GEOPHYSICS, VOL. 84, NO. 2 (MARCH-APRIL 2019); P. C75–C83, 5 FIGS., 1 TABLE.
10.1190/GEO2018-0474.1

Reflection moveout approximation for a P-SV wave in a moderately anisotropic homogeneous vertical transverse isotropic layer

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ABSTRACT

A description of the subsurface is incomplete without the use of S-waves. Use of converted waves is one way to involve S-waves. We have developed and tested an approximate formula for the reflection moveout of a wave converted at a horizontal reflector underlying a homogeneous transversely isotropic layer with the vertical axis of symmetry. For its derivation, we use the weak-anisotropy approximation; i.e., we expand the square of the reflection traveltime in terms of weak-anisotropy (WA) parameters. Traveltimes are calculated along reference rays of converted reflected waves in a reference isotropic medium. This requires the determination of the point of reflection (the conversion point) of the reference ray, at which the conversion occurs. This can be done either by a numerical solution of a quartic equation or by using a simple approximate solution. Presented tests indicate that the accuracy of the proposed moveout formula is comparable with the accuracy of formulas derived in a weak-anisotropy approximation for pure-mode reflected waves. Specifically, the tests indicate that the maximum relative traveltime errors are well below 1% for models with P- and SV-wave anisotropy of approximately 10% and less than 2% for models with P- and SV-wave anisotropy of 25% and 12%, respectively. For isotropic media, the use of the conversion point obtained by numerical solution of the quartic equation yields exact results. The approximate moveout formula is used for the derivation of approximate expressions for the two-way zero-offset traveltime, the normal moveout velocity and the quartic term of the Taylor series expansion of the squared traveltime.

INTRODUCTION

Converted reflected waves play an important role in the processing of multicomponent ocean-bottom measurements (Thomsen, 1999). Converted waves offer an additional information to that obtained from pure-mode reflected P-waves. Most of the existing studies are based on the Taylor series expansion of the squared traveltime of a converted wave in a transversely isotropic medium with a vertical axis of symmetry (VTI) with respect to the squared offset (see, e.g., Seriff and Sriram, 1991; Tsvankin and Thomsen, 1994; Granli et al., 1999; Thomsen, 1999; Tsvankin and Grechka, 2000; Li and Yuan, 2003; Hao and Stovas, 2016). Many details can be found in monographs of Tsvankin (2001) or Tsvankin and Grechka (2011). We concentrate on VTI media too, but we use an alternative procedure.

Following Farra and Pšenčík (2013, 2017a), Farra et al. (2016), and Pšenčík and Farra (2017), we derive a reflection moveout formula based on the combined use of the weak-anisotropy approximation and weak-anisotropy (WA) parameters. The derived approximate formula holds for P-SV and SV-P converted waves, and it offers a direct link between observed traveltimes and parameters of the medium.

For the derivation of moveout formulas for pure-mode reflected P or SV waves, Farra and Pšenčík (2013) or Farra et al. (2016) use actual rays of pure-mode P or SV waves reflected from a horizontal reflector, which coincided with one symmetry plane of the overlying anisotropic medium. For lower symmetry anisotropic media, for media with tilted symmetry elements or, for example, for converted waves, actual rays would have to be calculated by two-point ray tracing in the actual medium, which would make the procedure impractical. Fortunately, Pšenčík and Farra (2017) and Farra and Pšenčík (2017a) show that it is possible to derive simple and still sufficiently accurate moveout formulas even without knowledge of actual rays, and they extend their previous work to weakly anisotropic media of arbitrary symmetry and

Manuscript received by the Editor 9 July 2018; revised manuscript received 9 October 2018; published ahead of production 12 November 2018; published online 4 February 2019.
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orientation. An important step of their procedure is the replacement of the actual ray in a studied anisotropic medium by a reference ray in a reference isotropic medium. Because the reference ray of a pure-mode reflected wave is symmetric with respect to the reflector, its construction is straightforward.

In contrast to pure-mode reflected waves, construction of a reference ray of a converted reflected wave in an isotropic medium, and, especially, the determination of the conversion point, is a more complicated task. Fortunately, there were successful attempts in the past to find the conversion point in isotropic media. Probably the first of these attempts was made by Tessmer and Behle (1988). Improved version of their formula proposed by Thomsen (1999) is used in this paper (see also Farra and Pšenčík, 2017b).

The proposed converted-wave moveout formula depends on the position of the conversion point and, through it, on the parameters of the reference isotropic medium. Most importantly, the formula depends on the parameters (four WA parameters) specifying the actual VTI medium. The formula can be used, among other applications, for the estimate of parameters directly either from the formula itself or from the expressions for the normal moveout (NMO) velocity or the quartic term derived from it. Estimated WA parameters can be used for the approximate reconstruction of P- and SV-wave phase or ray velocities. In this paper, we test its accuracy by comparing its results with results of exact and commonly used formulas.

Let us mention that the formula proposed in this paper can be generalized for the case of a dipping reflector (see Farra and Pšenčík, 2018).

The paper has the following structure: After introducing two important approximations, we use them in the derivation of the approximate converted-wave moveout formula for VTI media. From this formula, we derive expressions for basic quantities of the Taylor series expansion of the squared traveltime in terms of the squared offset, the two-way zero-offset traveltime, NMO velocity, and quartic term in terms of the WA parameters. The accuracy of the proposed formula is compared with the accuracy of the commonly used rational approximation, which is presented in the section "Reference moveout formula." In the next section "Tests of accuracy," the results of the proposed formula and of the rational approximation are compared with exact results. The following "Conclusions" section summarizes the main results of the paper and indicates possible extensions. Appendix A contains description of possible ways of the determination of the conversion point in the reference isotropic medium. In Appendix B, WA parameters used in the study are defined. Appendix C contains derivations of expressions for the two-way zero-offset traveltime, NMO velocity, and the quartic term for the converted wave in a homogeneous VTI layer in terms of WA parameters.

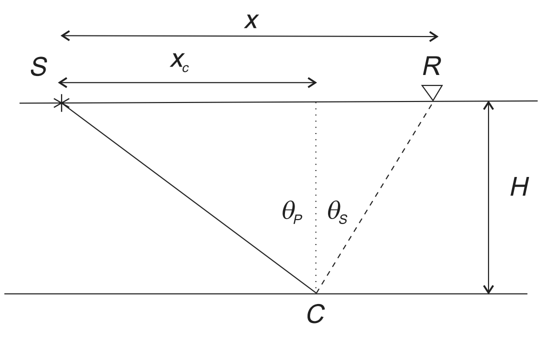

Figure 1. Schematic plot of a ray of a P-SV converted wave starting from the source S, reflected at the conversion point C, and terminating at the receiver R. The symbol x denotes the offset of the receiver from the source, and x_C denotes the offset of the conversion point. The symbol H denotes the depth of the reflector, and θ_P and θ_S are the angles of incidence and reflection, respectively.

TRAVELTIME FORMULA

We consider the Cartesian coordinate system, whose x_1- and x_2-axes are horizontal, the x_3-axis is vertical and positive downward. The coordinate system is right handed. We consider a homogeneous layer underlaid, at the depth H, by a horizontal reflector (see Figure 1). The layer is VTI. In this layer, we consider a P-SV converted wave. It propagates as a P-wave from the source S to the conversion point C at the reflector, and as an SV wave from C to the receiver R, see Figure 1 again. Without loss of generality, we can consider the profile along the x_1-axis, which means that the SV wave is polarized in the vertical (x_1, x_3) plane. The traveltime along the ray of the converted wave from S to R via C is $T = T_\mathrm{P} + T_\mathrm{SV}$, where T_P is the traveltime along the P-wave leg of the ray, and T_SV along the SV-wave ray leg. From the geometry of the ray of the converted wave, we can derive the following expressions for the squares of traveltimes along the P- and SV-wave ray legs:

$$T^2_\mathrm{P}(x) = \frac{x^2_C + H^2}{v^2_\mathrm{P}(\mathbf{N}^\mathrm{P})}, \qquad T^2_\mathrm{SV}(x) = \frac{(x - x_C)^2 + H^2}{v^2_\mathrm{SV}(\mathbf{N}^\mathrm{SV})}, \tag{1}$$

where x_C is the offset of the conversion point of the P-SV converted wave in the VTI medium. In equation 1, $v_\mathrm{P}(\mathbf{N}^\mathrm{P})$ and $v_\mathrm{SV}(\mathbf{N}^\mathrm{SV})$ denote the P- and SV-wave ray velocities in the VTI medium, respectively. They depend on vectors $\mathbf{N}^\mathrm{P}$ and $\mathbf{N}^\mathrm{SV}$, unit vectors parallel to the P- and SV-wave ray legs of the converted wave. We call the vectors $\mathbf{N}^\mathrm{P}$ and $\mathbf{N}^\mathrm{SV}$ ray vectors. Before proceeding further, let us introduce the normalized offset $\bar{x}$ and normalized offset of the conversion point $\bar{x}_C$

$$\bar{x} = \frac{x}{H}, \qquad \bar{x}_C = \frac{x_C}{H}. \tag{2}$$

In addition, let us introduce the one-way zero-offset traveltimes T_{0_P} and T_{0_S} in the reference isotropic medium with P- and S-wave velocities α and β

$$T_{0_\mathrm{P}} = \frac{H}{\alpha}, \qquad T_{0_\mathrm{S}} = \frac{H}{\beta}. \tag{3}$$

Taking into account equations 2 and 3, equation 1 reads

$$T^2_\mathrm{P}(\bar{x}) = T^2_{0_\mathrm{P}}\alpha^2\frac{1+\bar{x}^2_C}{v^2_\mathrm{P}(\mathbf{N}^\mathrm{P})}, \quad T^2_\mathrm{SV}(\bar{x}) = T^2_{0_\mathrm{S}}\beta^2\frac{1+(\bar{x}-\bar{x}_C)^2}{v^2_\mathrm{SV}(\mathbf{N}^\mathrm{SV})}. \tag{4}$$

If $\bar{x}_C$, v_P, and v_SV are exact, then the traveltime $T = T_\mathrm{P} + T_\mathrm{SV}$, constructed from T_P and T_SV given in equation 4, is also exact. At this point, we shall make two important approximations, which we also did in our previous studies.

The first approximation is related to the fact that the actual ray of the converted wave is unknown. As, for example, Pšenčík and Farra (2017) or Farra and Pšenčík (2017a), we replace the actual ray by a reference ray of the converted wave in the reference isotropic medium with P- and S-wave velocities α and β, respectively. The normalized offset $\bar{x}_C$ of the conversion point is thus sought in the reference isotropic medium, and it can be determined by the procedures described in Appendix A. In this way, we replace the actual ray by the ray whose deviation from the actual ray is of the first order, and the traveltime along it represents the first-order approximation of the actual traveltime (Fermat's principle). Because, in contrast to pure-mode reflected waves, the reference ray of a converted wave is composed of P- and S-wave ray legs, its form is affected by the ratio of P- and S-wave velocities. To minimize the deviations of the actual and reference raypaths, the ratio of the reference velocities $r = \beta/\alpha$ should be chosen close to the actual one. In the described approximation, the vectors $\mathbf{N}^{\mathrm{P}}$ and $\mathbf{N}^{\mathrm{SV}}$, parallel to the actual ray, are replaced by vectors $\mathbf{N}^{\mathrm{P}}$ and $\mathbf{N}^{\mathrm{S}}$ parallel to the P- and S-wave legs of the reference ray. Note that we keep the same notation for $\mathbf{N}^{\mathrm{P}}$ as in the case of an actual P-wave ray, but use $\mathbf{N}^{\mathrm{S}}$ instead of $\mathbf{N}^{\mathrm{SV}}$. Components of the ray vectors $\mathbf{N}^{\mathrm{P}}$ and $\mathbf{N}^{\mathrm{S}}$ are specified by the same formulas, as those used in the above-mentioned references. The components of the vector $\mathbf{N}^{\mathrm{P}}$ in the plane (x_1, x_3) are

$$N_1^{\mathrm{P}} = \frac{\bar{x}_C}{\sqrt{1+\bar{x}_C^2}}, \quad N_2^{\mathrm{P}} = 0, \quad N_3^{\mathrm{P}} = \frac{1}{\sqrt{1+\bar{x}_C^2}}. \tag{5}$$

The components of the vector $\mathbf{N}^{\mathrm{S}}$ in the same plane are

$$N_1^{\mathrm{S}} = \frac{\bar{x}-\bar{x}_C}{\sqrt{1+(\bar{x}-\bar{x}_C)^2}}, \ N_2^{\mathrm{S}} = 0, \ N_3^{\mathrm{S}} = -\frac{1}{\sqrt{1+(\bar{x}-\bar{x}_C)^2}}. \tag{6}$$

The negative sign in the expression for N_3^{S} indicates upgoing character of the S-wave ray leg.

The second approximation consists in the replacement of exact squares of ray velocities $v_{\mathrm{P}}^2(\mathbf{N}^{\mathrm{P}})$ and $v_{\mathrm{SV}}^2(\mathbf{N}^{\mathrm{SV}})$ in equation 4 by their approximations $\widetilde{v_{\mathrm{P}}^2}(\mathbf{N}^{\mathrm{P}})$ and $\widetilde{v_{\mathrm{SV}}^2}(\mathbf{N}^{\mathrm{S}})$. A tilde above the quantities indicates that they are of the first order in the WA parameters. As Pšenčík and Farra (2017) or Farra and Pšenčík (2017a), we approximate exact squares of ray velocities by the first-order approximations of squares of phase velocities in the corresponding directions $\mathbf{N}$. Using equation 24 of Pšenčík and Farra (2005) and generalization of equation 29 of Farra and Pšenčík (2013) for $\epsilon_z \neq 0$, we have, in the notation of this paper

$$\widetilde{v_{\mathrm{P}}^2}(\mathbf{N}^{\mathrm{P}}) \sim \widetilde{c_{\mathrm{P}}^2}(\mathbf{N}^{\mathrm{P}}) \sim \alpha^2\{1 + 2[\epsilon_x(N_1^{\mathrm{P}})^4 + \delta_y(N_1^{\mathrm{P}})^2(N_3^{\mathrm{P}})^2 + \epsilon_z(N_3^{\mathrm{P}})^4]\} \tag{7}$$

and

$$\widetilde{v_{\mathrm{SV}}^2}(\mathbf{N}^{\mathrm{S}}) \sim \widetilde{c_{\mathrm{SV}}^2}(\mathbf{N}^{\mathrm{S}}) \sim \beta^2[1 + 2\gamma_y + 2r^{-2}(\epsilon_x + \epsilon_z - \delta_y)(N_1^{\mathrm{S}})^2(N_3^{\mathrm{S}})^2]. \tag{8}$$

Symbols c_{P} and c_{SV} denote P- and SV-wave phase velocities, r is the ratio of the reference velocities α and β, $r = \beta/\alpha$. Symbols ϵ_x, ϵ_z, δ_y, and γ_y are the WA parameters defined in Appendix B. It is important to note that neither equation 7 nor 8 is dependent on α or β. To prove this, it is sufficient to insert the expressions for the WA parameters given in equation B-1 to equations 7 and 8. We can also see that equations 7 and 8 would describe an isotropic medium with P- and S-wave velocities α and β if $\epsilon_x = \epsilon_z = \delta_y = \gamma_y = 0$. Let us mention that we prefer to use the above approximations rather than the approximation of the slowness v^{-1} because we found that the use of the expressions 7 and 8 leads to more accurate (and independent of α and β) results (see Pšenčík and Farra, 2017).

Inserting equations 5 and 6 into equations 7 and 8, we obtain approximate expressions for squares of ray velocities expressed in terms of the normalized offset $\bar{x}$ and normalized conversion offset $\bar{x}_C$. The resulting expressions inserted to the traveltime formulas 4 yield

$$T_{\mathrm{P}}^2(\bar{x}) = T_{0_{\mathrm{P}}}^2 \frac{(1+\bar{x}_C^2)^3}{P_{\mathrm{P}}(\bar{x}_C)}, \quad T_{\mathrm{SV}}^2(\bar{x}) = T_{0_{\mathrm{S}}}^2 \frac{[1+(\bar{x}-\bar{x}_C)^2]^3}{P_{\mathrm{SV}}(\bar{x}-\bar{x}_C)}. \tag{9}$$

Symbols $P_{\mathrm{P}}(x)$ and $P_{\mathrm{SV}}(x)$ represent polynomials

$$P_{\mathrm{P}}(x) = (1+x^2)^2 + 2\epsilon_x x^4 + 2\delta_y x^2 + 2\epsilon_z \tag{10}$$

and

$$P_{\mathrm{SV}}(x) = (1+x^2)^2(1+2\gamma_y) + 2r^{-2}(\epsilon_x + \epsilon_z - \delta_y)x^2. \tag{11}$$

For the square of the total traveltime $T^2 = (T_{\mathrm{P}} + T_{\mathrm{SV}})^2$, we thus get

$$T^2(\bar{x}) = \left\{ T_{0\mathrm{P}} \frac{(1+\bar{x}_C^2)^{3/2}}{P_{\mathrm{P}}^{1/2}(\bar{x}_C)} + T_{0\mathrm{S}} \frac{[1+(\bar{x}-\bar{x}_C)^2]^{3/2}}{P_{\mathrm{SV}}^{1/2}(\bar{x}-\bar{x}_C)} \right\}^2. \tag{12}$$

This is the final approximate converted-wave moveout formula for VTI media. Through the polynomials $P_{\mathrm{P}}(x)$ and $P_{\mathrm{SV}}(x)$, equation 12 depends on four WA parameters ϵ_x, ϵ_z, δ_y, and γ_y.

Note that equation 12 depends on the choice of the parameters of the reference medium because $\bar{x}_C$ depends on $r = \beta/\alpha$ (see equation A-3 or equations A-4 and A-5). Also note that it would be possible to expand the square roots and terms in the denominators to get rid of them. The cost would be a significant loss in accuracy. Compare the results of traveltime approximations I and II in Pšenčík and Farra (2017), where a similar expansion was made.

In an isotropic medium with P- and S-wave velocities α and β, equation 12 reduces to

$$T^2(\bar{x}) = \{T_{0\mathrm{P}}(1+\bar{x}_C^2)^{1/2} + T_{0\mathrm{S}}[1+(\bar{x}-\bar{x}_C)^2]^{1/2}\}^2. \tag{13}$$

Because we are considering a VTI layer with a horizontal reflector, equation 12 can be rewritten into the form of the Taylor series expansion of the squared traveltime T^2 with respect to the squared offset x^2

$$T^2(x) = T^2(0) + v_{\mathrm{NMO}}^{-2}x^2 + A_4x^4 + \ldots. \tag{14}$$

For dipping reflector, equation 14 would also contain odd terms. From equation 14, it is possible to find the expressions for the two-way zero-offset traveltime $T(0)$, the NMO velocity v_{NMO}, and the quartic term A_4 in terms of WA parameters.

We start with the two-way zero-offset traveltime $T(0)$. Although the corresponding amplitude of the converted wave is zero, $T(0)$ is of practical use. If we take equation C-2 and insert into it expressions for $T_{0_{\mathrm{P}}}$, $T_{0_{\mathrm{S}}}$, ϵ_z, and γ_y, we obtain

$$T(0) = H(A_{33}^{-1/2} + A_{55}^{-1/2}). \tag{15}$$

The two-way zero-offset traveltime $T(0)$ is exact (parameters A_{33} and A_{55} represent squares of vertical P and SV ray velocities in the VTI medium), and it is thus independent of the choice of the reference velocities α and β.

Expressions for the NMO velocity and the quartic term of the Taylor series expansion of the squared traveltime of the converted wave can be obtained from equations given in Appendix C. The two quantities depend on four WA parameters ϵ_x, ϵ_z, δ_y, and γ_y and on the parameters of the reference medium through the ratio r of S- and P-wave velocities.

In the following, we use a special choice of reference velocities α and β, which leads to the simplification of the formulas. We choose

$$\alpha^2 = A_{33}, \qquad \beta^2 = A_{55}, \tag{16}$$

which yields

$$\epsilon_z = 0, \qquad \gamma_y = 0. \tag{17}$$

With the specification in equation 16, the following expressions for the NMO velocity and the quartic term depend on only two WA parameters ϵ_x and δ_y.

Inserting equations A-5, C-1, C-2, C-7, and C-8 to equation C-5, we obtain for the NMO velocity the expression

$$v_{\mathrm{NMO}}^{-2} = (\alpha\beta)^{-1}\left[1 - 2\frac{\delta_y(r-1)+\epsilon_x}{r(r+1)}\right]. \tag{18}$$

Equation 18 corresponds to equation 27 of Tsvankin and Grechka (2000) or equation 5.130 of Tsvankin (2001). Mentioned references use Thomsen's (1986)δ_T instead of δ_y used in equation 18 and defined in equation B-1.

In an isotropic layer, equation 18 reduces to

$$v_{\mathrm{NMO}}^{-2} = (\alpha\beta)^{-1}. \tag{19}$$

The approximate formula for the quartic term is obtained from equation C-13 in combination with equations A-5, C-1, C-2, C-7, C-8, C-14, and C-15

$$A_4 = -\frac{1}{4}T^{-2}(0)(\alpha\beta)^{-2}r^{-1} \times\left\{(1-r)^2 + \frac{4(1-r)}{1+r}[\epsilon_x - 2r^{-1}(1-r)(r\delta_y+\epsilon_x-\delta_y)] - \frac{4}{1+r}\left[3\delta_y^2 + 3r^{-1}(\epsilon_x-\delta_y)^2 + \frac{(r\delta_y+\epsilon_x-\delta_y)^2}{r(1+r)}\right]\right\}. \tag{20}$$

In an isotropic layer with P- and S-wave velocities α and β, respectively, the approximate expression in equation 20 reduces to

$$A_4 = -\frac{1}{4}T^{-2}(0)(\alpha\beta)^{-2}r^{-1}(1-r)^2. \tag{21}$$

The series expansion of equation 12 contains, in addition to the terms shown in equation 14, also higher order terms, which we do not present here. Use of the expansion 14 up to the quartic term instead of equation 12, thus leads to the reduction of accuracy of the moveout formula 12. The accuracy of equation 14 is usually enhanced by adding a denominator to the quartic term, the so-called rational approximation. See the following section.

REFERENCE MOVEOUT FORMULA

In the following, we also compare the accuracy of the moveout formulas 12 and 13 with the rational approximation of moveout formula proposed by Tsvankin and Thomsen (1994). It was later modified and generalized by, for example, Thomsen (1999) and Li and Yuan (2003). Hao and Stovas (2016) propose the generalized moveout approximation that includes all types of rational approximations as special cases. For comparison with equation 12 proposed in this paper, we use the formula

$$T^2(x) = T^2(0) + (v_{\mathrm{NMO}}^{ex})^{-2}x^2 + \frac{A_4^{\mathrm{ex}}x^4}{1+Bx^2}. \tag{22}$$

The symbol $T^2(0)$ in it is the exact two-way zero-offset traveltime 15, $v_{\mathrm{NMO}}^{\mathrm{ex}}$ is the exact NMO velocity

$$(v_{\mathrm{NMO}}^{\mathrm{ex}})^2 = \alpha_T\beta_T\left[1 + 2\frac{\delta_T(r_T-1)+\epsilon_T}{r_T(1+r_T)}\right]; \tag{23}$$

see Thomsen (1999), equation 23, and Hao and Stovas (2016), equation 26. Let us note that equation 18 derived from the proposed moveout formula represents the first-order weak-anisotropy approximation of the exact NMO velocity equation 23. The symbol A_4^{ex} in equation 22 represents the exact quartic term

$$A_4^{\mathrm{ex}} = -\frac{1}{4}T^{-2}(0)(v_{\mathrm{NMO}}^{\mathrm{ex}})^{-4}r_T^{-1}\frac{(1-r_T^2+2\epsilon_T)^2}{[1+r_T+2\delta_T+2r_T^{-1}(\epsilon_T-\delta_T)]^2}. \tag{24}$$

From comparison of equation 24 with equation 20, one can find that equation 20 represents the first-order weak-anisotropy approximation of the exact quartic term 24. In equations 23 and 24, ϵ_T and δ_T are Thomsen's (1986) parameters and r_T is the ratio of vertical velocities β_T and α_T, $r_T = \beta_T/\alpha_T = \sqrt{A_{55}/A_{33}}$. The factor B in equation 22 is given by

$$B = A_4^{\mathrm{ex}}\frac{A_{11}(v_{\mathrm{NMO}}^{\mathrm{ex}})^2}{(v_{\mathrm{NMO}}^{\mathrm{ex}})^2 - A_{11}}, \tag{25}$$

which follows, for example, from equations of Appendix A of Thomsen (1999). The parameter A_{11} can be expressed through the Thomsen's (1986) parameter, ϵ^T, $A_{11} = \alpha_T^2(1+2\epsilon^T)$.

TESTS OF ACCURACY

We test equations 13 and 12 for P-SV converted waves in the isotropic model with P-wave velocity $\alpha = 2.5$ km/s and varying ratio $r = \beta/\alpha$, in the limestone model, whose P- and SV-wave anisotropy are approximately 8% and 5%, respectively, and the Mesa Verde mud shale and the hard shale models with P-wave anisotropy of approximately 6% and 25%, respectively, and an SV-wave anisotropy of approximately 12%. The anisotropy percentage is defined as $2(c_{\max} - c_{\min})/(c_{\max} + c_{\min}) \times 100\%$, where c denotes the corresponding phase velocity. The reference velocities α and β are chosen equal the vertical P- and SV-wave phase velocities, which results in $\epsilon_z = \gamma_y = 0$; see Table 1.

In the following figures, we present plots of relative traveltime errors $(T - T_{\mathrm{cal}})/T_{\mathrm{cal}} \times 100\%$. Here, T is the traveltime calculated from equation 12, 13, or 22, and T_{cal} is the traveltime calculated using the package ANRAY (Gajewski and Pšenčík, 1990), which we take as an exact reference.

Isotropic model

In Figure 2, we show results of tests for the isotropic model. The dashed black curve in Figure 2a shows that the use of the numerical solution of equation A-3 in equation 13 leads to exact results. It is because equation A-3 is an equation for the exact determination of the conversion point in an isotropic medium. Use of the approximate expression A-4 instead of the solution of equation A-3 introduces certain errors (the colored solid curves corresponding to varying ratios $r = \beta/\alpha$). These errors however, do not exceed 0.5%. Figure 2b shows comparison of the dashed black curve for $r = 0.4$ (it corresponds to the solid black curve in Figure 2a) with colored curves obtained for varying values of r from the reference formula in equation 22, which represents the so-called rational approximation (Tsvankin and Thomsen, 1994). We can see that this formula yields satisfactory results (comparable with formula 13) only to the normalized offsets of $\bar{x} \sim 1-2$; the higher the ratio r, the larger the offset to which the formula in equation 22 yields satisfactory results. For larger offsets, the accuracy of the reference formula rapidly decreases, whereas the results of formula in equation 13 remain satisfactorily accurate.

Anisotropic models

Figures 3, 4, and 5 contain results of tests for the limestone, the Mesa Verde mud shale and the hard shale models, respectively. Each figure consists of two plots. The (a) plots contain two curves, both obtained from equation 12. The black curve is obtained with the conversion point determined from the approximate expression A-4, and the red curve is obtained by numerically solving the quartic equation A-3. In the plots (b) , we compare the performance of equation 12 with the performance of equation 22. The curve corresponding to equation 12 with the conversion point determined from the approximate expression A-4 is black, and the curve corresponding to the reference formula 22 with equations 15, 23–25 is red.

Limestone model

In Figure 3, we show the results for the weakly anisotropic limestone model. In Figure 3a, we can see that $\bar{x}_C$ determined by solving numerically quartic equation A-3 leads to relative traveltime errors less than 0.1% for normalized offsets from 0 to 8. The use of the normalized conversion offset $\bar{x}_C$ determined from the approximate equation A-4 leads to slightly larger errors, but they are still less than 0.2% for normalized offsets between 0 and 8. These relative traveltime errors are comparable with relative errors of the first-order formula for pure-mode reflected P waves, but they are less than errors of the first-order formula for pure-mode reflected SV waves. Compare Figure 3a of this paper with Figures 1 and 3 of Farra and Pšenčík (2013). It is also of interest to

Table 1. Parameters of the models used. The α and β — P- and S-wave reference velocities, ϵ_x, δ_y, ϵ_z, and γ_y — WA parameters.

Model	α(km/s)	β(km/s)	ϵ_x	δ_y	ϵ_z	γ_y
Limestone	3.0	1.707	0.076	0.133	0	0
Mesa Verde mud shale	4.53	2.703	0.034	0.184	0	0
Hard shale	3.0	1.914	0.252	0.034	0	0

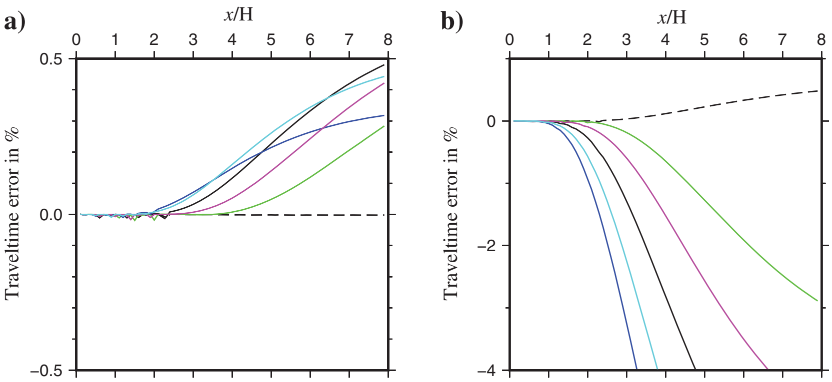

Figure 2. P-SV-wave moveout in the isotropic model. P-wave velocity $\alpha = 2.5$ km/s, varying ratio $r = \beta/\alpha$, $r = 0.2$ (blue), $r = 0.3$ (light blue), $r = 0.4$ (black), $r = 0.5$ (pink), and $r = 0.6$ (green). Variation with the normalized offset $\bar{x} = x/H$ of the relative traveltime error of (a) the approximate equation 13 with the conversion point estimated for varying r from the relation A-4 (colored solid) and the conversion point determined by the numerical solution of equation A-3 for $r = 0.4$ (black dashed); (b) the approximate equation 13 with the conversion point estimated from the relation A-4 for $r = 0.4$ (black dashed) and the reference equation 22 for varying r (colored solid).

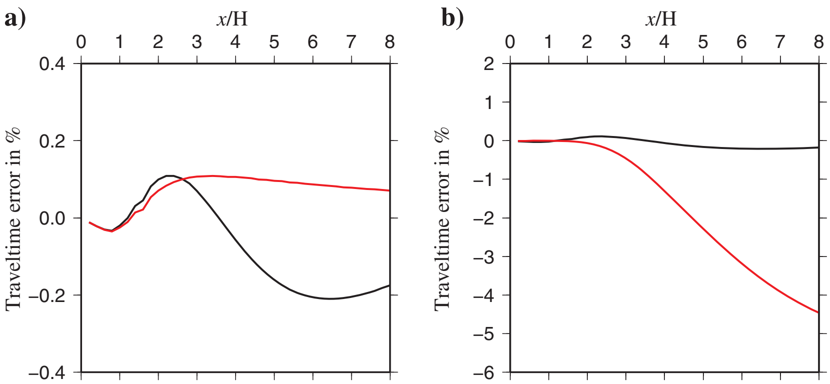

Figure 3. P-SV-wave moveout in the limestone model, P-wave anisotropy of approximately 8%, SV-wave anisotropy of approximately 5%. Variation with the normalized offset $\bar{x} = x/H$ of the relative traveltime error of (a) the approximate equation 12 with the conversion point determined from equation A-4 (black) and the conversion point determined by the numerical solution of equation A-3 (red) and (b) the approximate equation 12 with the conversion point determined from equation A-4 (black) and of the reference equation 22 (red).

compare exact and approximate values of NMO velocity in the limestone model. From equations 23 and 18, we get $v_{\mathrm{NMO}}^{\mathrm{ex}} = 2.296$ km/s and $v_{\mathrm{NMO}} = 2.312$ km/s, respectively. Thus, equation 18 yields quite accurate results. In Figure 3b, we compare the black curve corresponding to the black curve in Figure 3a, obtained from formula 12, with the red curve obtained from the reference formula in equation 22. The latter formula yields very accurate results up to the normalized offset $\bar{x} \sim 2$. Then, its accuracy strongly decreases. The accuracy of the formula in equation 12 remains high.

Mesa Verde mud shale model

Figure 4 shows relative traveltime errors of equation 12 applied to the Mesa Verde mud shale model. In Figure 4a, the errors are slightly larger than in Figure 3a (the S-wave anisotropy is stronger), but they do not exceed 0.5% for the normalized offsets between 0 and 8. These errors are substantially smaller than the errors of the first-order formula for the pure-mode reflected SV wave, compare Figure 4a with Figure 4 of Farra and Pšenčík (2013). The difference between the approximate value of the NMO velocity obtained from equation 18 $v_{\mathrm{NMO}} = 3.359$ km/s and the exact value $v_{\mathrm{NMO}}^{\mathrm{ex}} = 3.306$ km/s obtained from equation 23 is again quite small despite stronger anisotropy. For normalized offsets $\bar{x}$ less than $\bar{x} \sim 3$, the reference formula in equation 22 represented by the red curve in Figure 4b, yields better results than the black curve obtained from formula in equation 12 with the conversion point determined from the approximate expression A-4. For larger offsets, however, its accuracy rapidly decreases. The formula in equation 12, although also more inaccurate (but with errors not exceeding 0.5%) than in Figure 3, yields more satisfactory results.

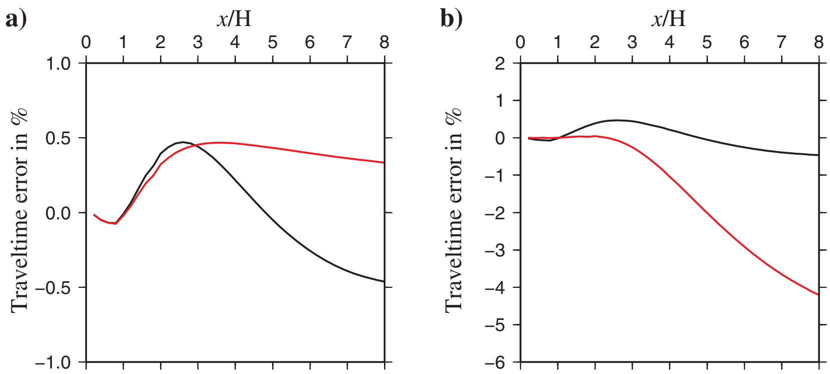

Figure 4. P-SV-wave moveout in the Mesa Verde mud shale model, P-wave anisotropy of approximately 6%, and SV-wave anisotropy of approximately 12%. Variation with the normalized offset $\bar{x} = x/H$ of the relative traveltime error of (a) the approximate equation 12 with the conversion point determined from equation A-4 (black) and the conversion point determined by the numerical solution of equation A-3 (red) and (b) the approximate equation 12 with the conversion point determined from equation A-4 (black) and of the reference equation 22 (red).

Hard shale model

Because of the stronger P-wave anisotropy of the hard shale, relative traveltime errors of equation 12 in Figure 5 are larger. They nearly reach 2% around the normalized offset $\bar{x} \sim 2$, see Figure 5a. For the remaining offsets, the errors are, however, smaller, and they tend to zero for increasing offsets. In this case, the accuracy of equation 12 is higher than the accuracy of the first-order formula for the pure-mode reflected SV wave. This follows from comparison of Figure 5a with Figure 5 of Farra and Pšenčík (2013). The accuracy of the approximate NMO velocity formula in equation 18 is, however, lower than in the previous models. Equation 18 yields $v_{\mathrm{NMO}} = 3.257$ km/s, whereas the exact value obtained from equation 23 is $v_{\mathrm{NMO}}^{\mathrm{ex}} = 2.893$ km/s. The reasons are stronger anisotropy and also strong positive value of the difference $\epsilon_x - \delta_y$, which was not the case in the previous models. In Figure 5b, we can see that the reference formula in equation 22 yields much better results (errors of less than 1%) than formula in equation 12 for the normalized offsets less than $\bar{x} \sim 4$. For larger offsets, the accuracy of formula in equation 22 decreases, whereas the formula in equation 12 yields results that converge to zero error with the increasing offset. The increased errors of formula in equation 12 for small and intermediate offsets are obviously caused by strong anisotropy, the strong positive difference $\epsilon_x - \delta_y$, and, as in our previous studies, by the deviation of ray $\mathbf{N}$ and phase $\mathbf{n}$ vectors (also related to the strength of the anisotropy). The phase vector $\mathbf{n}$ is a unit vector in the direction of the slowness vector.

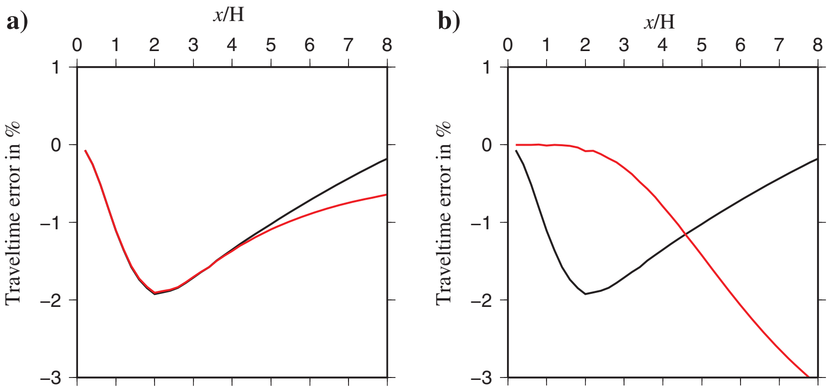

Figure 5. P-SV-wave moveout in the hard shale model, P-wave anisotropy of approximately 25%, SV-wave anisotropy of approximately 12%. Variation with the normalized offset $\bar{x} = x/H$ of the relative traveltime error of (a) the approximate equation 12 with the conversion point determined from equation A-4 (black) and the conversion point determined by the numerical solution of equation A-3 (red) and (b) the approximate equation 12 with the conversion point determined from equation A-4 (black) and of the reference equation 22 (red).

CONCLUSION

We derived an approximate, explicit, and relatively simple reflection-moveout formula for a converted wave in a weakly or moderately anisotropic homogeneous VTI layer. The formula relates, in a simple and transparent way, traveltimes calculated along a reference ray of the converted wave to the parameters of the medium represented by WA parameters. Along a profile, the formula depends on four WA parameters and the ratio r of S- and P-wave velocities of the reference medium. The ratio r controls the form of the reference ray. The reference ray approximates the actual one well when the ratio r does not differ significantly from the actual velocity ratio.

Although the derivations and tests were performed for the P-SV converted wave, due to its kinematic reciprocity, the formula holds also for the SV-P converted wave.

Performed tests indicate that the accuracy of the moveout formula is close to the accuracy of formulas derived in a similar way earlier for pure-mode reflected P or SV waves. As in similar previous studies, the moveout formula yields satisfactory results for all offsets without necessity to trace rays in actual anisotropic medium to finite offsets as in some moveout approximations to increase their accuracy.

The tests also show that the moveout formula can be used not only for weakly but also for moderately anisotropic media. In isotropic media, the proposed moveout formula yields results of high accuracy; when numerical solution of quartic equation is used for the determination of the conversion point, the moveout formula yields exact results.

The procedure described in this paper can be extended to waves reflected at the bottom of a stack of VTI layers of moderate anisotropy. As in this paper, two basic approximations will be used. The approximate traveltimes will be calculated along the reference rays of reflected waves in a reference medium composed of isotropic layers. For the evaluation of traveltimes, the weak-anisotropy approximation of the ray velocity will be used. Another possible extension of the presented moveout formula is to the so-called dip-constrained TI medium, whose symmetry axis is perpendicular to the reflector. Using the concept of a common S-wave, the moveout formula for the converted wave could probably be generalized for anisotropic media of arbitrary symmetry and orientation.

ACKNOWLEDGMENTS

We thank Y. Sripanich, A. Stovas and an anonymous referee for useful comments. We are grateful to the Research Project 16-05237S of the Grant Agency of the Czech Republic and the project “Seismic waves in complex 3-D structures” (SW3D) for support.

DATA AND MATERIALS AVAILABILITY

Data associated with this research are available and can be obtained by contacting the corresponding author.

APPENDIX A

DETERMINATION OF THE CONVERSION POINT IN THE REFERENCE ISOTROPIC MEDIUM

Let us consider a homogeneous isotropic layer underlaid by a horizontal reflector. On the surface of the layer, we consider source S and receiver R. These two points are connected by the ray of a P-S converted wave with the conversion point C at the reflector (see Figure 1). In the following, we follow closely the derivation of Thomsen (1999), but we use slightly different notation corresponding to the notation, which we used in our previous studies. We denote the P- and S-wave velocities by α and β, respectively, and their ratio $r=\beta/\alpha$. The angles of incidence and reflection of a ray of the P-S converted wave are denoted by θ_P and θ_S. By x, we denote the source-receiver offset and by x_C, the offset of the conversion point. The depth of the layer is H.

Elementary trigonometry considerations based on the sketch in Figure 1 yield for $\sin\theta_P$ and $\sin\theta_S$

$$\sin\theta_P=\frac{x_C}{\sqrt{x_C^2+H^2}},\quad \sin\theta_S=\frac{x-x_C}{\sqrt{(x-x_C)^2+H^2}}. \tag{A-1}$$

Combination of equation A-1 with the Snell law, $\sin\theta_P/\alpha=\sin\theta_S/\beta$, leads to the equation:

$$(x-x_C)^2(x_C^2+H^2)r^{-2}=x_C^2[(x-x_C)^2+H^2]. \tag{A-2}$$

Equation A-2 is equivalent to equation 14 of Thomsen (1999). Normalizing it by H^4, using notation introduced in equation 2 and rearranging equation A-2 to the form of a polynomial equation, we get

$$\bar{x}_C^4-2\bar{x}\bar{x}_C^3+(1+\bar{x}^2)\bar{x}_C^2-\frac{2\bar{x}\bar{x}_C}{1-r^2}+\frac{\bar{x}^2}{1-r^2}=0. \tag{A-3}$$

This is a quartic polynomial equation for the normalized offset of the conversion point $\bar{x}_C$. For the pure-mode reflected wave ($r=1$), equation A-3 yields the expected $\bar{x}_C=\bar{x}/2$. Equation A-3 can be solved analytically using, for example, the so-called Ferrari procedure. It can also be solved numerically. Tessmer and Behle (1988) derive an approximate explicit formula for the determination of the offset of the conversion point. The formula is improved by Thomsen (1999). Taking into account the normalization specified in equation 2, we use here the approximate formula of Thomsen (1999) in the form

$$\bar{x}_C\sim\bar{x}\left(C_0+C_2\frac{\bar{x}^2}{1+C_3\bar{x}^2}\right), \tag{A-4}$$

where

$$C_0=\frac{1}{1+r},\quad C_2=\frac{r}{2}\frac{1-r}{(1+r)^3},\quad C_3=\frac{1}{2}\frac{1-r}{(1+r)^2}. \tag{A-5}$$

From equations A-4 and A-5, we can see that the normalized offset $\bar{x}_C$ of the conversion point depends on the parameters of the reference medium only through the ratio r of the S- and P-wave velocities. For a detailed study of the accuracy of the expression A-4, see Thomsen (1999).

APPENDIX B

WA PARAMETERS USED IN THE STUDY

WA parameters ϵ_x, ϵ_z, δ_y, and γ_y used in equations 7, 8, 10, and 11 of the main text are defined as

$$\epsilon_x=\frac{A_{11}-\alpha^2}{2\alpha^2},\quad \epsilon_z=\frac{A_{33}-\alpha^2}{2\alpha^2},$$
$$\delta_y=\frac{A_{13}+2A_{55}-\alpha^2}{\alpha^2},\quad \gamma_y=\frac{A_{55}-\beta^2}{2\beta^2}, \tag{B-1}$$

where A_{11}, A_{13}, A_{33}, and A_{55} denote density-normalized elastic moduli in the Voigt notation, and α and β are the P- and S-wave velocities in the reference isotropic medium.

APPENDIX C

TWO-WAY ZERO OFFSET TRAVELTIME $T(0)$, NMO VELOCITY v_{NMO} AND QUARTIC TERM A_4

From equations 9 to 11 specified for the zero-offset case, we get

$$T_{\mathrm{P}}(0)=T_{0_{\mathrm{P}}}(1+2\epsilon_z)^{-1/2} \text{ and } T_{\mathrm{SV}}(0)=T_{0_{\mathrm{S}}}(1+2\gamma_y)^{-1/2}. \tag{C-1}$$

From equation C-1, we get the expressions for the two-way zero-offset traveltime $T(0)$:

$$T(0) = T_{0_{\mathrm{P}}}(1+2\epsilon_z)^{-1/2} + T_{0_{\mathrm{S}}}(1+2\gamma_y)^{-1/2}. \tag{C-2}$$

Note that $T_{\mathrm{P}}(0)$, $T_{\mathrm{SV}}(0)$, and $T(0)$ do not depend on the choice of the parameters of the reference medium. It can be proved by substituting equations 3 and B-1 to equations C-1 and C-2.

Inverse square of the NMO velocity v_{NMO} of the P-SV converted wave is given by the standard expression

$$v_{\mathrm{NMO}}^{-2} = \left.\frac{dT^2}{dx^2}\right|_{x=0} = \left.\frac{d(T_{\mathrm{P}}+T_{\mathrm{SV}})^2}{dx^2}\right|_{x=0} = 2T(0)\left.\left(\frac{dT_{\mathrm{P}}}{dx^2}+\frac{dT_{\mathrm{SV}}}{dx^2}\right)\right|_{x=0}. \tag{C-3}$$

Let us introduce NMO velocities $v_{\mathrm{NMO}}^{\mathrm{P}}$ and $v_{\mathrm{NMO}}^{\mathrm{SV}}$ related to the P- and SV-wave ray legs of the converted wave in the following way:

$$(v_{\mathrm{NMO}}^{\mathrm{P}})^{-2} = \left.\frac{dT_P^2}{dx_C^2}\right|_{x_C=0}, \qquad (v_{\mathrm{NMO}}^{\mathrm{SV}})^{-2} = \left.\frac{dT_{SV}^2}{d(x-x_C)^2}\right|_{x-x_C=0}. \tag{C-4}$$

Using the definitions C-4, equation C-3 can be rewritten in the form

$$v_{\mathrm{NMO}}^{-2} = T(0)\left[\frac{C_0^2(v_{\mathrm{NMO}}^{\mathrm{P}})^{-2}}{T_{\mathrm{P}}(0)} + \frac{(1-C_0)^2(v_{\mathrm{NMO}}^{\mathrm{SV}})^{-2}}{T_{\mathrm{SV}}(0)}\right]. \tag{C-5}$$

In the derivation of equation C-5, we used the relations:

$$\left.\frac{dx_C^2}{dx^2}\right|_{x=0} = C_0^2 \text{ and } \left.\frac{d(x-x_C)^2}{dx^2}\right|_{x=0} = (1-C_0)^2 \tag{C-6}$$

resulting from the differentiation of equation A-4 and the specification of the result for the zero offset $x=0$.

To evaluate equation C-5, we need to specify expressions for the NMO velocities $v_{\mathrm{NMO}}^{\mathrm{P}}$ and $v_{\mathrm{NMO}}^{\mathrm{SV}}$. Differentiation of formulas in equation 9 with respect to x_C^2 and $(x-x_C)^2$, respectively, yields expressions

$$(v_{\mathrm{NMO}}^{\mathrm{P}})^{-2} = \alpha^{-2}\frac{1+6\epsilon_z-2\delta_y}{(1+2\epsilon_z)^2} \tag{C-7}$$

and

$$(v_{\mathrm{NMO}}^{\mathrm{SV}})^{-2} = \beta^{-2}\frac{1+2\gamma_y-2r^{-2}(\epsilon_x+\epsilon_z-\delta_y)}{(1+2\gamma_y)^2}. \tag{C-8}$$

Note that NMO velocities $v_{\mathrm{NMO}}^{\mathrm{P}}$ and $v_{\mathrm{NMO}}^{\mathrm{SV}}$ do not depend on the parameters of the reference medium. The NMO velocity v_{NMO} in equation C-5, however, depends on them through the factor C_0 (see equation A-5), which depends on the ratio $r=\beta/\alpha$.

The derivation of the quartic term A_4 is similar, but slightly more involved. The quartic term A_4 is given by the expression

$$A_4 = \frac{1}{2}\frac{d}{dx^2}\left.\left(\frac{dT^2}{dx^2}\right)\right|_{x=0}. \tag{C-9}$$

We have

$$\begin{aligned}\frac{d}{dx^2}\left(\frac{dT^2}{dx^2}\right) &= \frac{1}{2}\left(T_{\mathrm{P}}^{-1}\frac{dT_{\mathrm{P}}^2}{dx^2} + T_{\mathrm{SV}}^{-1}\frac{dT_{\mathrm{SV}}^2}{dx^2}\right)^2 \\ &+ (T_{\mathrm{P}}+T_{\mathrm{SV}})\left[T_{\mathrm{P}}^{-1}\frac{d}{dx^2}\left(\frac{dT_{\mathrm{P}}^2}{dx^2}\right) + T_{\mathrm{SV}}^{-1}\frac{d}{dx^2}\left(\frac{dT_{\mathrm{SV}}^2}{dx^2}\right)\right. \\ &\left.-\frac{1}{2}T_{\mathrm{P}}^{-3}\left(\frac{dT_{\mathrm{P}}^2}{dx^2}\right)^2 - \frac{1}{2}T_{\mathrm{SV}}^{-3}\left(\frac{dT_{\mathrm{SV}}^2}{dx^2}\right)^2\right].\end{aligned} \tag{C-10}$$

Taking into account equations 2, 3, A-4, C-7 to equation C-9, we can express the term A_4 in the following way:

$$\begin{aligned}A_4 &= \frac{1}{4}\left[\frac{C_0^2}{(v_{\mathrm{NMO}}^{\mathrm{P}})^2T_{\mathrm{P}}(0)} + \frac{(1-C_0)^2}{(v_{\mathrm{NMO}}^{\mathrm{SV}})^2T_{\mathrm{SV}}(0)}\right]^2 \\ &+ \frac{1}{2}T(0)\left[T_{\mathrm{P}}^{-1}(0)\frac{d}{dx^2}\left.\left(\frac{dT_{\mathrm{P}}^2}{dx^2}\right)\right|_{x=0} + T_{\mathrm{SV}}^{-1}(0)\frac{d}{dx^2}\left.\left(\frac{dT_{\mathrm{SV}}^2}{dx^2}\right)\right|_{x=0}\right. \\ &\left.-\frac{C_0^4}{2(v_{\mathrm{NMO}}^{\mathrm{P}})^4T_{\mathrm{P}}^3(0)} - \frac{(1-C_0)^4}{2(v_{\mathrm{NMO}}^{\mathrm{SV}})^4T_{\mathrm{SV}}^3(0)}\right].\end{aligned} \tag{C-11}$$

In a way similar to the introduction of NMO velocities $v_{\mathrm{NMO}}^{\mathrm{P}}$ and $v_{\mathrm{NMO}}^{\mathrm{SV}}$ in equation C-4, we introduce quartic terms A_4^{P} and A_4^{SV}:

$$\begin{aligned}A_4^P &= \frac{1}{2}\frac{d}{dx_C^2}\left.\left(\frac{dT_{\mathrm{P}}^2}{dx_C^2}\right)\right|_{x_C=0}, \\ A_4^{\mathrm{SV}} &= \frac{1}{2}\frac{d}{d(x-x_C)^2}\left.\left(\frac{dT_{\mathrm{SV}}^2}{d(x-x_C)^2}\right)\right|_{x-x_C=0}.\end{aligned} \tag{C-12}$$

Using equations C-5 and C-12, and with the help of equation A-4, we can rewrite equation C-11 into the form

$$\begin{aligned}A_4 &= \frac{1}{4}v_{\mathrm{NMO}}^{-4}T^{-2}(0) + \frac{1}{2}T(0)\left[\frac{4C_0C_2}{(v_{\mathrm{NMO}}^{\mathrm{P}})^2T_{\mathrm{P}}(0)H^2}\right. \\ &-\frac{C_0^4}{2(v_{\mathrm{NMO}}^{\mathrm{P}})^4T_{\mathrm{P}}^3(0)} + \frac{2C_0^4A_4^P}{T_{\mathrm{P}}(0)} \\ &\left.-\frac{4(1-C_0)C_2}{(v_{\mathrm{NMO}}^{\mathrm{SV}})^2T_{\mathrm{SV}}(0)H^2} - \frac{(1-C_0)^4}{2(v_{\mathrm{NMO}}^{\mathrm{SV}})^4T_{\mathrm{SV}}^3(0)} + \frac{2(1-C_0)^4A_4^{\mathrm{SV}}}{T_{\mathrm{SV}}(0)}\right].\end{aligned} \tag{C-13}$$

To express the quartic term A_4 in equation C-13 in terms of parameters of the medium, we need to evaluate A_4^{P} and A_4^{SV} in equation C-12. From equations 9 to 11, we get

$$A_4^P = \frac{2}{\alpha^4 T_{0_P}^2 (1+2\epsilon_z)^3}[(1+2\epsilon_z)(\delta_y - \epsilon_x - \epsilon_z) + 2(\delta_y - 2\epsilon_z)^2] \tag{C-14}$$

and

$$A_4^{SV} = \frac{2}{\beta^4 T_{0_S}^2 (1+2\gamma_y)^3}[r^{-2}(1+2\gamma_y)(\epsilon_x + \epsilon_z - \delta_y) + 2r^{-4}(\epsilon_x + \epsilon_z - \delta_y)^2]. \tag{C-15}$$

Inserting equations C-14 and C-15 into equation C-13, we get the first-order expression for the quartic term A_4 of the converted wave. Note that the terms A_4^P and A_4^{SV} in equations C-14 and C-15 are independent of the choice of the reference medium, but the term A_4 in equation C-13 depends on it through the terms C_0 and C_2, which depend on the ratio $r = \beta/\alpha$.

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