EXTENSION OF RAY THEORY TO ANISOTROPIC VISCOELASTIC MEDIA

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Attenuation is a very important phenomenon in wave propagation, and is essential whenever the intensity of waves matters. It is thus fundamental to extend the ray theory from anisotropic elastic media to anisotropic viscoelastic media.

The idea of the extension looks simple: The eikonal equation in an attenuating medium has the form of a complex–valued Hamilton–Jacobi equation, which would generate complex–valued rays. Since we know the velocity model in real space rather than in complex space, we have to trace the real–valued reference rays using the reference Hamiltonian function, and calculate the complex–valued travel time right in real space by the perturbation from the reference travel time calculated along real–valued reference rays to the complex–valued travel time defined by the complex–valued Hamilton–Jacobi equation (Klimeš, 2002; 2016). Analogously for the corresponding amplitudes (Klimeš, 2006a). For this purpose, Klimeš & Klimeš (2011) designed the construction of the optimum real–valued reference Hamiltonian function corresponding to a given complex–valued Hamiltonian function.

However, we have encountered various more or less expected problems. The ray tracing equations and the corresponding equations of geodesic deviation (Červený, 1972) are often formulated in terms of the eigenvectors of the Christoffel matrix (Klimeš, 2006b). Unfortunately, a complex–valued Christoffel matrix need not have all three eigenvectors at an S–wave singularity (Klimeš, 2021). We thus have to formulate the ray tracing equations and the corresponding equations of geodesic deviation using the characteristic values of the complex–valued Christoffel matrix, without the eigenvectors of the Christoffel matrix (Klimeš, 2020). The resulting equations for the real–valued reference P–wave rays and real–valued reference common S–wave rays are applicable everywhere, including S–wave singularities.

The eigenvectors of the complex–valued Christoffel matrix are normalized to unit complex–valued pseudonorm with respect to pseudoscalar product $a_i b_i$ of vectors a_i and b_i rather than to unit real–valued norm with respect to scalar product $a_i^* b_i$, because the eigenvectors are pseudoorthogonal with respect to the pseudoscalar product (Klimeš, 2018). Consequently, the pseudonormal S–wave eigenvectors frequently diverge when approaching an S–wave singularity (Klimeš, 2022). This divergence does not occur in elastic media. As a result, the phase–space derivatives of the anisotropic–ray–theory Hamiltonian function used to trace the anisotropic–ray–theory rays may also diverge when approaching an S–wave singularity. Fortunately, the phase–space derivatives of the reference Hamiltonian function used to trace the reference common S–wave rays are smooth at S–wave singularities (Klimeš, 2020).

If the S–wave eigenvectors of the complex–valued Christoffel matrix diverge at an S–wave singularity, the corresponding anisotropic–ray–theory vectorial amplitudes diverge at the S–wave singularity too. This divergence does not occur in elastic media where the anisotropic–ray–theory vectorial amplitudes diverge at caustics only.

Since the variation of the S–wave eigenvectors along the reference ray plays a decisive role in the coupling equations (Bulant & Klimeš, 2002; Klimeš, 2013), the divergence of S–wave eigenvectors in a vicinity of an S–wave singularity represents a considerably challenging problem. The coupling equations for viscoelastic media contain the complex–valued angle of rotation of the S–wave eigenvectors about the P–wave eigenvector instead of the real–valued angle of rotation used in elastic media. Fortunately, these coupling equations for viscoelastic media compensate the divergence of S–wave eigenvectors and yield vectorial amplitudes smoothly varying through S–wave singularities. Whereas the coupling ray theory corrects just the S–wave polarization in elastic media, it corrects also the S–wave amplitudes at S–wave singularities in viscoelastic media which may diverge in the anisotropic ray theory.

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