P-wave reflection-moveout approximation for horizontally layered media of tilted moderate orthorhombic symmetry

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Summary

An approximate nonhyperbolic P-wave moveout formula applicable to horizontally layered media of moderate or weak orthorhombic or higher anisotropy symmetry of arbitrary orientation is presented and tested. Weak-anisotropy approximation is used for its derivation, in which the square of the reflection traveltime is expanded in terms of weak-anisotropy (WA) parameters specifying the anisotropy of the layers. Due to it, the proposed formula is applicable to any offset. Its accuracy decreases with increasing strength of anisotropy. The relation between traveltimes and WA parameters in the formula is transparent and relatively simple. Along a chosen single surface profile, the formula depends in each layer on its thickness, on the auxiliary reference P-wave velocity, on, at most, six WA parameters describing P-wave orthorhombic or transverse isotropy symmetry, and on three Euler angles specifying the orientation of symmetry elements in the layer. Performed tests indicate that the maximum relative traveltime error does not exceed 2.5% for P-wave anisotropy about 25%. For most offsets, however, the errors are considerably lower.

Introduction

The majority of studies of reflection moveout is based on the Taylor-series traveltime expansion around the zero offset. It concerns moveout formulae not only for a single homogeneous anisotropic layer, but also for stacks of layers of, mostly, higher-symmetry anisotropy, see, e.g., Tsvankin and Thomsen (1994), Tsvankin (2001), Li and Yuan (2003), Ursin and Stovas (2006), Tsvankin and Grechka (2011), Hao and Stovas (2016), Sripanich and Fomel (2016), Sripanich et al. (2017), Koren and Ravve (2017), Ravve and Koren (2017). Most of these studies focus on media, in which one of the symmetry planes is horizontal, or even more, in which the source-receiver profile is situated in the vertical plane representing the symmetry plane. Often, a special parameterization related just to the studied type of seismic anisotropy is used.

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In this paper, we use an approach based on the results of Farra and Pšenčík (2019). The used approach is applicable to weak or moderate anisotropy of any symmetry and/or orientation. In this paper, we apply it to layered media whose layers might be orthorhombic (OR), transversely isotropic (TI) or isotropic (ISO). Anisotropy in layers may have an arbitrary tilt, i.e., we consider tilted OR (TOR) or TI (TTI) layers. Tilt may vary from layer to layer.

In the following, we repeat only the most important basic formulae. For details, we refer to the above-mentioned paper of Farra and Pšenčík (2019). The paper also contains, as a byproduct, formulae for the two-way zero-offset traveltime, NMO velocity and the quartic term of the Taylor-series traveltime expansion expressed in terms of the WA parameters. We do not repeat them here.

The proposed reflection-moveout formula is based on two important approximations. First, we replace the actual ray connecting the source S and the receiver R, both situated on a surface profile, and transmitted through a stack of horizontal layers of arbitrary, but weak or moderate TOR ot TTI symmetry, by a nearby reference ray between S and R, transmitted through a stack of isotropic layers with reference velocities closely approximating the velocities of the actual anisotropic layers. Second, as mentioned above, we use the weak-anisotropy approximation. Instead of the exact ray-velocities along individual elements of reference rays, we use their first-order weak-anisotropy approximations. Specifically, we replace squares of the exact ray velocities by first-order approximations of the squared phase velocities calculated in the same direction. Due to this, the approximation is less accurate in situations, in which the two velocities differ significantly. Thanks to the first of the two approximations, we are dealing with reference rays, which, although they may consist of many elements, are strictly 2D (actual rays are generally 3D). This simplifies considerably the two-point ray-tracing procedure, required if traveltimes of a reflected wave are sought from the source point S to the prescribed receiver point R. The second approximation leads to a simple traveltime formula along an arbitrarily chosen surface profile, with transparent relation between WA parameters and traveltimes.

For the application of the moveout formula, two Cartesian coordinate systems are necessary. The first, which we call *profile coordinate system* z_j has z_3 axis vertical, positive down. The z_1 and z_2 axes are situated in a horizontal plane, and make the coordinate system z_j right-handed. Besides the profile coordinate system, we also use, separately in each layer, a *local coordinate system*, which can be arbitrarily rotated with respect to the profile coordinate system. The rotation may vary from layer to layer. Local coordinate systems are introduced for easy specification of media of arbitrary anisotropy symmetry in each layer. Coordinate planes of the local coordinate systems are chosen so that they coincide with symmetry planes of the considered anisotropy in each layer.

Traveltime formula

Let us consider an arbitrary surface profile over a stack of horizontal TOR, TTI or ISO layers, anisotropic layers being of weak or moderate anisotropy. Let us choose the profile coordinate system so that its z_1 axis is parallel to the profile. On the profile, we consider the source S and the receiver R, separated by the offset x. Approximate expression for the square of the traveltime $T(x)$ of the unconverted reflected P-wave generated at the source S, reflected from the bottom of the stack of layers, and recorded at the receiver R

reads:

$$T^2(x) = [\sum_{i=1}^{N} T_i(x_i)]^2 \ , \qquad x = \sum_{i=1}^{N} x_i \ . \tag{1}$$

When evaluating equation (1), the actual ray in a layered medium composed of homogeneous horizontal TOR, TTI or ISO layers is replaced by the reference ray in a nearby reference isotropic medium with P-wave velocities $\bar{\alpha}_i$. The deviation of the actual ray from the reference ray is considered to be of the first order, and thus the traveltime along it represents the first-order approximation of the actual traveltime (Fermat's principle). In equation (1), N is the number of layers, through which the wave propagates. The actual and reference rays consist of N couples of down- and up-going elements associated with each layer. The symbol $T_i(x_i)$ denotes the traveltime along the down- and up-going elements of the reference ray in the i-th layer. The offset corresponding to the couple of down- and up-going elements in this layer is x_i.

In the following, instead of offsets x_i, we use the normalized offsets $\bar{x}_i$, $\bar{x}_i = x_i/(2h_i)$, where h_i is the thickness of the i-th layer. The traveltime T_i can be then expressed in the following way:

$$T_i^2(\bar{x}_i) = T_{0i}^2 \, \frac{(1+\bar{x}_i^2)^3}{P(\bar{x}_i)} \ . \tag{2}$$

In equation (2), the symbol T_{0i} is the two-way zero-offset traveltime in the i-th layer of the reference isotropic medium, $T_{0i} = 2h_i/\alpha_i$. Here, α_i is the P-wave velocity of the reference isotropic medium in the i-th layer. The velocity α_i is used for the definition of weak-anisotropy (WA) parameters, see Appendix A. WA parameters represent a generalization of Thomsen's parameters (Thomsen, 1986); for more details, see, e.g., Farra and Pšenčík (2017). The velocity α_i generally differs from the velocity $\bar{\alpha}_i$ specifying the isotropic medium, in which reference rays are constructed.

The symbol P in equation (2) represents the polynomial

$$P(\bar{x}) = (1+\bar{x}^2)^2 + 2\epsilon_x \bar{x}^4 + 2\delta_y \bar{x}^2 + 2\epsilon_z \ . \tag{3}$$

The symbols ϵ_x, ϵ_z, and δ_y are the *profile P-wave WA parameters* of the considered layer. They are specified in the profile coordinate system z_j and are related to the vertical plane (z_1, z_3). For their definition, see equations (A-2) in Appendix A. These WA parameters can be obtained by a linear transform, see equations (A-5)-(A-6), from *local P-wave WA parameters* (A-1) specified in the local coordinate system. The coordinate planes or axes of the local coordinate systems coincide with the symmetry planes of the TOR or TTI media in the layer, respectively. The relation between the profile and local coordinate systems is specified by the rotation matrix $\mathbf{R}$ given in equations (A-3) and (A-4). Thus the traveltimes along a given profile depend in each layer on the normalized offset $\bar{x}_i$, on the six local WA parameters, ϵ_x^{OR}, ϵ_y^{OR}, ϵ_z^{OR}, δ_x^{OR}, δ_y^{OR} and δ_z^{OR}, specifying orthorhombic symmetry (three local WA parameters ϵ_x^{TI}, ϵ_z^{TI} and δ_y^{TI}, specifying transverse isotropy) in the layer, and on three (two) Euler angles φ, θ and ν, (φ and θ) specifying the transformation from the local to the profile coordinate system. The traveltimes also depend on the choice of the velocity $\bar{\alpha}$ used for the construction of reference rays, but they are independent of the velocity α used for the definition of WA parameters.

For the evaluation of equation (2), we need to know the normalized offset $\bar{x}_i$ in the i-th layer. It can be determined from a simple 2D two-point ray-tracing procedure in the reference isotropic medium specified by the P-wave velocities $\bar{\alpha}_i$. Detailed description of the procedure can be found in Farra and Pšenčík (2019). With equations (1)-(3) and equations of Appendix A, we can approximately evaluate P-wave moveout along an arbitrarily chosen profile over the stack of horizontal layers of orthorhombic or transversely isotropic symmetry of arbitrary orientation. The anisotropy symmetry and its orientation may vary from layer to layer. Of course, some layers might also be isotropic. Let us mention that the proposed reflection moveout formula yields exact results for a stack of isotropic layers. For a single layer, equation (1) reduces to the single-layer moveout formula derived earlier (Farra and Pšenčík, 2017).

Accuracy tests

Here we test the accuracy of the approximate traveltime formula (1) by comparing its results with results calculated using the package ANRAY (Gajewski and Pšenčík, 1990), which we consider as an exact reference. We present the results of the tests in the form of plots of relative traveltime errors $(T - T_{ex})/T_{ex}\times$ 100%, where T denotes traveltimes calculated from equation (1), and T_{ex} denotes the traveltimes obtained from ANRAY. Traveltime errors are shown as functions of the normalized offset $\bar{x} = x/(2H)$, where H is the thickness of the whole stack of layers, $H = h_1 + h_2 + ... + h_N$.

We test equation (1) on three models of layered media. They consist of orthorhombic (OR), transversely isotropic (TI) and isotropic layers. We use the orthorhombic model of Schoenberg and Helbig (1997) with varying tilts of symmetry planes, and transversely isotropic Greenhorn shale model (Fomel, 2004; Stovas, 2010) with varying tilt of the axis of symmetry. The WA parameters of the OR model are $\epsilon_x^{OR} = 0.2576$, $\epsilon_y^{OR} = 0.3283$, $\epsilon_z^{OR} = 0$, $\delta_x^{OR} = 0.0774$, $\delta_y^{OR} = -0.0825$, $\delta_z^{OR} = 0.3401$. The upper indices OR indicate that we deal with WA parameters of orthorhombic symmetry specified in the local coordinate system with coordinate planes coinciding with the planes of symmetry. The above WA parameters correspond to the reference velocity $\alpha = 2.4372$ km/s. The WA parameters of the TI model are $\epsilon_x^{TI} = 0.256$, $\epsilon_z^{TI} = 0$ and $\delta_y^{TI} = -0.0523$. The upper indices TI indicate that the WA parameters specify the TI symmetry in the local coordinate system. The WA parameters correspond to the reference velocity $\alpha = 3.094$ km/s. It represents again the phase velocity along the vertical axis of the local coordinate system. For this reason, $\epsilon_z^{TI} = 0$. The P-wave velocity of the isotropic layer is 3 km/s. The P-wave anisotropy of both OR and TI media is $\sim$ 25%. The anisotropy strength is defined as $2(c_{max} - c_{min})/(c_{max} + c_{min})\times$ 100%, where c_{max} and c_{min} denote maximum and minimum phase velocities.

The accuracy of formula (1) depends on the choice of the P-wave reference velocities $\bar{\alpha}_i$ used for the construction of reference rays. Since its choice as the the phase velocity along the source-receiver profile proved to be a good choice in most cases in the previous study (Farra and Pšenčík, 2019), we use this choice here too. Equation (1) can be applied along any arbitrarily chosen surface profile, which we identify with the z_1 axis of the profile coordinate system. The tilt of the local coordinate systems is in each layer specified by three Euler angles φ_i, θ_i and ν_i, introduced in Appendix A, see Equation (A-4).

The curves in Figure 1 show the variation of the relative traveltime errors with the normalized offset in the model consisting of two orthorhombic layers of equal thickness.

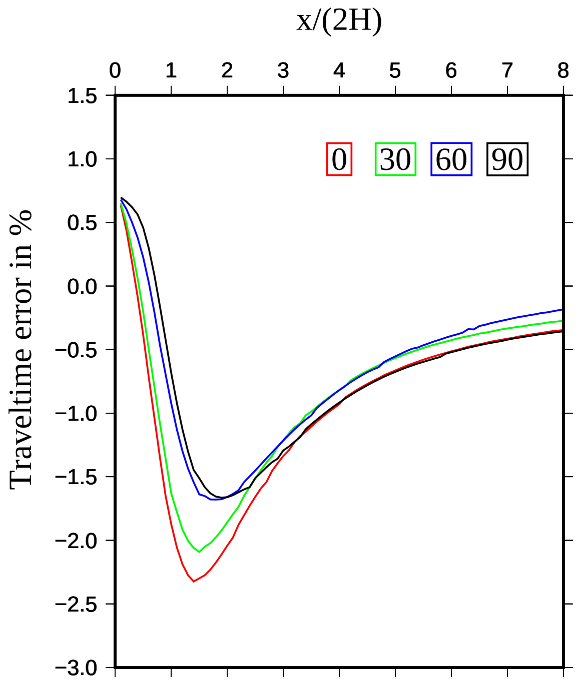

Figure 1: P-wave reflection from the bottom of the model consisting of two orthorhombic layers of equal thickness. Schoenberg and Helbig (1997) model used in both layers. In the upper layer, tilted orthorhombic symmetry is specified by varying azimuth angle $\varphi_1 = 0°$ (red), $30°$ (green), $60°$ (blue) and $90°$ (black), polar angle $\theta_1 = 60°$ and angle $\nu_1 = 0°$. In the bottom layer, symmetry planes coincide with coordinate planes, i.e., $\varphi_2 = \theta_2 = \nu_2 = 0°$. Variation with the normalized offset $\bar{x} = x/(2H)$ (H is the depth to the bottom of the model) of the relative traveltime error of the approximate equation (1). Reference velocities $\bar{\alpha}_i$ chosen as phase velocities along the source-receiver line.

The bottom layer is formed by the orthorhombic medium of Schoenberg and Helbig (1997), whose planes of symmetry are parallel with coordinate planes of the profile coordinate system, i.e., $\varphi_2 = \theta_2 = \nu_2 = 0$. The upper layer is formed by the same orthorhombic medium, but with varying tilt. The angles θ_1 and ν_1 are constant, $\theta_1 = 60°$ and $\nu_1 = 0°$. The azimuth angle φ_1 varies, $\varphi_1 = 0°$ (red), $\varphi_1 = 30°$ (green), $\varphi_1 = 60°$ (blue) and $\varphi_1 = 90°$ (black). All the presented curves have character of curves observed in application of the moveout formula to single-layer media, see Farra and Pšenčík (2017). The maximum relative traveltime error is less than 2.5%, but mostly less than 1%. The maximum errors are the consequence of the significant deviation of phase- and ray-velocity vectors at normalized offsets $\bar{x} \sim 1 - 3$ (equation (1) is based on the assumption that the mentioned two vectors are equal). The error of the two-way zero-offset traveltime is $\sim 0.7\%$. It is caused by differences between the reference (vertical) rays with velocities along them and their actual counterparts. Using equation (3.1.27) of Červený (2001) with $p_1 = p_2 = 0$ specifying the exact zero offset ray, we can see that the exact traveltime along the zero-offset ray depends only on vertical phase velocities in individual layers. By inspecting equation (B-2) of Farra and Pšenčík (2019) for the approximate two-way zero-offset traveltime $T(0)$, we can see that it depends on velocities related to $A_{33}^{(i)}$ elements of the stiffness matrix in the Voigt notation. In fact, the term $(A_{33}^{(i)})^{1/2}$ represents the first-order approximation of the vertical phase velocity. Since the exact and approximate vertical phase velocities differ, and are independent of the angle φ_i, the errors of the two-way zero-offset traveltimes in Figure 1 are the same for all choices of the angle φ_i.

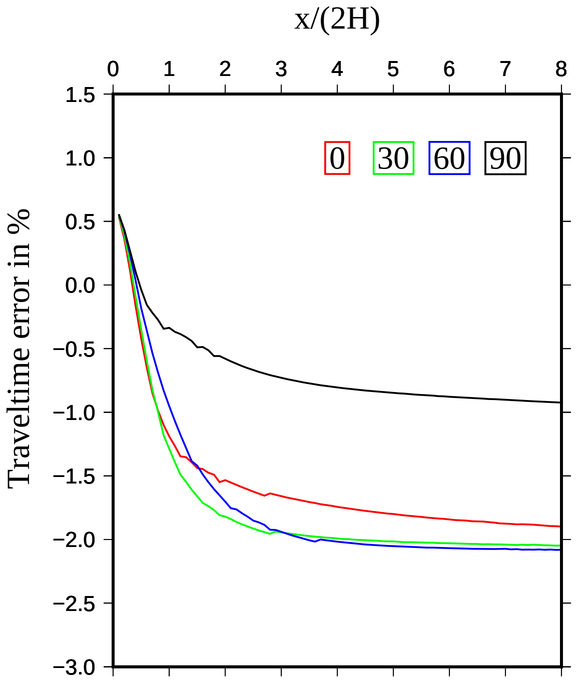

Figure 2: P-wave reflection from the bottom of the model consisting of two orthorhombic layers of equal thickness. Schoenberg and Helbig (1997) model used in both layers. In the upper layer, tilted orthorhombic symmetry is specified by varying azimuth angle $\varphi_1 = 0°$ (red), 30° (green), 60° (blue) and 90° (black), polar angle $\theta_1 = 60°$ and angle $\nu_1 = 45°$. In the bottom layer, $\varphi_2 = 45°$, $\theta = 30°$ and $\nu_2 = 30°$. Variation with the normalized offset $\bar{x} = x/(2H)$ (H is the depth to the bottom of the model) of the relative traveltime error of the approximate equation (1). Reference velocities $\bar{\alpha}_i$ chosen as phase velocities along the source-receiver line.

A similar two-layer model is considered in Figure 2. The top layer resembles the top layer in Figure 1. The only difference is the nonzero angle ν_1. In the model in Figure 2, it makes $\nu_1 = 45°$. Anisotropy in the bottom layer is, in contrast to the model in Figure 1, tilted. The tilt is fixed and specified by the Euler angles $\varphi_2 = 45°$, $\theta_2 = 30°$ and $\nu_2 = 30°$. We can see that the tilt in the bottom layer changes the character of the relative traveltime error curves. The errors are slightly reduced, around 2% (around 1% for $\varphi_1 = 90°$), but they extend to larger offsets. As in Figure 1, the error of the two-way zero-offset traveltime is the same for all azimuths φ_1. The reasons are the same as above.

In Figure 3, we present results of tests of the accuracy of equation (1) for the four-layer model composed of isotropic (top), tilted TI Greenhorn shale (below the isotropic layer) and two tilted OR layers (bottom). All layers are of the same thickness. For the P-wave

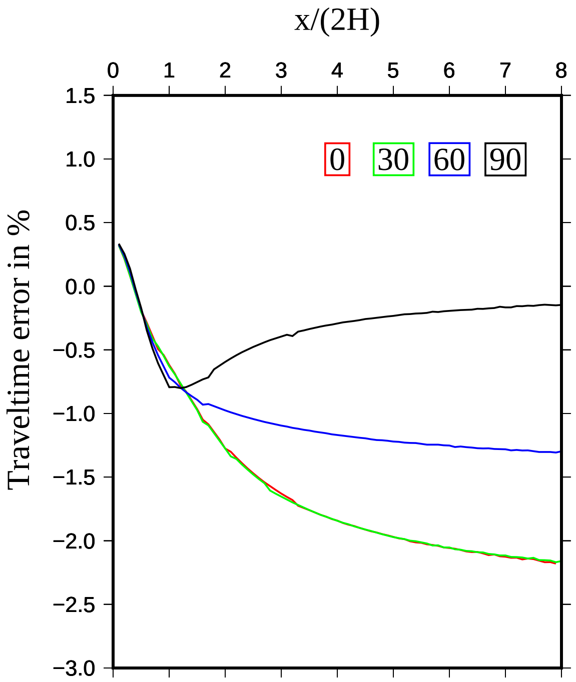

Figure 3: P-wave reflection from the bottom of the model consisting of four layers of equal thickness. The top layer is isotropic with P-wave velocity of 3 km/s. In the layer below it, tilted transversely isotropic Greenhorn shale is specified by varying azimuth angle $\varphi_2 = 0°$ (red), 30° (green), 60° (blue) and 90° (black), and polar angle $\theta_2 = 30°$. The two bottom layers are of tilted orthorhombic symmetry. The third layer is specified by azimuth angle $\varphi_3 = 0°$, polar angle $\theta_3 = 60°$, and angle $\nu_3 = 45°$. The fourth layer is specified by azimuth angle $\varphi_4 = 45°$, polar angle $\theta_4 = 30°$, and angle $\nu_4 = 30°$. Variation with the normalized offset $\bar{x} = x/(2H)$ (H is the depth to the bottom of the model) of the relative traveltime error of the approximate equation (1). Reference velocities $\bar{\alpha}_i$ chosen as phase velocities along the source-receiver line.

velocity of the isotropic layer and for WA parameters specifying the remaining layers, see the text above. The tilts of the OR layers are specified by the Euler angles $\varphi_3 = 0°$, $\theta_3 = 60°$, and $\nu_3 = 45°$ in the third layer, and by $\varphi_4 = 45°$, $\theta_4 = 30°$ and $\nu_4 = 30°$ in the fourth layer. The tilt of the axis of symmetry in the TI layer is $\theta_2 = 30°$. The azimuth angle φ_2 of the axis of symmetry in the TI layer varies, $\varphi_2 = 0°$ (red), $\varphi_2 = 30°$ (green), $\varphi_2 = 60°$ (blue) and $\varphi_2 = 90°$ (black). Despite the increased number of layers, the maximum relative traveltime errors only slightly exceed 2%, as in the case of two-layered models. As in Figure 2, the errors depend on the azimuth φ_2 of the axis of symmetry of the TI layer. They are below 1% for $\varphi_2 = 90°$, they slightly exceed 1% for $\varphi_2 = 60°$, and they are identical and reach $\sim 2\%$ for $\varphi_2 = 0°$ and 30°. The equal error of the two-way zero-offset traveltime has again the same explanation as in previous figures.

Conclusions

The proposed and tested formula is applicable to an arbitrarily chosen single profile over the horizontally layered medium composed of weakly or moderately anisotropic TOR or TTI layers, or isotropic layers. The formula is based on two simplifying assumptions. First, actual rays in the studied medium are replaced by the reference rays in a reference isotropic medium. Second, instead of the exact ray velocity, its first-order weak-anisotropy approximation is used.

In each layer, the formula depends on its thickness, on the reference P-wave velocity used for the construction of the reference rays, on, at most, six P-wave WA parameters specifying the anisotropy of the layer, and on three Euler angles specifying the orientation of symmetry elements of the anisotropic medium.

The tests, which we performed indicate that the proposed moveout formula yields results of acceptable accuracy even for anisotropy of more than 20%. The maximum relative traveltime error in such a case does not exceed 2.5%. For most offsets, it is considerably less. The errors depend strongly on the choice of the reference velocity $\bar{\alpha}_i$. The choice of $\bar{\alpha}_i$ as the phase velocity along the source-receiver direction seems to yield better results in most cases.

The presented approximate moveout formula could be generalized for converted waves or even S waves if the common S-wave concept is used. In contrast to this study, in the moveout formula for converted waves, the down- and up-going elements of the S wave would have to be treated separately in each layer.

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Appendix A

Weak-anisotropy parameters and their transformation

In the main text, we introduced local Cartesian coordinate systems in each layer, whose coordinate planes coincide with the symmetry planes of orthorhombic media. In each layer, the local coordinate systems may have a different orientation with respect to the profile Cartesian coordinate system z_j. For simplicity, we focus on one selected layer, and, therefore, we omit its index throughout this Appendix. Use of the local coordinate systems allows us to specify orthorhombic layers by only six local P-wave WA parameters, ϵ_x^{OR}, ϵ_y^{OR}, ϵ_z^{OR}, δ_x^{OR}, δ_y^{OR} and δ_z^{OR}, and transversely isotropic layers by only three local P-wave WA parameters, ϵ_x^{TI}, ϵ_z^{TI} and δ_y^{TI}. Their definitions are following:

$$\epsilon_x^{OR} = \frac{A_{11}^{OR} - \alpha^2}{2\alpha^2} , \quad \epsilon_y^{OR} = \frac{A_{22}^{OR} - \alpha^2}{2\alpha^2} , \quad \epsilon_z^{ORL} = \frac{A_{33}^{OR} - \alpha^2}{2\alpha^2} ,$$

$$\delta_x^{OR} = \frac{A_{23}^{OR} + 2A_{44}^{OR} - \alpha^2}{\alpha^2} \ , \quad \delta_y^{OR} = \frac{A_{13}^{OR} + 2A_{55}^{OR} - \alpha^2}{\alpha^2} \ , \quad \delta_z^{OR} = \frac{A_{12}^{OR} + 2A_{66}^{OR} - \alpha^2}{\alpha^2} \ .$$

and

$$\epsilon_x^{TI} = \frac{A_{11}^{TI} - \alpha^2}{2\alpha^2} \ , \quad \epsilon_z^{TI} = \frac{A_{33}^{TI} - \alpha^2}{2\alpha^2} \ , \quad \delta_y^{TI} = \frac{A_{13}^{TI} + 2A_{55}^{TI} - \alpha^2}{\alpha^2} \ . \qquad (A-1)$$

The symbol α in equation (A-1) denotes the P-wave velocity in the reference isotropic medium. It generally differs from the reference velocity $\bar{\alpha}$ used for the calculation of reference rays. The symbols $A_{\alpha\beta}^{OR}$ and $A_{\alpha\beta}^{TI}$ denote density-normalized elastic moduli in the local coordinate system in the Voigt notation. The upper indices OR or TI indicate that the WA parameters specify the orthorhombic or TI symmetry.

The profile WA parameters ϵ_x, ϵ_z, and δ_y specified in the profile coordinate system, appearing in equation (3), are defined as:

$$\epsilon_x = \frac{A_{11} - \alpha^2}{2\alpha^2} \ , \quad \epsilon_z = \frac{A_{33} - \alpha^2}{2\alpha^2} \ , \quad \delta_y = \frac{A_{13} + 2A_{55} - \alpha^2}{\alpha^2} \ . \qquad (A-2)$$

Here, α is identical with α used in equation (A-1). The symbols $A_{\alpha\beta}$ denote density-normalized elastic moduli in Voigt notation in the profile coordinate system. The profile WA parameters (A-2) can be obtained from the parameters (A-1) by a linear transform involving the rotation matrix $\mathbf{R}$ from local to profile coordinates.

We consider the rotation matrix $\mathbf{R}$ in the form:

$$\mathbf{R} = \begin{pmatrix} n_1 & e_1 & t_1 \\ n_2 & e_2 & t_2 \\ n_3 & e_3 & t_3 \end{pmatrix} \ . \qquad (A-3)$$

The vectors $\mathbf{n}$, $\mathbf{e}$ and $\mathbf{t}$ in equation (A-3) are unit, mutually perpendicular vectors. They specify three mutually perpendicular intersections of planes of symmetry of an orthorhombic medium. Their orthonormality yields $n_1 = e_2t_3 - t_2e_3$, $n_2 = e_3t_1 - t_3e_1$, $n_3 = e_1t_2 - t_1e_2$.

The matrix $\mathbf{R}$ in equation (A-3) can be rewritten using three Euler angles φ, θ and ν. The Euler angles φ and θ represent the azimuth and polar angles specifying the orientation of one of the axes of symmetry (intersection of two planes of symmetry). The angle ν represents a rotation around the axis of symmetry. The matrix $\mathbf{R}$ then reads:

$$\mathbf{R} = \begin{pmatrix} \cos\varphi\cos\theta\cos\nu - \sin\varphi\sin\nu & -\cos\varphi\cos\theta\sin\nu - \sin\varphi\cos\nu & \cos\varphi\sin\theta \\ \sin\varphi\cos\theta\cos\nu + \cos\varphi\sin\nu & -\sin\varphi\cos\theta\sin\nu + \cos\varphi\cos\nu & \sin\varphi\sin\theta \\ -\sin\theta\cos\nu & \sin\theta\sin\nu & \cos\theta \end{pmatrix} \ . \qquad (A-4)$$

We use the above specification in numerical examples.

With the use of the rotation matrix $\mathbf{R}$ in the form (A-3), the three profile WA parameters ϵ_x, ϵ_z and δ_y from equation (A-2) can be expressed in terms of six local WA parameters ϵ_x^{OR}, ϵ_y^{OR}, ϵ_z^{OR}, δ_x^{OR}, δ_y^{OR} and δ_z^{OR} from equation (A-1) in the following way:

$$\epsilon_x = \epsilon_x^{OR}n_1^4 + \epsilon_y^{OR}e_1^4 + \epsilon_z^{OR}t_1^4 + \delta_x^{OR}e_1^2t_1^2 + \delta_y^{OR}n_1^2t_1^2 + \delta_z^{OR}n_1^2e_1^2 \ ,$$

$$\epsilon_z = \epsilon_x^{OR}n_3^4 + \epsilon_y^{OR}e_3^4 + \epsilon_z^{OR}t_3^4 + \delta_x^{OR}e_3^2t_3^2 + \delta_y^{OR}n_3^2t_3^2 + \delta_z^{OR}n_3^2e_3^2 \ ,$$

$$\delta_y = 6\epsilon_x^{OR} n_1^2 n_3^2 + 6\epsilon_y^{OR} e_1^2 e_3^2 + 6\epsilon_z^{OR} t_1^2 t_3^2$$
$$+\delta_x^{OR}(D_{21}^2 + 2e_1e_3t_1t_3) + \delta_y^{OR}(D_{22}^2 + 2t_1t_3n_1n_3) + \delta_z^{OR}(D_{23}^2 + 2n_1n_3e_1e_3) \ . \qquad (A-5)$$

In the above equations, we used the following notation:

$$D_{21} = e_1t_3 + e_3t_1, \qquad D_{22} = t_1n_3 + t_3n_1, \qquad D_{23} = n_1e_3 + n_3e_1. \qquad (A-6)$$

In TI layers, the rotation matrix $\mathbf{R}$ in equation (A-3) reduces to

$$\mathbf{R} = \begin{pmatrix} t_1t_3/D & -t_2/D & t_1 \\ t_2t_3/D & t_1/D & t_2 \\ -D & 0 & t_3 \end{pmatrix}, \qquad (A-7)$$

see equation (B-4) of Pšenčík and Farra (2017). In equation (A-7):

$$D = (t_1^2 + t_2^2)^{1/2} \ . \qquad (A-8)$$

In the TI symmetry, the angle ν representing the rotation around the axis of symmetry looses its meaning and can be set $\nu = 0$. Equation (A-4) then reduces to:

$$\mathbf{R} = \begin{pmatrix} \cos\varphi\cos\theta & -\sin\varphi & \cos\varphi\sin\theta \\ \sin\varphi\cos\theta & \cos\varphi & \sin\varphi\sin\theta \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \ , \qquad (A-9)$$

and transformation equations (A-5) to:

$$\epsilon_x = \epsilon_x^{TI}(t_2^2 + t_3^2)^2 + \epsilon_z^{TI} t_1^4 + \delta_y^{TI} t_1^2 (t_2^2 + t_3^2) \ ,$$
$$\epsilon_z = \epsilon_x^{TI}(t_1^2 + t_2^2)^2 + \epsilon_z^{TI} t_3^4 + \delta_y^{TI} t_3^2 (t_1^2 + t_2^2) \ ,$$
$$\delta_y = 2\epsilon_x^{TI}(3t_1^2t_3^2 + t_2^2) + 6\epsilon_z^{TI} t_1^2 t_3^2 + \delta_y^{TI}(t_1^2 + t_3^2 - 6t_1^2t_3^2) \ , \qquad (A-10)$$

see equation (B-6) of Pšenčík and Farra (2017).

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