Reference transversely isotropic medium approximating a given generally anisotropic medium

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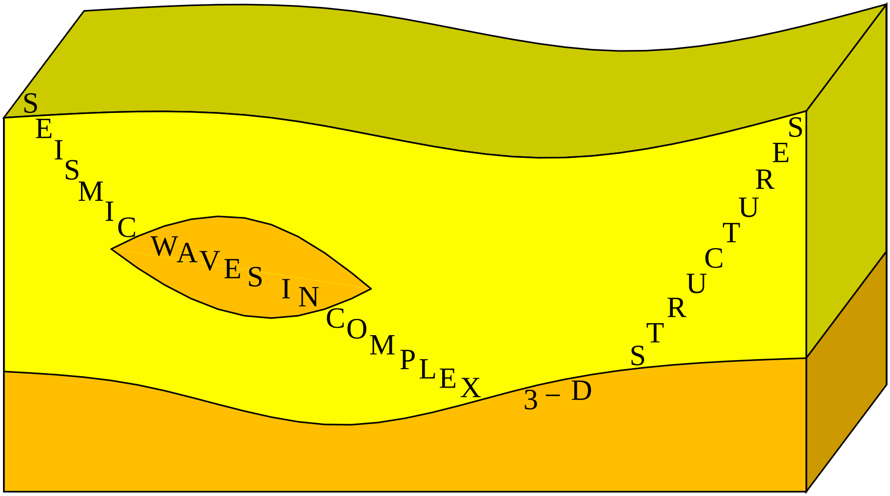

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For a given stiffness tensor (tensor of elastic moduli) of a generally anisotropic medium, we can estimate to which extent the medium is transversely isotropic and determine the direction of its reference symmetry axis using the method of Klimeš (2015). If we find that the medium is approximately transversely isotropic (approximately uniaxial), we can determine the reference transversely isotropic (uniaxial) medium which approximates the given generally anisotropic medium.

The stiffness tensor of a transversely isotropic medium is independent of the rotation around the symmetry axis. We thus take the reference symmetry axis determined using the method of Klimeš (2015), rotate the given stiffness tensor about this reference symmetry axis, and determine the reference transversely isotropic stiffness tensor as the average of the rotated stiffness tensor over all angles of rotation.

The lower–case Roman indices take values 1, 2 and 3. The Einstein summation over repetitive lower–case Roman indices is used hereinafter.

We denote the density–reduced stiffness tensor of a given generally anisotropic medium by a_{ijkl}.

The unit reference symmetry vector t_i in the direction of the reference symmetry axis can be obtained using the method of Klimeš (2015).

Projection matrix onto the reference symmetry vector:

$$Z_{ia} = t_i t_a \tag{1}$$

Projection matrix onto the plane perpendicular to the reference symmetry vector:

$$C_{ia} = \delta_{ia} - t_i t_a \tag{2}$$

Minus the generator matrix of the rotation about the reference symmetry vector:

$$S_{ia} = \varepsilon_{iar} t_r \tag{3}$$

Here Kronecker delta δ_{in} represents the elements of the identity matrix, and ε_{ijk} is the Levi–Civita symbol.

Transformation matrix corresponding to the rotation of vectors about the given reference symmetry vector t_i by angle φ:

$$R_{ia}(\varphi) = Z_{ia} + C_{ia}\cos(\varphi) - S_{ia}\sin(\varphi) \tag{4}$$

The rotated stiffness tensor:

$$\tilde{a}_{ijkl}(\varphi) = R_{ia}(\varphi)\, R_{jb}(\varphi)\, R_{kc}(\varphi)\, R_{ld}(\varphi)\, a_{abcd} \tag{5}$$

Stiffness tensor of the reference transversely isotropic medium:

$$\overline{a}_{ijkl} = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\varphi\, \tilde{a}_{ijkl}(\varphi) \tag{6}$$

$$\begin{aligned}
\overline{a}_{ijkl} = \Big\{ & Z_{ia}Z_{jb}Z_{kc}Z_{ld} \\
& + \tfrac{1}{2}\big[Z_{ia}Z_{jb}C_{kc}C_{ld} + Z_{ia}C_{jb}Z_{kc}C_{ld} + Z_{ia}C_{jb}C_{kc}Z_{ld} \\
& \quad + C_{ia}Z_{jb}Z_{kc}C_{ld} + C_{ia}Z_{jb}C_{kc}Z_{ld} + C_{ia}C_{jb}Z_{kc}Z_{ld}\big] \\
& + \tfrac{1}{2}\big[Z_{ia}Z_{jb}S_{kc}S_{ld} + Z_{ia}S_{jb}Z_{kc}S_{ld} + Z_{ia}S_{jb}S_{kc}Z_{ld} \\
& \quad + S_{ia}Z_{jb}Z_{kc}S_{ld} + S_{ia}Z_{jb}S_{kc}Z_{ld} + S_{ia}S_{jb}Z_{kc}Z_{ld}\big] \\
& + \tfrac{1}{8}\big[C_{ia}C_{jb}S_{kc}S_{ld} + C_{ia}S_{jb}C_{kc}S_{ld} + C_{ia}S_{jb}S_{kc}C_{ld} \\
& \quad + S_{ia}C_{jb}C_{kc}S_{ld} + S_{ia}C_{jb}S_{kc}C_{ld} + S_{ia}S_{jb}C_{kc}C_{ld}\big] \\
& + \tfrac{3}{8}\big[C_{ia}C_{jb}C_{kc}C_{ld} + S_{ia}S_{jb}S_{kc}S_{ld}\big]\Big\}\, a_{abcd}
\end{aligned} \tag{22}$$

Conclusions

The proposed method of determining the reference transversely isotropic stiffness tensor for the given stiffness tensor of a generally anisotropic medium has been coded as a new option of program `tiaxis.for` of software package FORMS (Bucha & Bulant, 2016).

References:

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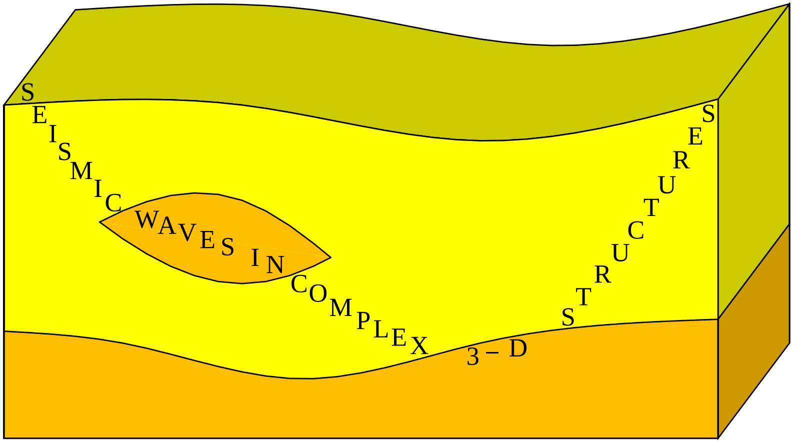

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