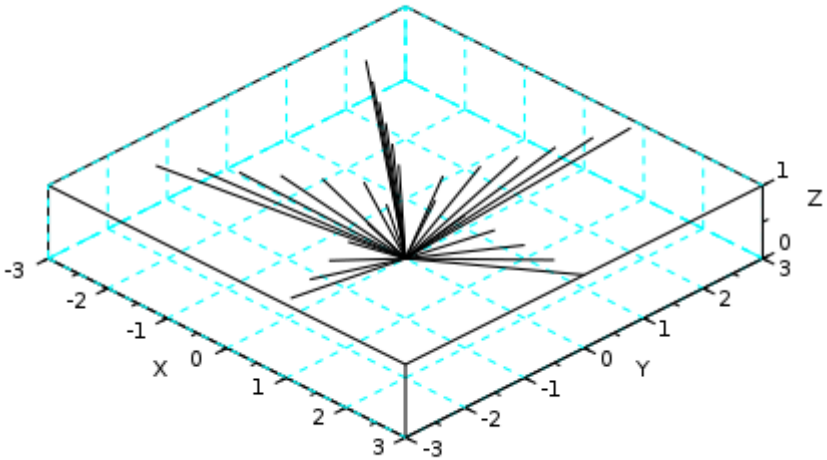
Normalized covariance matrix




Angular data coverage




Measurement geometry



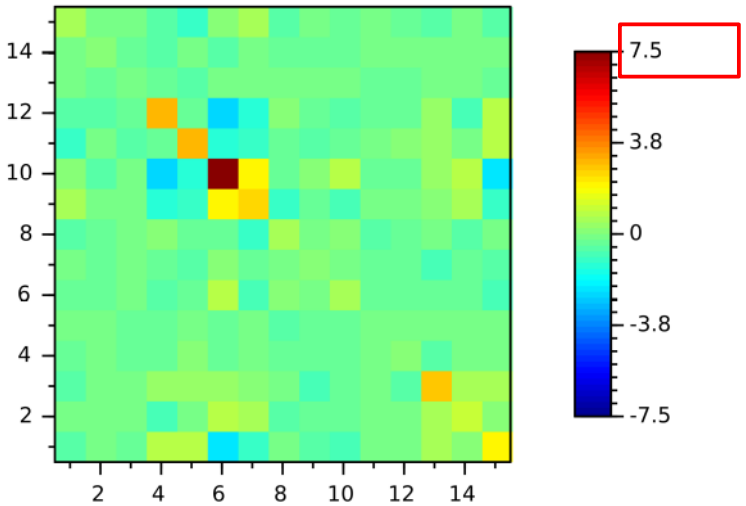

RMS(diag(Cm))= 3.855

# Discussion of the model covariance matrix - testing the inversion accuracy

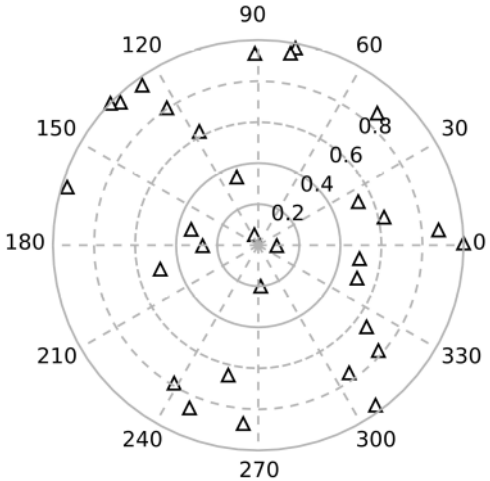
## 30 randomly oriented rays

$$m = \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \delta_x \\ \delta_y \\ \delta_z \\ \epsilon_{16} \\ \epsilon_{15} \\ \epsilon_{24} \\ \epsilon_{26} \\ \epsilon_{35} \\ \epsilon_{34} \\ \chi_x \\ \chi_y \\ \chi_z \end{bmatrix}$$

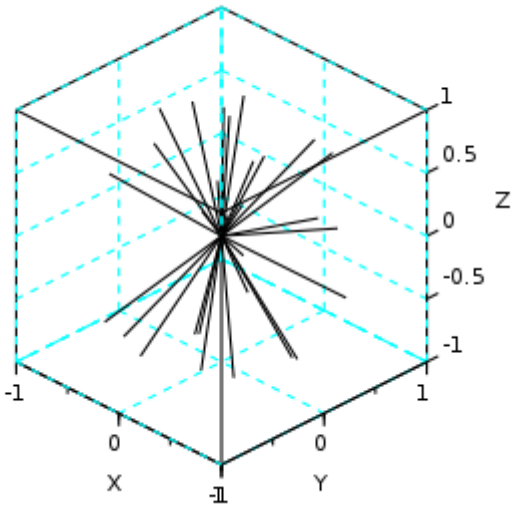
Normalized covariance matrix




Angular data coverage




Measurement geometry




```
RMS(diag(Cm))= 1.139
```

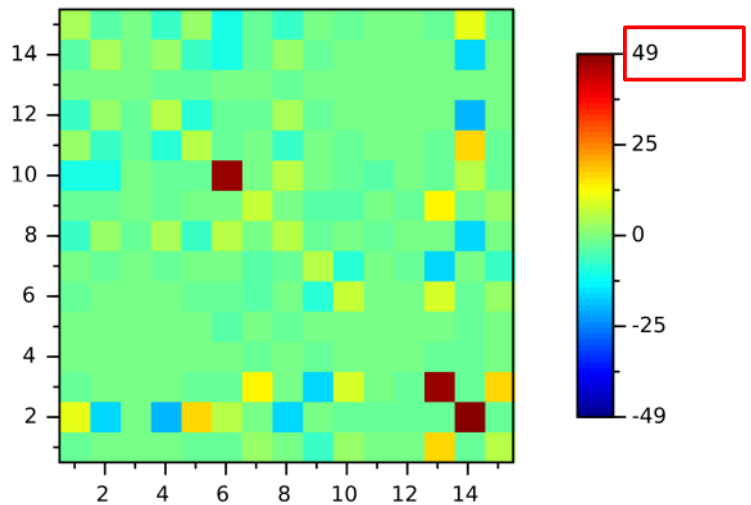
*RMS(diag(Cm))= 3.855*

# Discussion of the model covariance matrix - testing the inversion accuracy

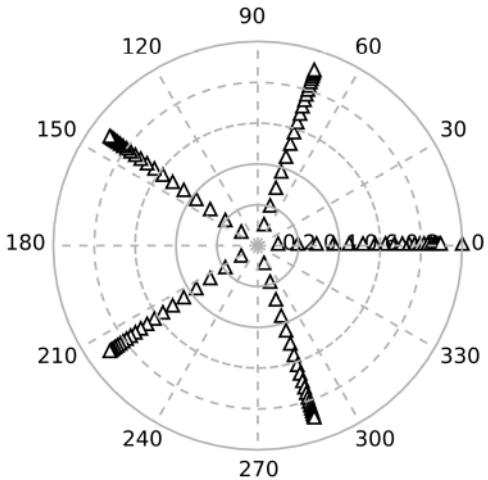
## 100 rays / 5 profiles

$$m = \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \delta_x \\ \delta_y \\ \delta_z \\ \epsilon_{16} \\ \epsilon_{15} \\ \epsilon_{24} \\ \epsilon_{26} \\ \epsilon_{35} \\ \epsilon_{34} \\ \chi_x \\ \chi_y \\ \chi_z \end{bmatrix}$$

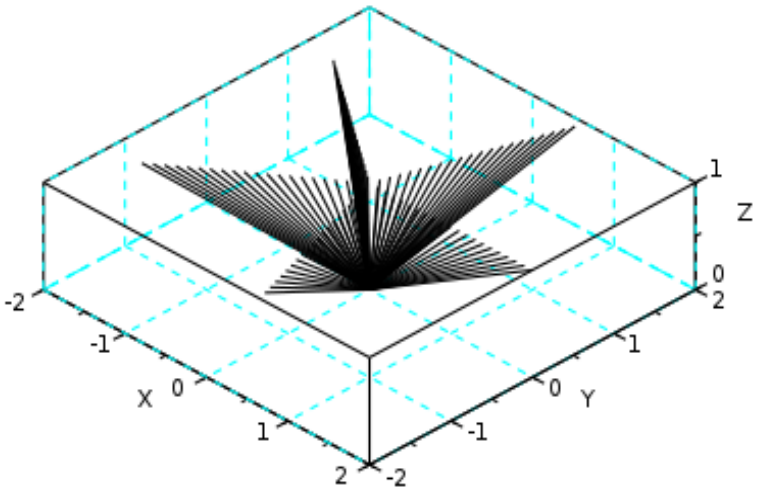
Normalized covariance matrix




Angular data coverage




Measurement geometry



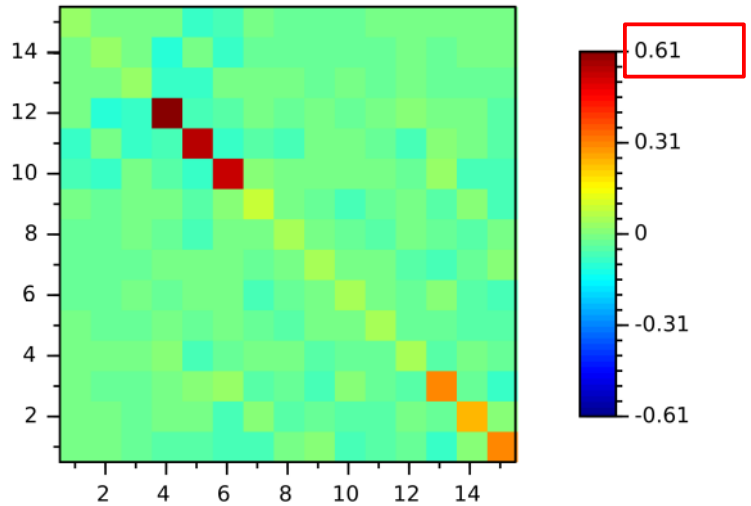

RMS(diag(Cm))= 3.735

# Discussion of the model covariance matrix - testing the inversion accuracy

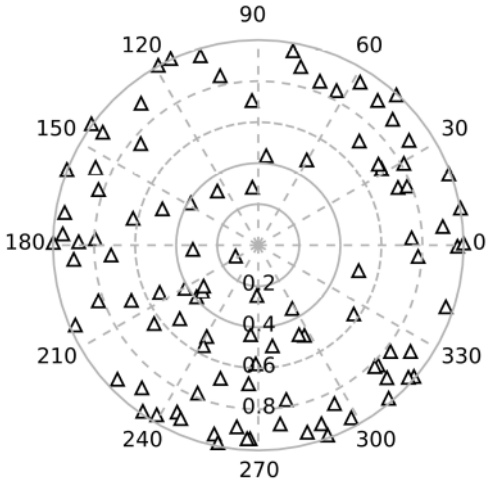
## 100 randomly oriented rays

$$m = \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \delta_x \\ \delta_y \\ \delta_z \\ \epsilon_{16} \\ \epsilon_{15} \\ \epsilon_{24} \\ \epsilon_{26} \\ \epsilon_{35} \\ \epsilon_{34} \\ \chi_x \\ \chi_y \\ \chi_z \end{bmatrix}$$

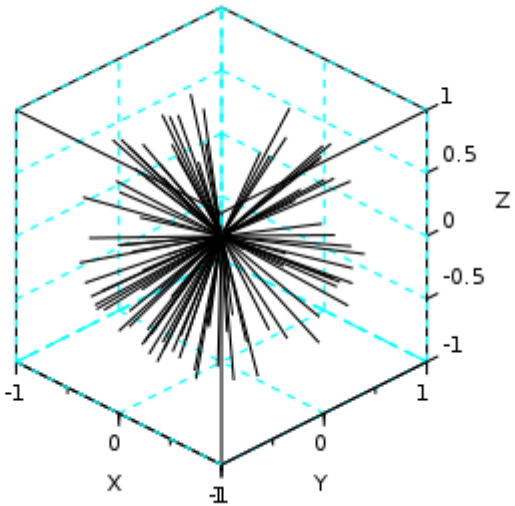
Normalized covariance matrix




Angular data coverage




Measurement geometry




**RMS(diag(Cm)) = 0.468**

*RMS(diag(Cm)) = 3.735*

# Discussion of the model covariance matrix - testing the inversion accuracy

Random directional distribution of rays outperforms profile-aligned measurements

# Inversion results

2 measurement setups

2 velocity models

2 velocity formulas

5 levels of noise

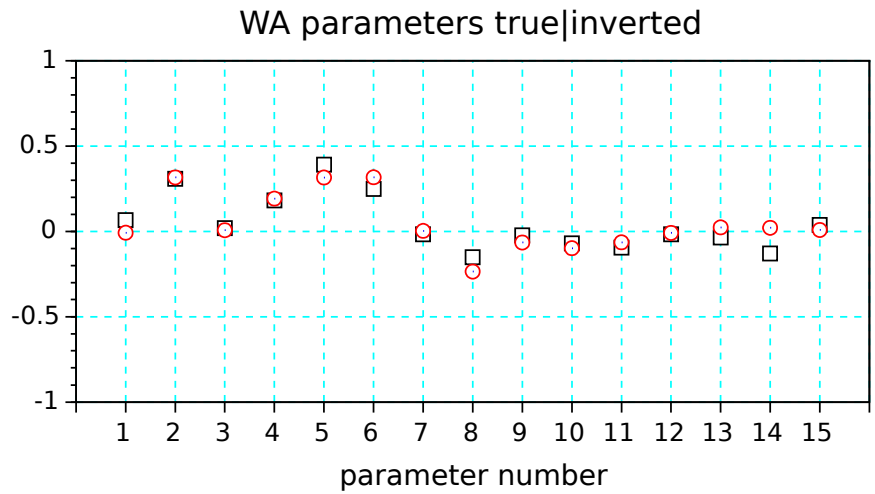
40 < number of inverse problems to discuss

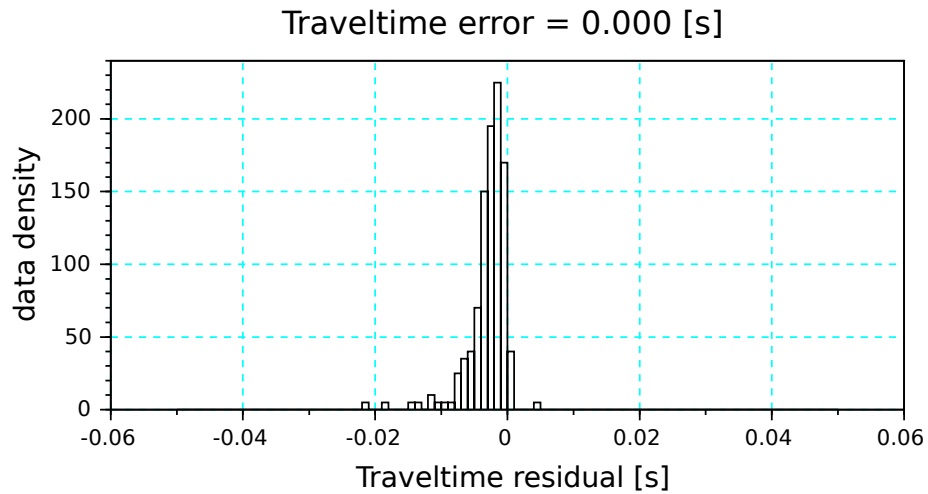
| | measurement setup | velocity model | forward formula | noise [ms] |
|---|---|---|---|---|
| 1,2,3 | profile | strong | $v^2$ | 0,5,10 |
| 4 | random | strong | $v^2$ | 10 |
| 5 | random | strong | $v^{-1}$ | 10 |
| 6,7 | profile | strong | $v^{-1}$, $\sim\alpha_0$ | 5 |
| 8 | profile/3r | strong | $v^2$ | 5 |
| 9 | rand/3r | strong | $v^2$ | 5 |

**Note**: *in the following slides, WA parameters are numbered according this table*

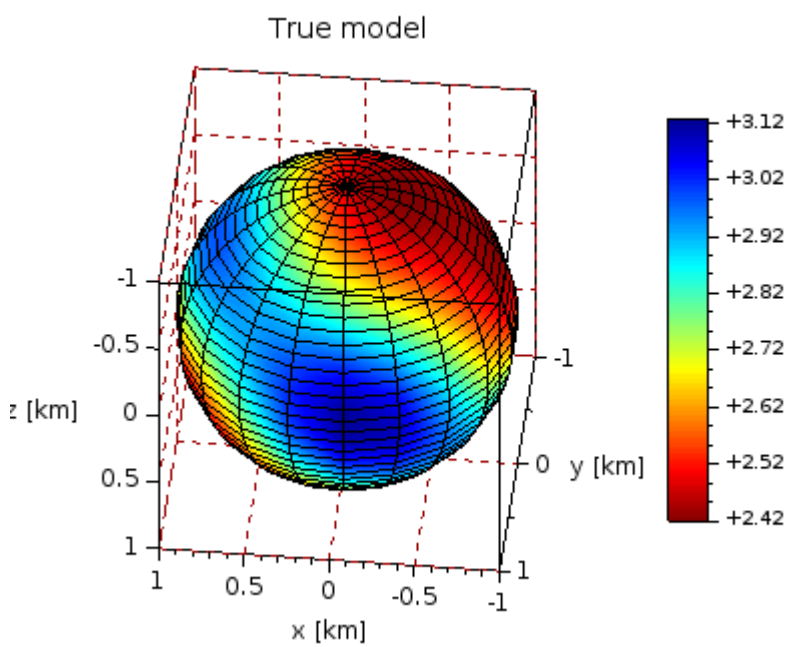
$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ \epsilon_x & \epsilon_y & \epsilon_z & \delta_x & \delta_y & \delta_z & \epsilon_{16} & \epsilon_{15} & \epsilon_{24} & \epsilon_{26} & \epsilon_{35} & \epsilon_{34} & \chi_x & \chi_y & \chi_z \end{bmatrix}$$

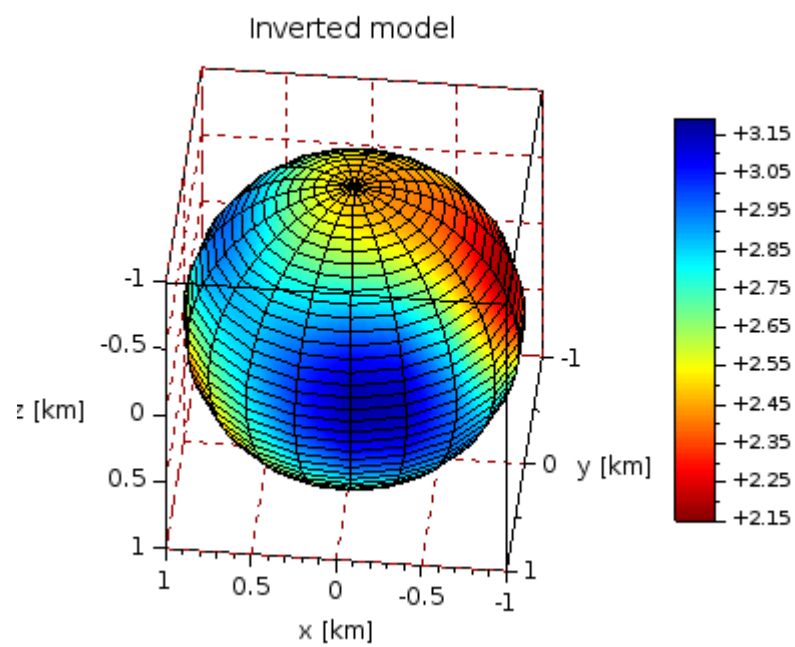
# Inversion results



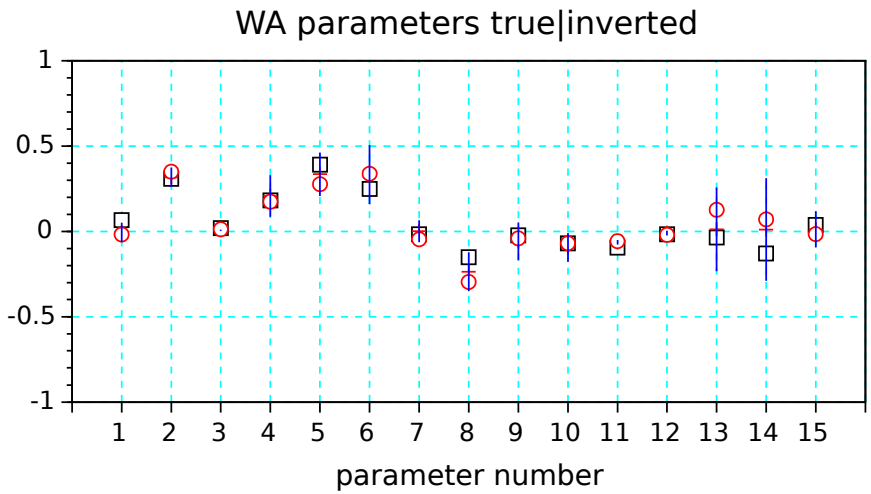



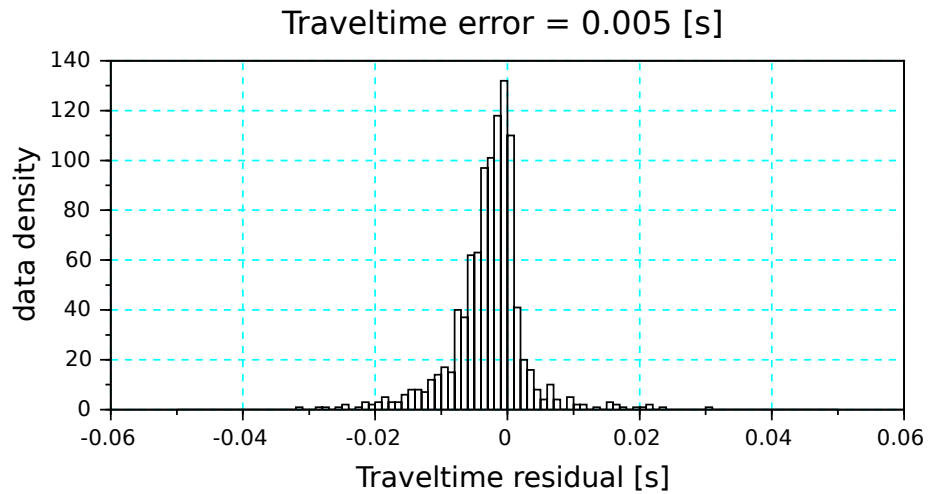

| Model | Config | Formula | Noise [ms] | Δm | Δv [%] |
|---|---|---|---|---|---|
| Strong | Profile | $v^2$ | 0 | **0.06** | **3** |



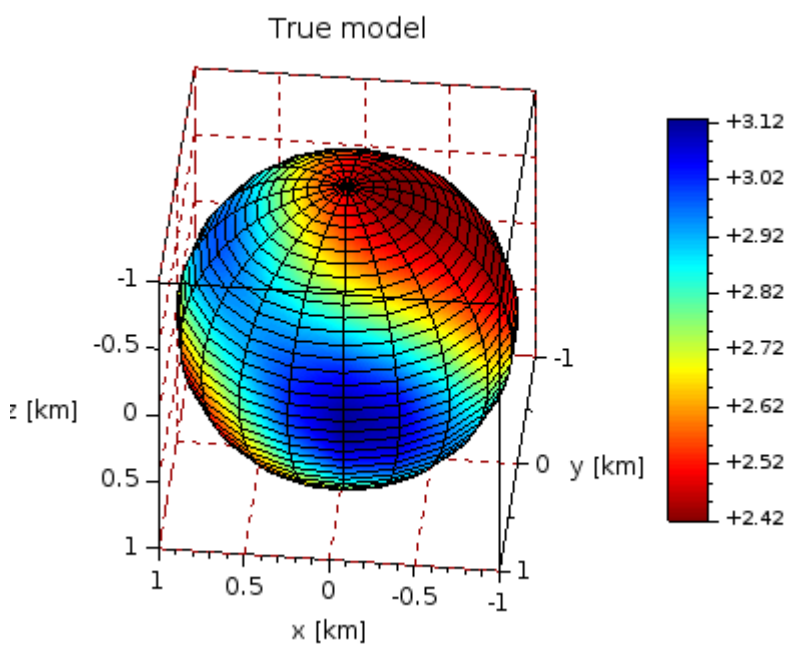



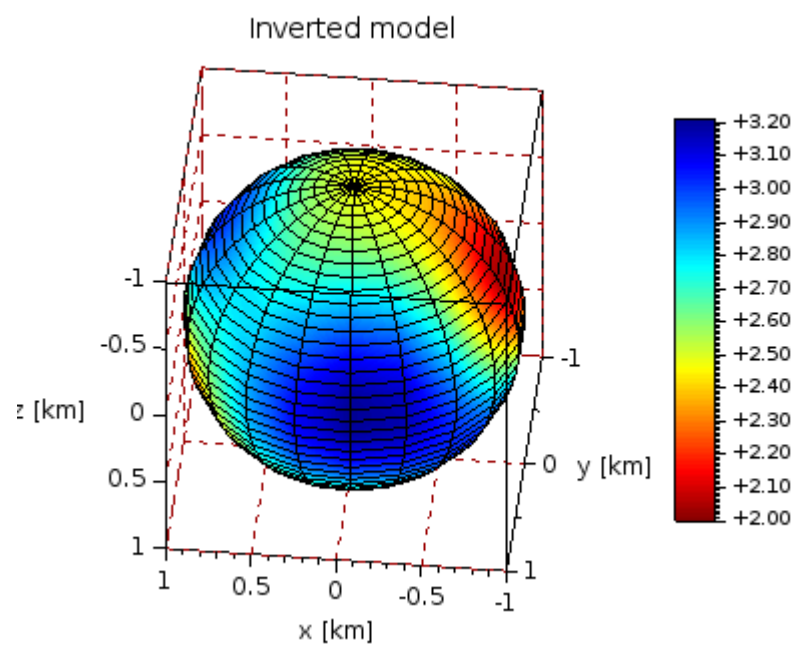
# Inversion results



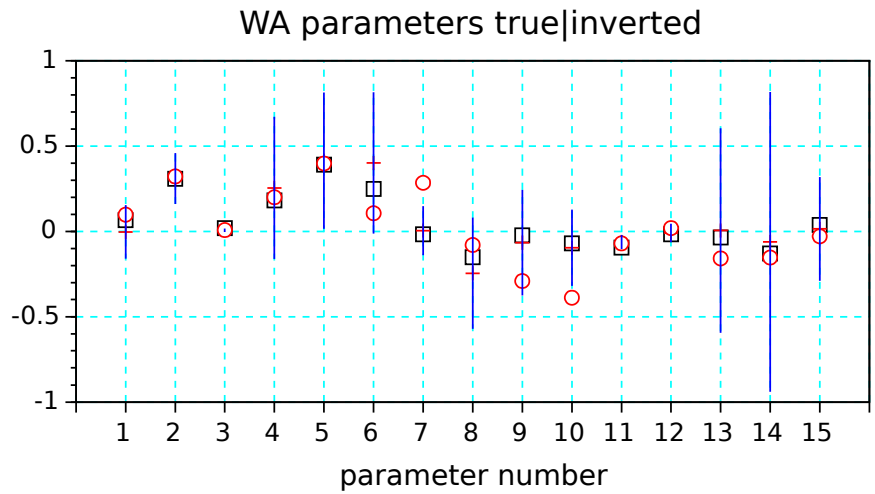



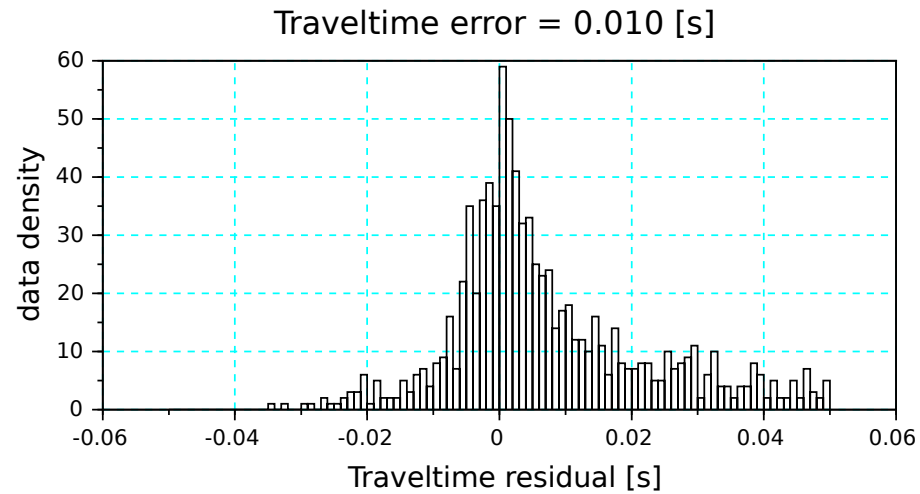

| Model | Config | Formula | Noise [ms] | Δm | Δv [%] |
|---|---|---|---|---|---|
| Strong | Profile | $v^2$ | **5** | **0.15** | **6** |



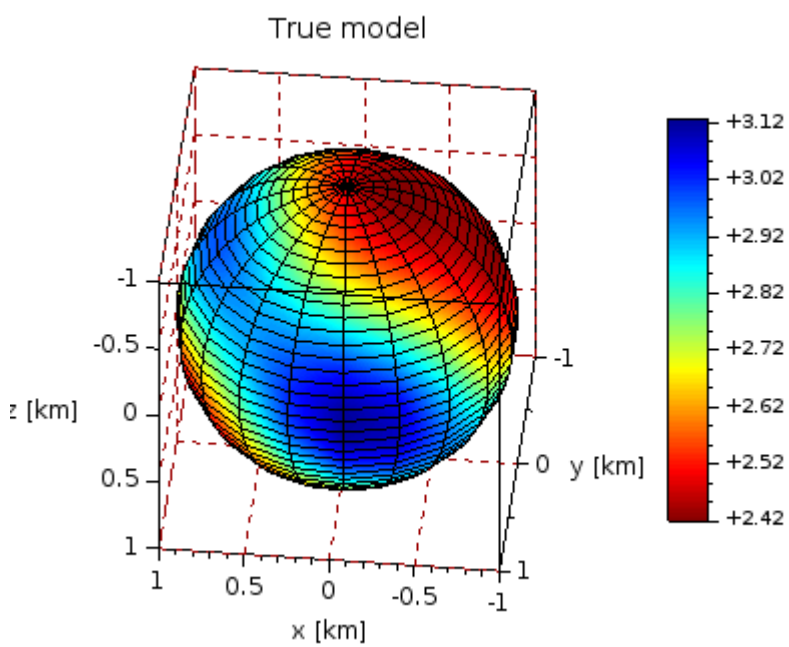



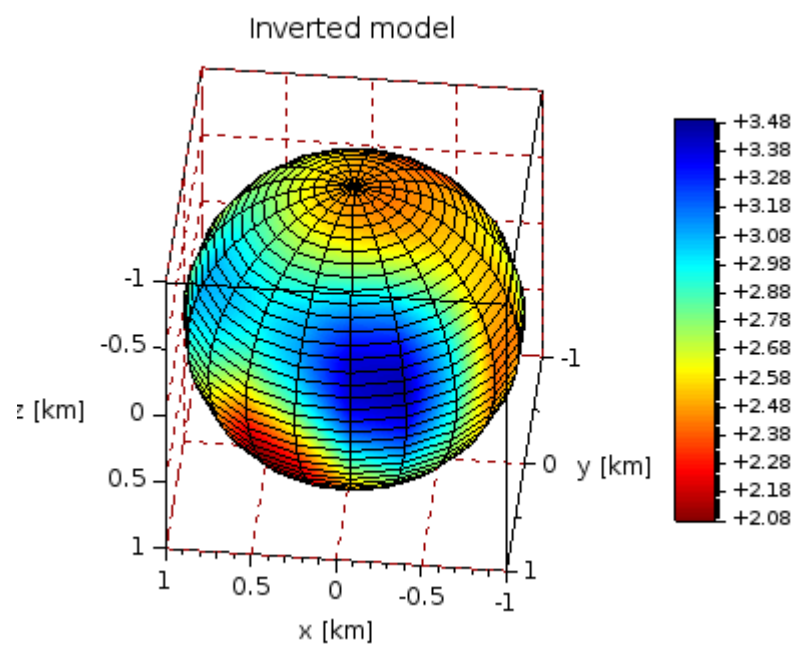
# Inversion results



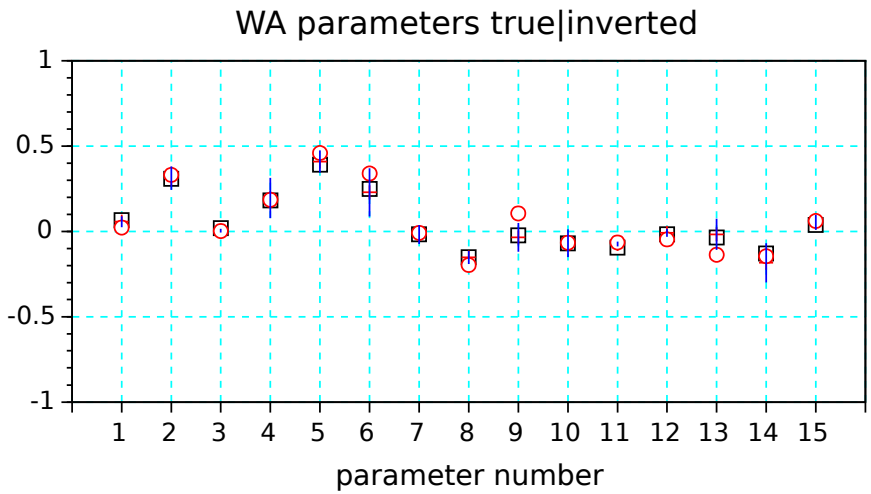



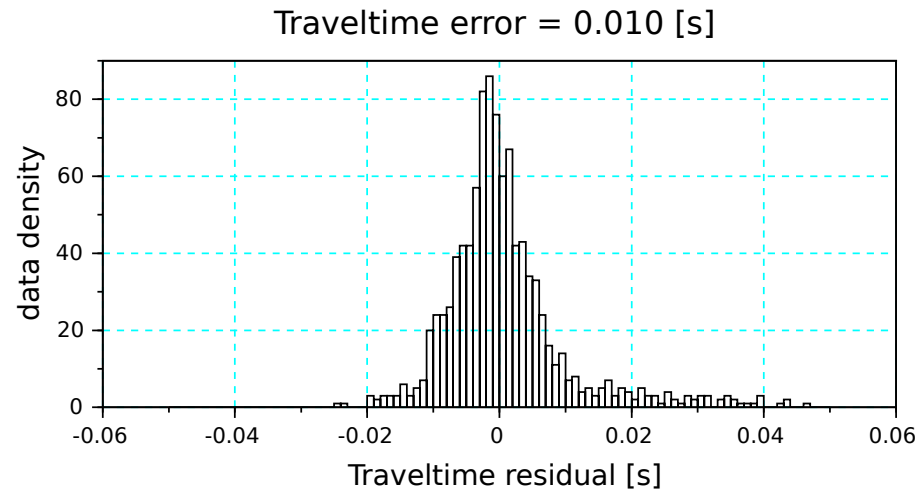

| Model | Config | Formula | Noise [ms] | Δm | Δv [%] |
|---|---|---|---|---|---|
| Strong | Profile | $v^2$ | **10** | **0.37** | **14** |



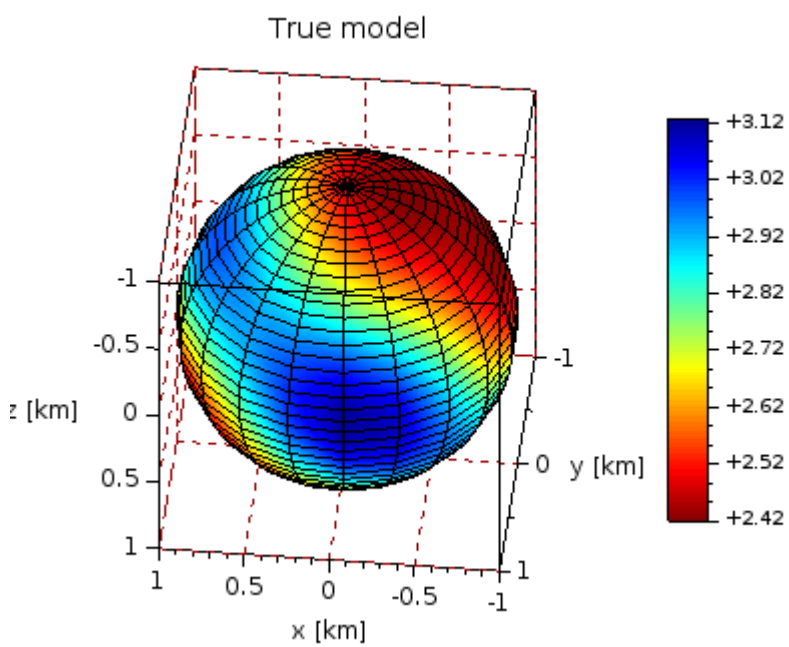



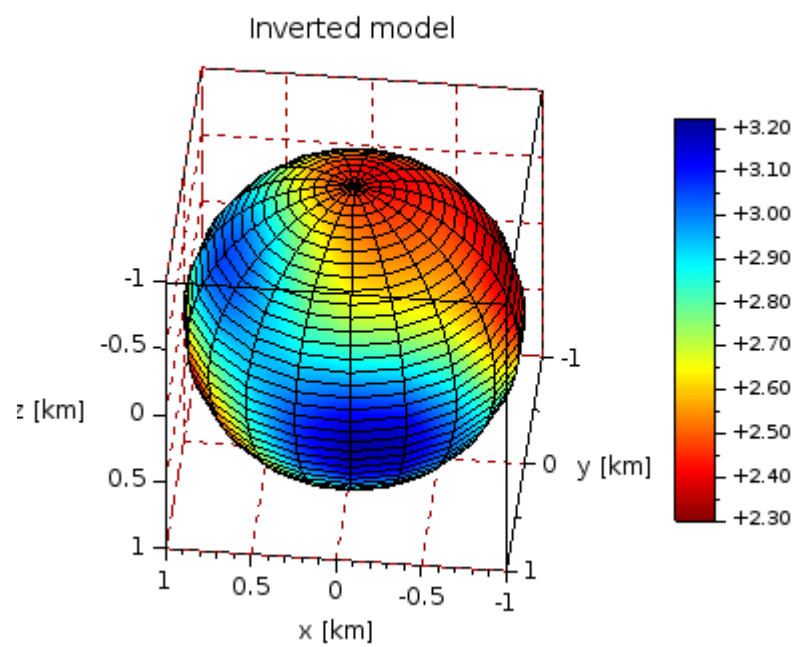
# Inversion results



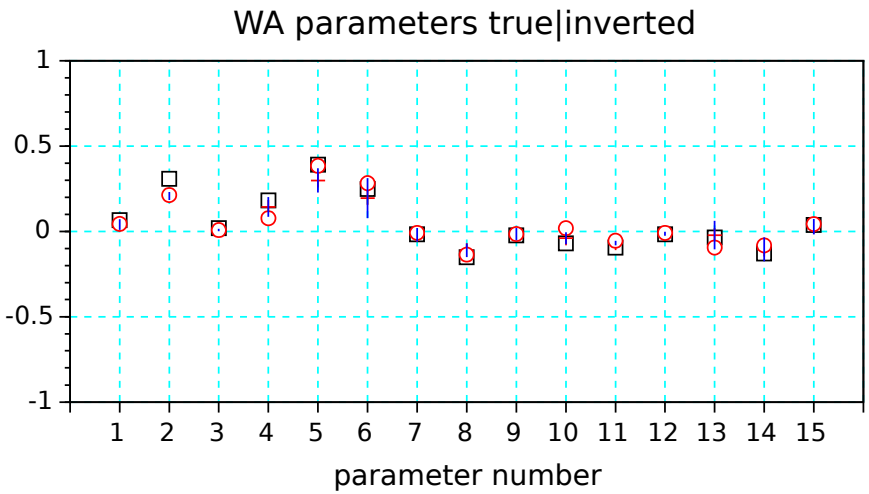



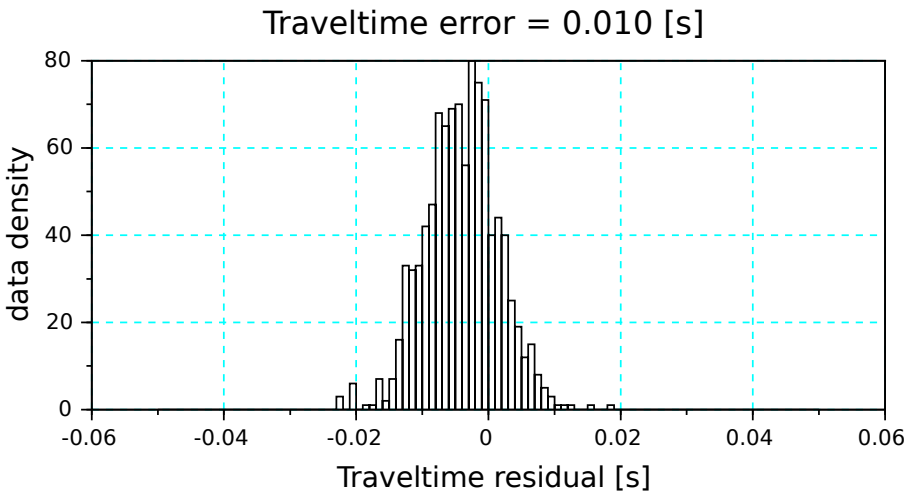

| Model | Config | Formula | Noise [ms] | Δm | Δv [%] |
|---|---|---|---|---|---|
| Strong | **Random** | $v^2$ | 10 | **0.08** | **3** |



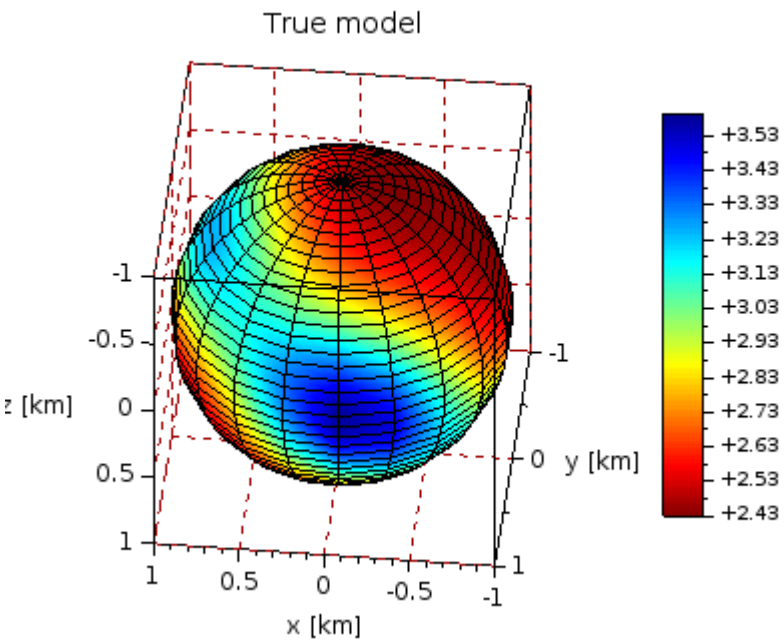



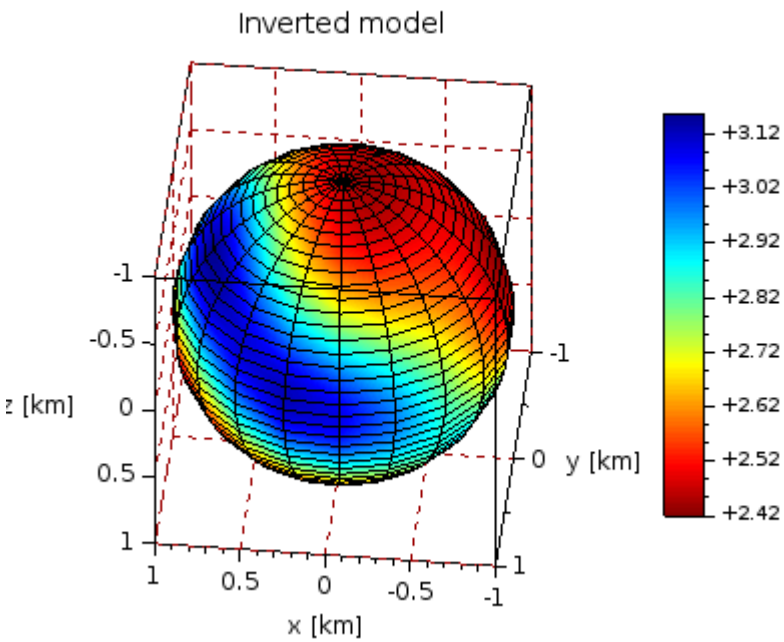
# Inversion results



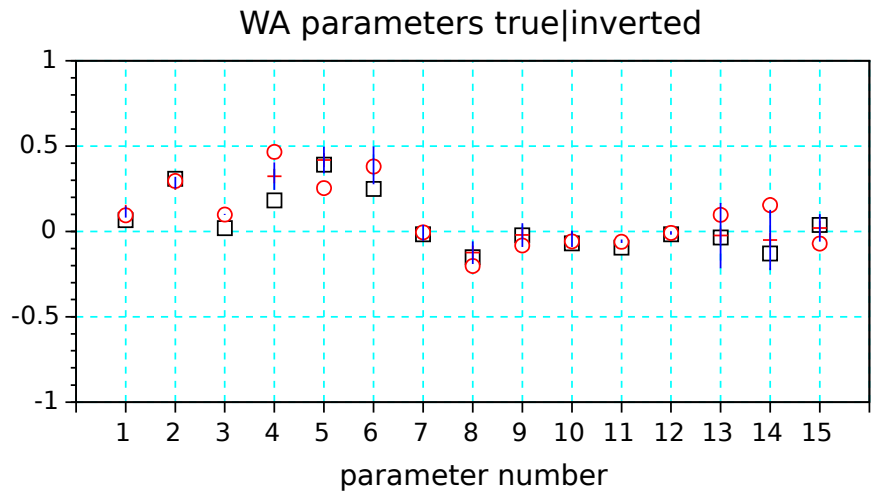



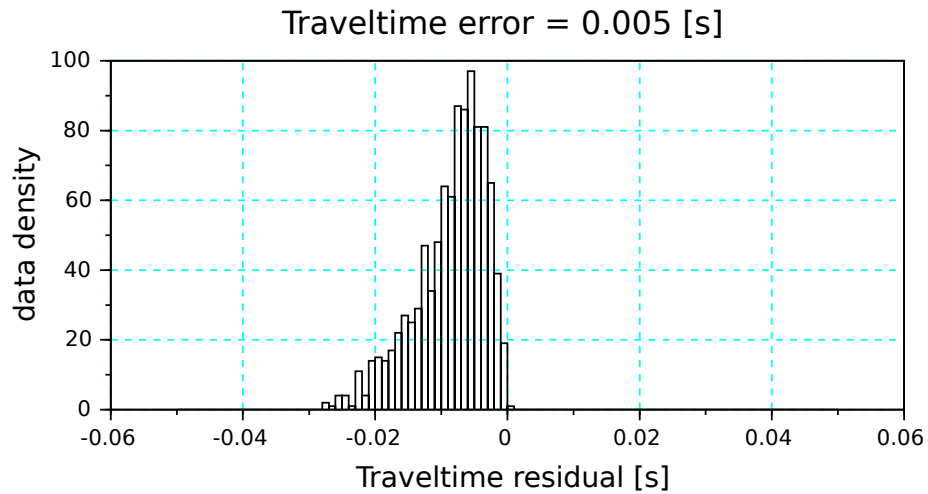

| Model | Config | Formula | Noise [ms] | Δm | Δv [%] |
|---|---|---|---|---|---|
| Strong | Random | $v^{-1}$ | 10 | 0.07 | 7 |



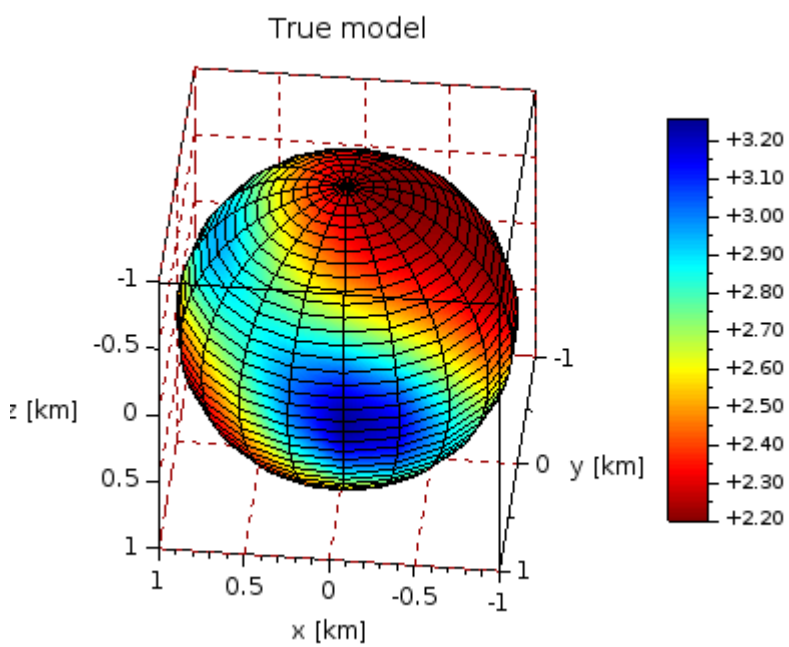



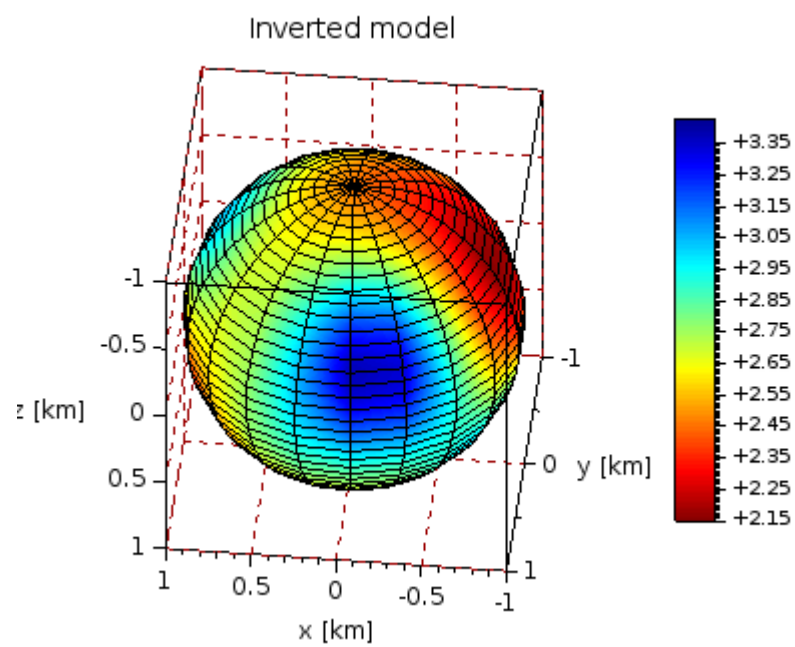
# Inversion results



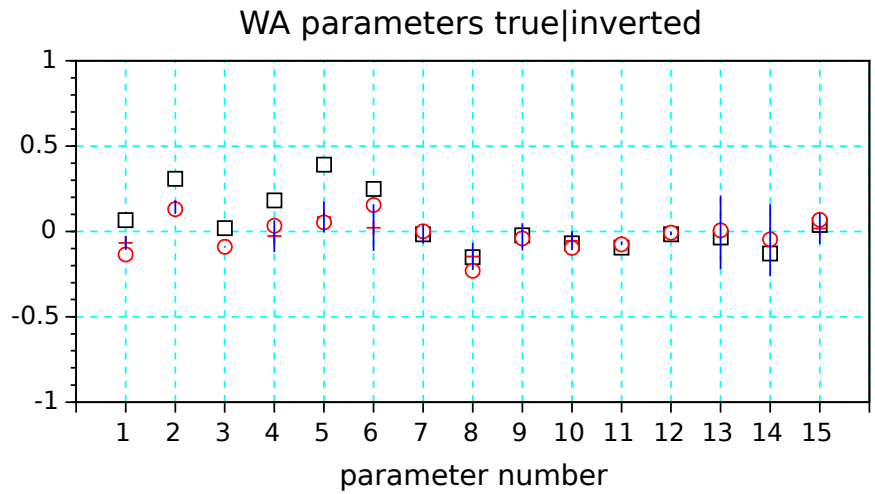



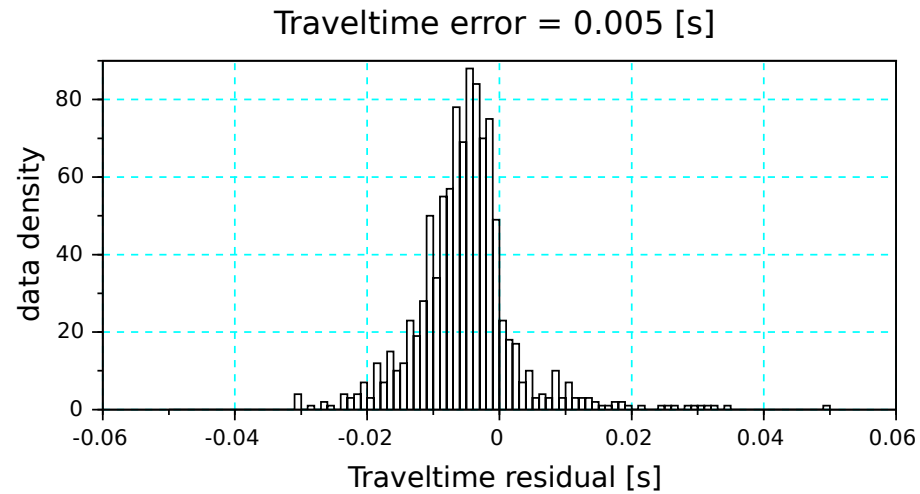

| Model | Config | Formula | Noise [ms] | Δm | Δv [%] |
|---|---|---|---|---|---|
| Strong | Profile | $v^{-1}$, $\alpha_0$ low | 5 | 0.11 | 8 |



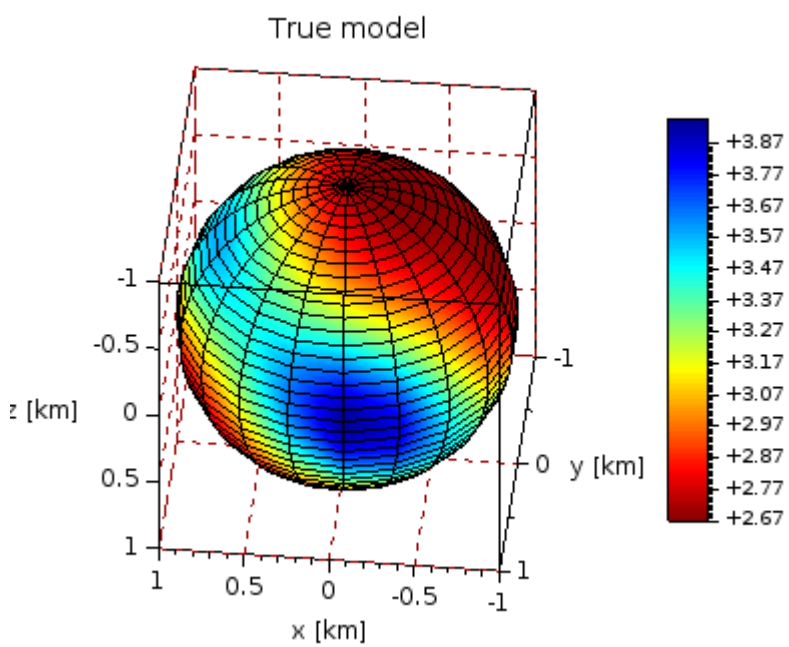



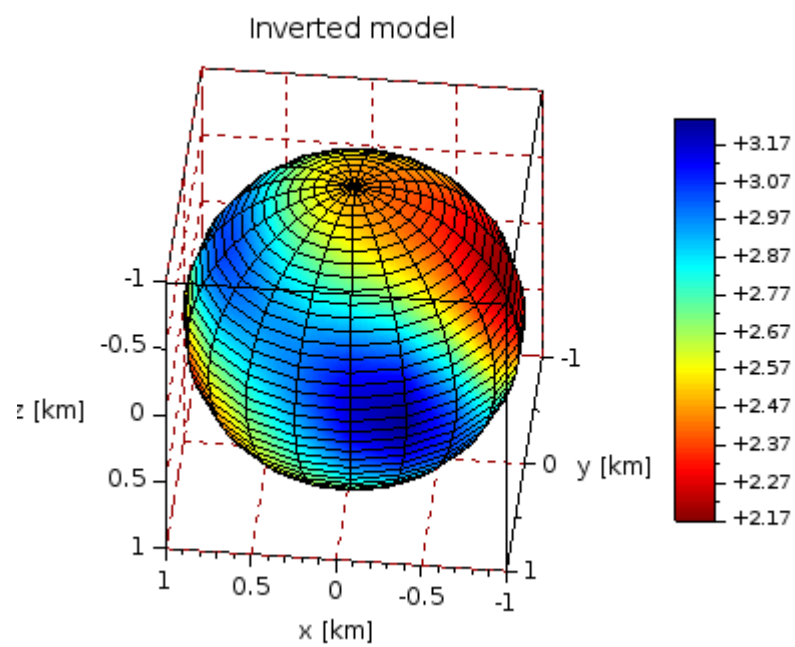
# Inversion results



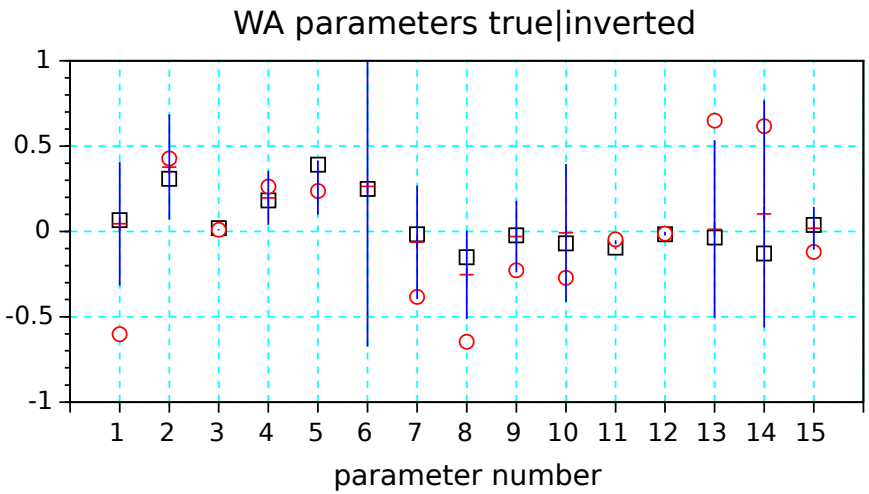



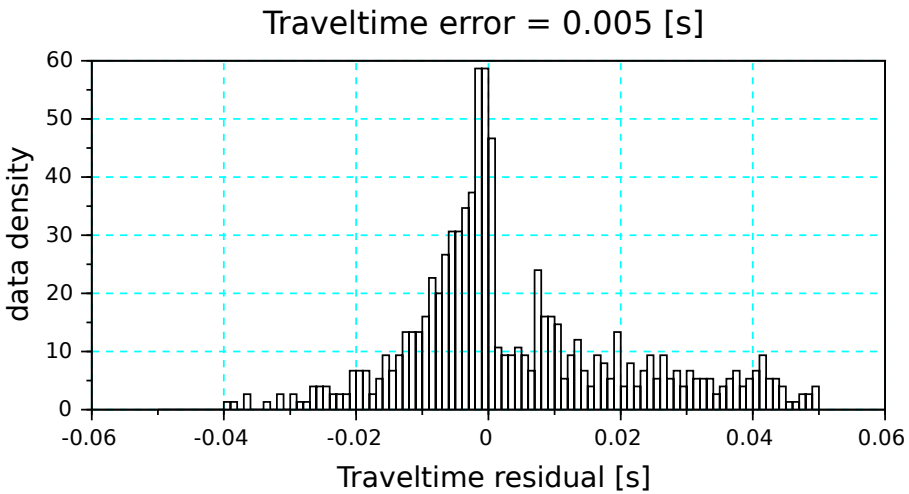

| Model | Config | Formula | Noise [ms] | Δm | Δv [%] |
|---|---|---|---|---|---|
| Strong | Profile | $v^{-1}$,**$\alpha^0$ high** | 5 | **0.17** | **15** |



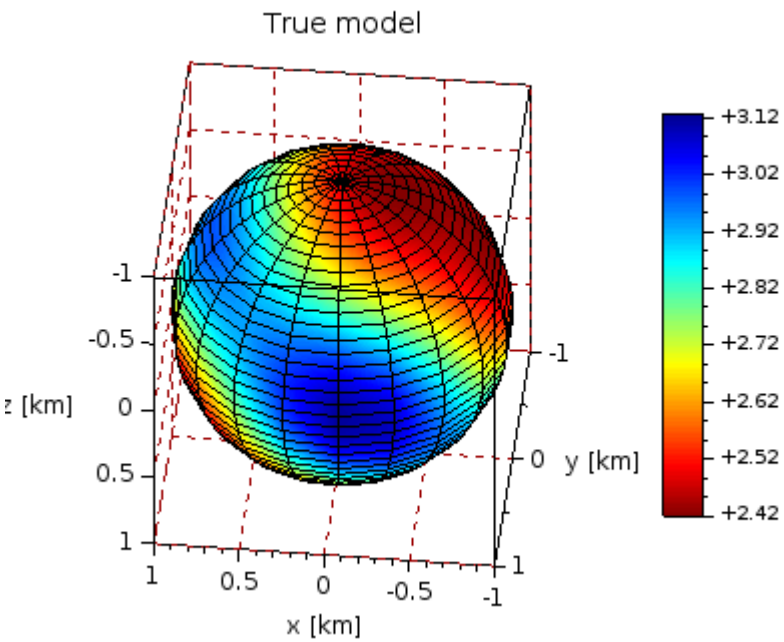



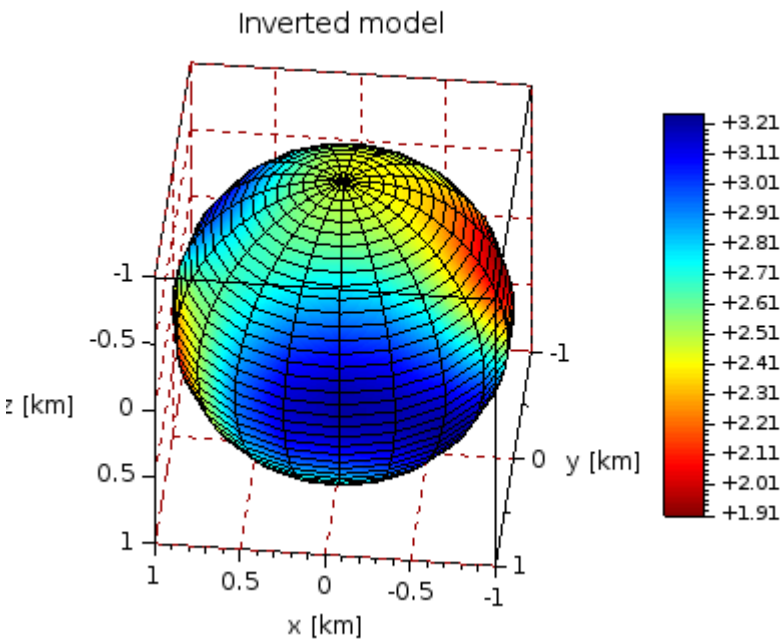
# Inversion results



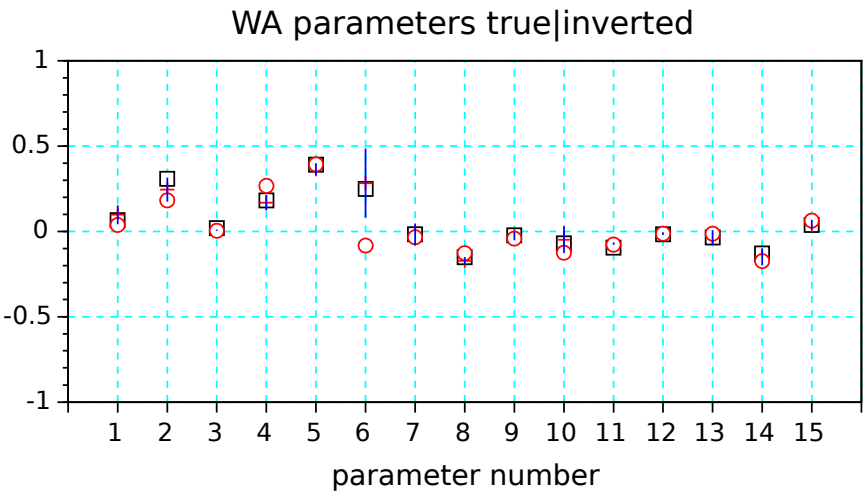



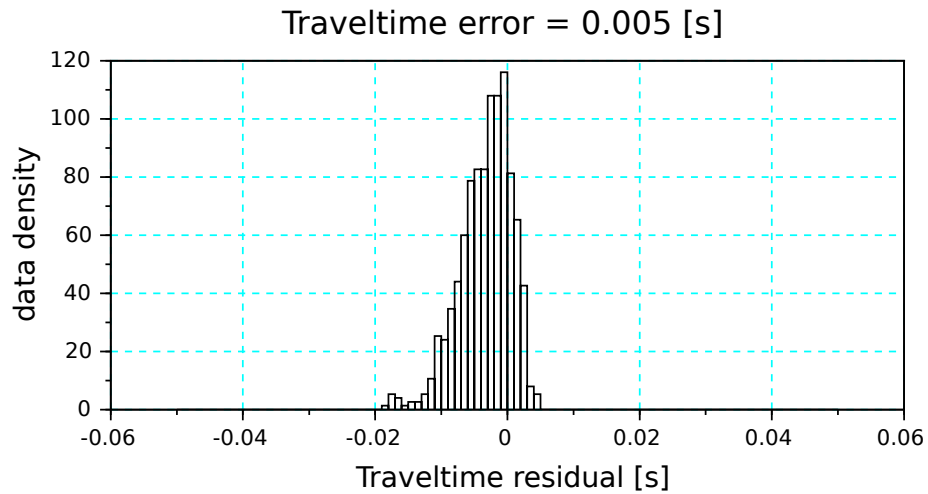

| Model | Config | Formula | Noise [ms] | Δm | Δv [%] |
|---|---|---|---|---|---|
| Strong | **Profile/3r** | $v^2$ | 5 | **0.40** | **18** |



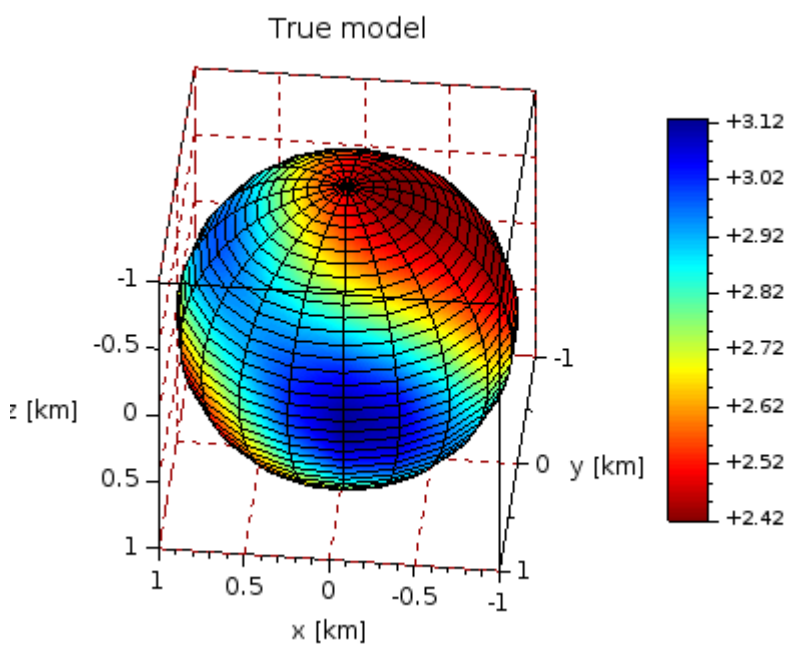



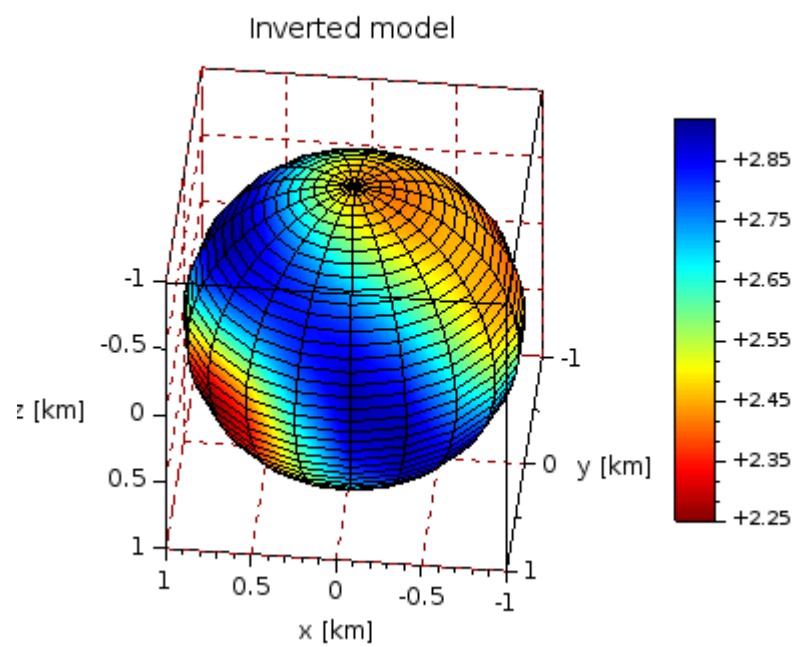
# Inversion results

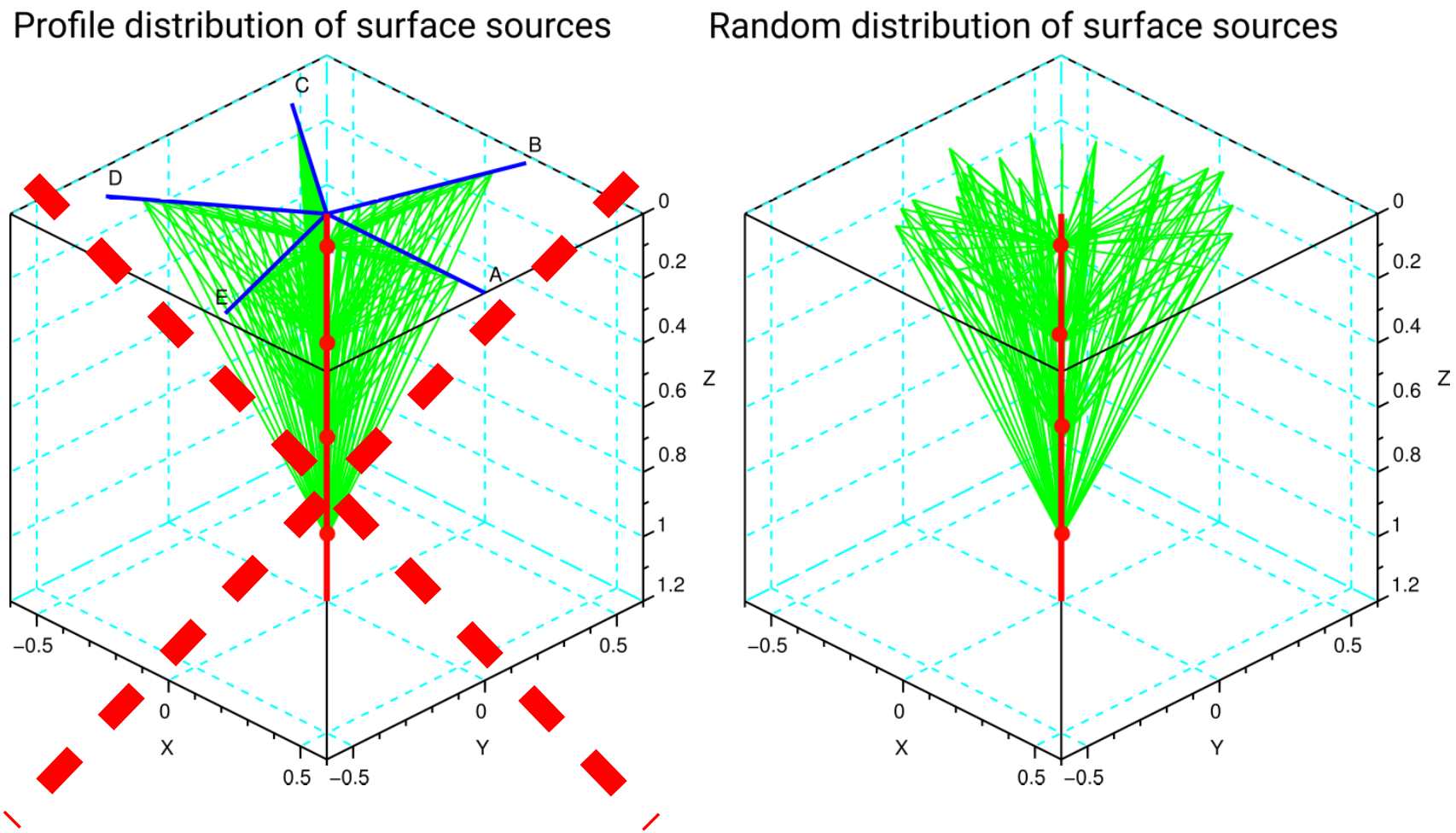


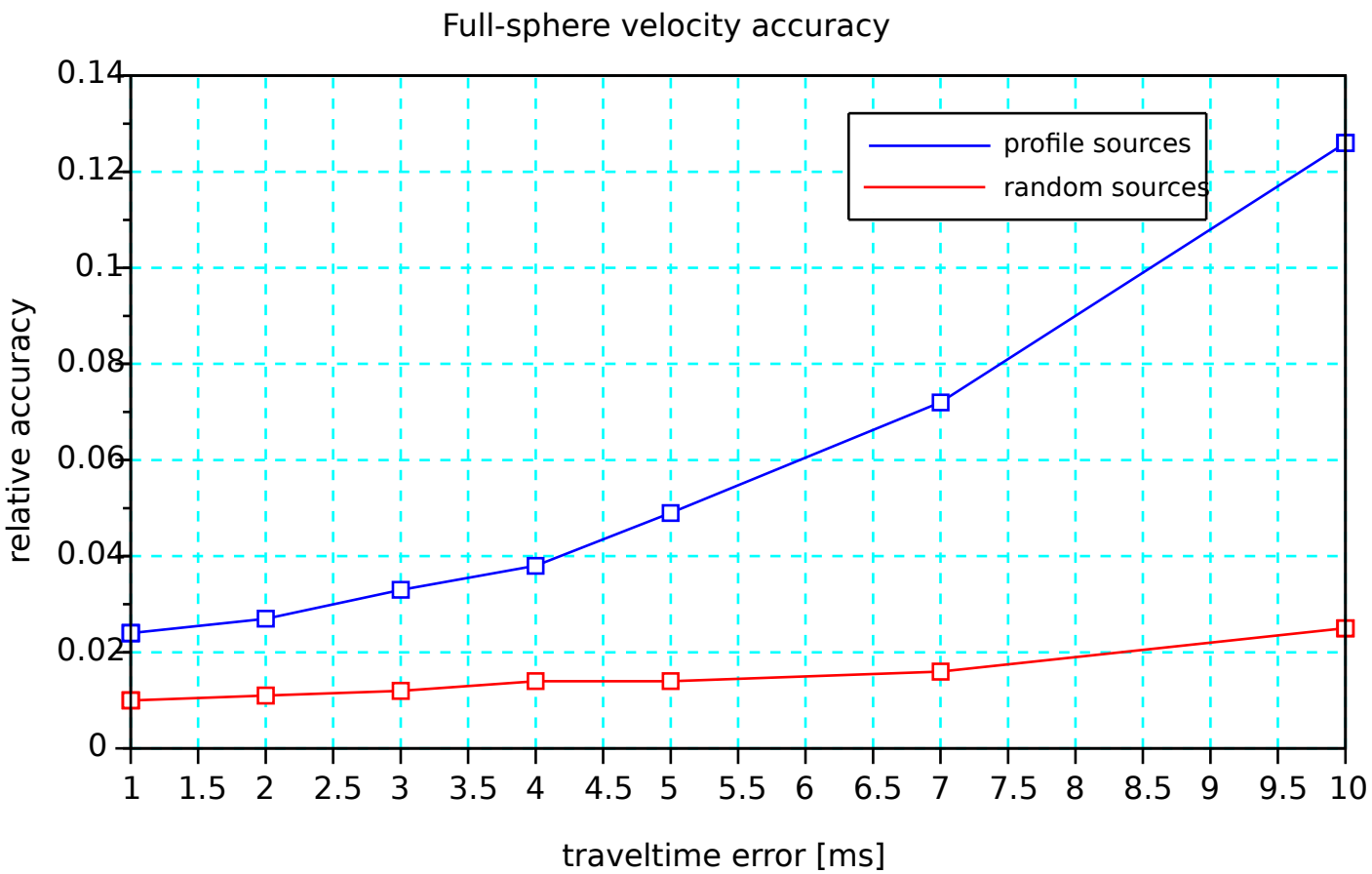

| Model | Config | Formula | Noise [ms] | Δm | Δv [%] |
|---|---|---|---|---|---|
| Strong | **Rand/3r** | $v^2$ | 5 | **0.07** | **4** |




Profile distribution of surface sources
Random distribution of surface sources
A
B
C
D
E
X
Y
Z
−0.5
0
0.5
0.2
0.4
0.6
0.8
1
1.2

# Measurement geometry + observational errors = key issue



# Conclusions

- need for general anisotropy - WA parameterization suitable

- P-wave traveltime inversion - 15 P-wave WA parameters sufficient

- first-order formulae for traveltime sufficient

- $v^{-1}$ formula slightly better than $v^2$; $v^2$ independent of $\alpha_0$

- dependence of accuracy on configuration and noise level

- random distribution of sources better than profile arrangement

- applicability to moderate anisotropy (e.g. $\sim$ 26% anisotropy)

# Future plans

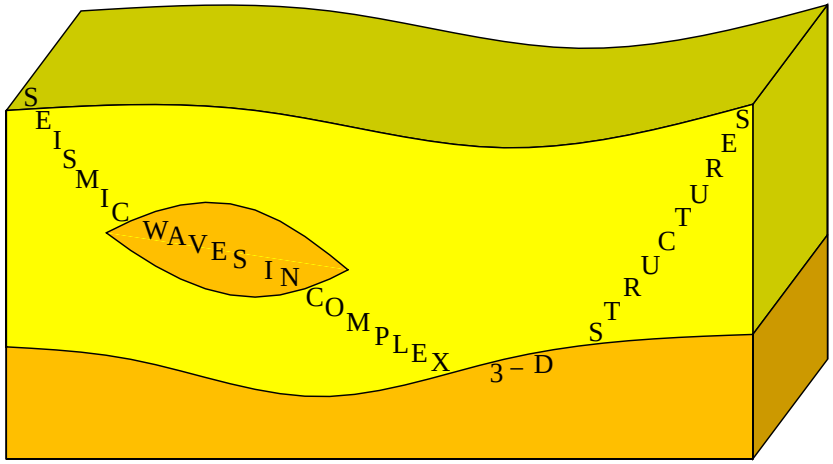
Continuing tests in homogeneous media

- estimate of type of anisotropic symmetry
- use of S-wave traveltimes
- combination with inversion reflection moveout data

Tests in inhomogeneous anisotropic media

- study of resolution and stability
- use of an additional independent information
- use of S-wave traveltimes
- use of P- and S-wave polarizations

# Acknowledgements

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