Green function as an integral superposition of Gaussian beams in inhomogeneous anisotropic layered structures in Cartesian coordinates

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Outline

Introduction

Integral superposition

Conclusions

Introduction

Wave modelling in inhomogeneous anisotropic media

- ray method
- coupling ray method
- paraxial ray approximations, paraxial Gaussian beams
- **weighted summation** of paraxial ray approximations
  or **paraxial Gaussian beams** in ray-centred
  or **Cartesian coordinates**

Integral superposition

$$G_{ij}(R,S,\omega) = \frac{\omega}{2\pi}\int\int_{\mathcal{D}} \bar{A}(P) g_i(P) g_j(S) |det\boldsymbol{\mathcal{P}}(S)|^{1/2} [-\det \boldsymbol{\mathcal{N}}(P)]^{1/2} \times \exp[\mathrm{i}\omega\theta(R,P)] d\gamma_1 d\gamma_2$$

$\mathbf{G}(R,S,\omega)$ - Green function

ω - circular frequency

$\mathbf{g}$ - polarization vector

γ_1, γ_2 - ray parameters defined on $\mathcal{D}$, specifying the ray Ω

R, S, P - receiver, source and a point on the ray Ω,
in a vicinity of R

Integral superposition

$$G_{ij}(R,S,\omega) = \frac{\omega}{2\pi}\int\int_{\mathcal{D}} \bar{A}(P)g_i(P)g_j(S)|det\boldsymbol{\mathcal{P}}(S)|^{1/2}[-\det\boldsymbol{\mathcal{N}}(P)]^{1/2} \times \exp[\mathrm{i}\omega\theta(R,P)]d\gamma_1 d\gamma_2$$

$\bar{A}(P)$ - spreading-free scalar ray-theory amplitude

$\boldsymbol{\mathcal{P}}(S)$ - 2×2 paraxial ray matrix

$\boldsymbol{\mathcal{N}}(P)$ - 2×2 weighting function matrix

$\theta(R,P)$ - traveltime function

Integral superposition

$$G_{ij}(R,S,\omega) = \frac{\omega}{2\pi} \int\int_{\mathcal{D}} \bar{A}(P) g_i(P) g_j(S) |det\boldsymbol{\mathcal{P}}(S)|^{1/2} [-\det \boldsymbol{\mathcal{N}}(P)]^{1/2} \times \exp[\mathrm{i}\omega\theta(R,P)] d\gamma_1 d\gamma_2$$

Spreading-free scalar ray-theory amplitude $\bar{A}(P)$

$$\bar{A}(P) = \frac{\mathcal{R}^C(P,S)\exp[\mathrm{i}T^G(P,S)]}{4\pi[\rho(S)\rho(P)C(S)C(P)]^{1/2}}$$

$\mathcal{R}^C$ ... complete normalized energy R/T coefficient

$T^G(P,S)$ ... complete phase shift due to caustics

$C(S)$, $C(P)$, $\rho(S)$, $\rho(P)$ - phase velocities and densities at S and P

Integral superposition

$$G_{ij}(R,S,\omega) = \frac{\omega}{2\pi}\int\int_{\mathcal{D}} \bar{A}(P) g_i(P) g_j(S) |det\boldsymbol{\mathcal{P}}(S)|^{1/2}[-\det\boldsymbol{\mathcal{N}}(P)]^{1/2} \times \exp[\mathrm{i}\omega\theta(R,P)] d\gamma_1 d\gamma_2$$

Paraxial ray matrix $\boldsymbol{\mathcal{P}}(S)$

$$P_{IK}(S) = H_{mI}(S) P^{(x)}_{mK}(S)$$

$\mathbf{P}^{(x)}(S)$ - 3×3 paraxial ray matrix specified at S

$\mathbf{H}$ - 3×3 transformation matrix

Integral superposition

$$G_{ij}(R,S,\omega) = \frac{\omega}{2\pi}\int\int_{\mathcal{D}} \bar{A}(P)g_i(P)g_j(S)|det\boldsymbol{\mathcal{P}}(S)|^{1/2}[-\det\boldsymbol{\mathcal{N}}(P)]^{1/2} \times \exp[\mathrm{i}\omega\theta(R,P)]d\gamma_1 d\gamma_2$$

Weighting function $\boldsymbol{\mathcal{N}}(P)$

$$\mathcal{N}_{IJ} = H_{mI}(P^{(x)}_{mJ} - \eta_m p_j Q^{(x)}_{jJ}) - M_{IL}\bar{H}_{Lj}Q^{(x)}_{jJ}$$

$\mathbf{Q}^{(x)}(P)$, $\mathbf{P}^{(x)}(P)$ - 3×3 paraxial matrices obtained from DRT

$\mathbf{M}(P)$ - 2×2 matrix of Gaussian-beam parameters (to be chosen)

$\mathbf{H}$, $\bar{\mathbf{H}}$ - 3×3 transformation matrices

$\mathbf{p}$, η - slowness and eta vectors (from RT)

Integral superposition

$$G_{ij}(R,S,\omega) = \frac{\omega}{2\pi}\int\int_{\mathcal{D}} \bar{A}(P) g_i(P) g_j(S) |det\boldsymbol{\mathcal{P}}(S)|^{1/2}[-\det\boldsymbol{\mathcal{N}}(P)]^{1/2} \times \exp[\mathrm{i}\omega\theta(R,P)]d\gamma_1 d\gamma_2$$

Traveltime function $\theta(R,P)$

$$\theta(R,P) = \tau(P) + (x_k^R - x_k^P)p_k(P) + \tfrac{1}{2}(x_k^R - x_k^P)\mathcal{M}_{kl}(P)(x_l^R - x_l^P)$$

$\tau(P)$ - traveltime at P (from RT)

$\mathbf{p}$ - slowness vector (from RT)

$\boldsymbol{\mathcal{M}}(P)$ - 3×3 matrix

Integral superposition

$$G_{ij}(R,S,\omega)=\frac{\omega}{2\pi}\int\int_{\mathcal{D}}\bar{A}(P)g_i(P)g_j(S)|det\boldsymbol{\mathcal{P}}(S)|^{1/2}[-\det\boldsymbol{\mathcal{N}}(P)]^{1/2}\times\exp[\mathrm{i}\omega\theta(R,P)]d\gamma_1 d\gamma_2$$

$\mathbf{3}\times\mathbf{3}$ matrix $\boldsymbol{\mathcal{M}}(P)$

$$\boldsymbol{\mathcal{M}}=\mathbf{H}\mathbf{M}\bar{\mathbf{H}}+\mathbf{p}\boldsymbol{\eta}^T+\boldsymbol{\eta}\mathbf{p}^T-\mathbf{p}(\boldsymbol{\mathcal{U}}^T\boldsymbol{\eta})\mathbf{p}^T$$

$\mathbf{M}(P)$ - 2×2 matrix of Gaussian-beam parameters (to be chosen)

$\mathbf{H}$, $\bar{\mathbf{H}}$ - 3×3 transformation matrices

$\mathbf{p}$, $\boldsymbol{\eta}$, $\boldsymbol{\mathcal{U}}$ - slowness, eta and ray-velocity vectors (from RT)

Integral superposition

$$G_{ij}(R,S,\omega)=\frac{\omega}{2\pi}\int\int_{\mathcal{D}}\bar{A}(P)g_i(P)g_j(S)|det\boldsymbol{\mathcal{P}}(S)|^{1/2}[-\det\boldsymbol{\mathcal{N}}(P)]^{1/2}\times\exp[\mathrm{i}\omega\theta(R,P)]d\gamma_1 d\gamma_2$$

3×3 transformation matrices $\mathbf{H}$, $\bar{\mathbf{H}}$

$\mathbf{H}$, $\bar{\mathbf{H}}$ - obtained as a solution

of a vectorial ordinary differential equation along Ω

$$\mathbf{H}\cdot\bar{\mathbf{H}}=\mathbf{I}$$

$\mathbf{I}$ - $3\times$ 3 identity matrix

Conclusions

- applicable to 3D inhomogeneous anisotropic media
  with curved structural interfaces

- applicable to separate P, S1 and S2 waves

- applicable to coupled S waves in weak anisotropy
  or in vicinities of S-wave singularities

- applicable to moment-tensor point sources

Conclusions

- DRT performed in Cartesian coordinates

- 3×2 parts of 3×3 paraxial matrices sufficient

- solution of a vectorial ordinary differential equation
  for H_{3i} along the ray Ω

- no need for two-point ray tracing

- removes or smoothes singularities of standard ray theory

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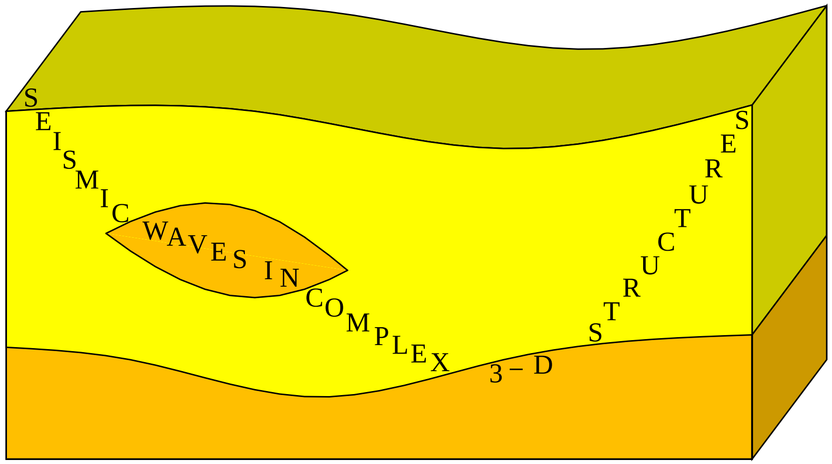

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