P-WAVE REFLECTION MOVEOUT, SPREADING AND REFLECTION COEFFICIENT IN A HORIZONTALLY LAYERED MEDIUM OF ARBITRARY ANISOTROPY

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We present and test approximate formulae for P-wave reflection moveout, geometrical spreading and reflection coefficient in horizontally layered anisotropic media of arbitrary symmetry and orientation. The formulae are based on weak-anisotropy approximation and expressed in terms of A-parameters (Pšenčík et al., 2018). A-parameters form a set of 21 parameters, which represent an alternative to 21 independent elements of the stiffness tensor c_{ijkl} or elastic parameters $C_{\alpha\beta}$ in the Voigt notation. For the derivation of the approximate reflection coefficient of the unconverted P wave, weak-contrast approximation is also used.

The approximate moveout formula of the unconverted P wave reflected at the bottom of a stack of horizontal layers and recorded along an arbitrarily selected profile has the form (Farra and Pšenčík, 2020):

$$T^2(x) = [\sum_{i=1}^{N} T_i(x_i)]^2 \ , \qquad x = \sum_{i=1}^{N} x_i \ . \tag{1}$$

Here T is the traveltime of the reflected wave at the offset x. N is the number of layers. T_i and x_i are the traveltime and offset corresponding to the down- and up-going elements of the ray of the reflected wave in the i-th layer. With the use of two approximations: (1) the replacement of the actual ray by a ray in a reference isotropic medium and (2) the replacement of the exact ray velocity by its first-order weak-anisotropy approximation, the traveltime T_i reads:

$$T_i^2(\bar{x}_i) = T_{0i}^2(1+\bar{x}_i^2)^3/P(\bar{x}_i) \ . \tag{2}$$

Here, $T_{0i} = 2h_i/\alpha_i$ and $\bar{x}_i = x_i/2h_i$ are two-way zero-offset traveltime and normalized offset in the i-th layer, h_i and α_i are the thickness and the P-wave reference velocity of the i-th layer. The symbol $P(\bar{x}_i)$ in equation (2) denotes the polynomial:

$$P(\bar{x}) = (1+\bar{x}^2)^2 + 2\epsilon_x^P\bar{x}^4 + 2(\eta_y^P + \epsilon_x^P + \epsilon_z^P)\bar{x}^2 + 2\epsilon_z^P \ . \tag{3}$$

In equation (3), ϵ_x^P, ϵ_z^P and η_y^P are the profile A-parameters, which can be expressed as linear functions of 9 global A-parameters defined in the global coordinate system or functions of (number dependent on the symmetry of the considered anisotropy) crystal A-parameters defined in the coordinate system coinciding with the symmetry elements of the considered anisotropy.

As shown by Farra and Pšenčík (2021), the geometrical spreading of unconverted P wave reflected at the bottom of a stack of horizontal layers of varying arbitrary anisotropy can be expressed in the first-order weak-anisotropy approximation in the factorized form:

$$L(x,\varphi) = L_{\parallel}(x,\varphi) L_{\perp}(x,\varphi) \ . \tag{4}$$

Here

$$L_{\parallel}(x,\varphi) = \cos\psi\Big(\frac{\partial^2 T}{\partial x^2}\Big)^{-1/2} \ , \qquad L_{\perp}(x,\varphi) = x\Big(\frac{\partial^2 T}{\partial \varphi^2} + x\frac{\partial T}{\partial x}\Big)^{-1/2} \ . \tag{5}$$

Here φ is the azimuth of the source-receiver profile and ψ is the ray angle of the reference ray. $T = T(x,\varphi)$ is the traveltime given by equations (1)-(3).

Approximate reflection coefficient of unconverted P wave has the form (Pšenčík and Farra, 2022):

$$R_{PP}(\theta_i) = R^{iso}_{PP}(\theta_i) + \frac{1}{2}\Delta\epsilon^P_z + \frac{1}{2}(\Delta\delta^P_y - 8\frac{\bar{\beta}^2}{\bar{\alpha}^2}\Delta\gamma^P_y - \Delta\epsilon^P_z)\sin^2\theta_i + \frac{1}{2}\Delta\epsilon^P_x \sin^2\theta_i \tan^2\theta_i \ . \tag{6}$$

The symbol R^{iso}_{PP} denotes the P-wave reflection coefficient in the reference model, in which the two half-spaces are isotropic. It reads:

$$R^{iso}_{PP}(\theta_i) = \frac{1}{2}\frac{\Delta Z}{\bar{Z}} + \frac{1}{2}[\frac{\Delta\alpha}{\bar{\alpha}} - 4\frac{\bar{\beta}^2}{\bar{\alpha}^2}\frac{\Delta G}{\bar{G}}]\sin^2\theta_i + \frac{1}{2}\frac{\Delta\alpha}{\bar{\alpha}}\sin^2\theta_i\tan^2\theta_i \ . \tag{7}$$

The angle θ_i is the angle of incidence, symbols Z and G are $Z = \rho\alpha$ and $G = \rho\beta^2$, respectively. The symbol Δ denotes contrast across the reflector, bar symbol an average.

The moveout formula $T(x)$ and the in-line spreading $L_{\parallel}(x,\varphi)$ depend on 3, the out-of-plane spreading $L_{\perp}(x,\varphi)$ depends on 6 profile A-parameters in each layer. Through the Bond transformation, the moveout and spreading formulae depend on 9 global A-parameters in each layer. P-wave reflection coefficient depends on contrast of 4 profile A-parameters or, through the Bond transformation, on contrast of 12 global A-parameters. No non-physical approximation such as acoustic approximation is used. On numerical tests, we illustrate the accuracy of the given approximate formulae.

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