Comparison of ray–matrix and finite–difference methods in a simple 1-D model

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Summary

Ray–matrix synthetic seismograms are compared with finite–difference synthetic seismograms in a simple 1-D model, consisting of a low–velocity layer covering a homogeneous halfspace. The numerical results demonstrate the efficiency of the combination of the ray–methods with the matrix methods.

1 Introduction

Ray methods, especially those designed to calculate synthetic wavefields, require velocity models composed of several smooth blocks separated by interfaces covered by smooth surfaces. The smaller the number of blocks and surfaces, the better. Such models miss possible fine layering in the vicinities of the source and receivers or along structural interfaces. Since the frequency–dependent response of the stacks of fine layers may considerably influence synthetic wavefields, the ray–theory synthetic seismograms are too distorted if the stacks of fine layers are neglected.

This drawback of the ray methods can be approximately corrected if the "thin" stacks of fine layers are locally approximated by 1-D stacks, the wave is locally approximated by the plane wave and a matrix method is used to calculate the local response of each stack. The ray–theory elementary Green functions are then combined with the frequency–dependent matrix–method responses. For more details refer, e.g., to the papers by Daley & Hron (1982), Červený (1989), Červený & Andrade (1992), Červený & Aranha (1992) and Jílek & Červený (1996a, 1996b).

Thomson (1998a, 1998b) developed program package RMATRIX for calculating the frequency–dependent responses of the stacks of fine isotropic or anisotropic layers. One of the versions of program package RMATRIX is included on compact disk SW3D–CD–2 (Klimeš 1998b). Program `greenss.for` of the compact disk can optionally call the subroutines of package RMATRIX calculating the response of the transmission through a stack of fine horizontal layers at each receiver location. The responses of fine layers at the source or along structural interfaces may be included in the future. The 2-D second–order elastic finite–difference program is also included on the compact disk and linked to the same 3-D model specification software as the ray tracing program.

A simple 1-D model, named RM, has been designed to debug the software and to compare the ray–matrix synthetic seismograms with the finite–difference synthetic seismograms. The test calculations are briefly discussed in this paper. For more details refer to the detailed description of the data and calculations by Klimeš (1998a) and to the documentation in Klimeš (1998b). For the comparison of the ray method with finite differences in a more complex model refer to Bucha & Klimeš (1999).

2 Model, source, receivers

A very simple 1-D model consists of a low–velocity layer covering a homogeneous halfspace. Vertical axis x_3 points downwards, the free Earth surface is situated at the depth $x_3 = 0.000\,\mathrm{km}$. The thickness of the low–velocity layer is $0.010\,\mathrm{km}$, with material properties $v_\mathrm{P} = 1.0\,\mathrm{km\,s^{-1}}$, $v_\mathrm{S} = 0.5\,\mathrm{km\,s^{-1}}$, $\rho = 1.6\,\mathrm{kg\,dm^{-1}}$. The velocities in real shallow sediments might be smaller, but would imply a denser grid for finite differences. The homogeneous halfspace has material properties $v_\mathrm{P} = 3.2\,\mathrm{km\,s^{-1}}$, $v_\mathrm{S} = 2.0\,\mathrm{km\,s^{-1}}$, $\rho = 2.5\,\mathrm{kg\,dm^{-1}}$.

The explosive point source is located at the depth $x_3 = 0.200\,\mathrm{km}$, below the origin of the Cartesian coordinates. The profile of 19 receivers spaced at $0.020\,\mathrm{km}$ on the Earth surface is centred above the source.

The sedimentary layer represents the fine local layered structures at the receiver locations and normally would not be contained within the macro model. Here we need the sedimentary layer in the model for the finite differences, and we wish to use the same model also for ray tracing. The two–point rays are thus caught at the top of the homogeneous halfspace, exactly below the receivers. In the ray–matrix method, the plane–wave approximation is used to extrapolate the wavefield from the endpoint of the two–point ray to the bottom of the local layer stack and from the top of the layer stack to the receiver point.

We selected the Küpper (also known as Müller) signal with 2 local maxima because we use the finite difference program, which does not support any other source time function. The reference frequency of 20Hz corresponds to the P–wave wavelength of $0.050\,\mathrm{km}$ in the sedimentary layer, i.e. 5 times the thickness of the layer.

We wish to approximate the point source by the line source excited by the Küpper signal in the 2-D FD program. We thus take the halfth derivative of the Küpper signal as the source time function describing the time dependence of the seismic moment of the explosive point source. The point and line sources then generate wavelets of the same form in the far–field approximation.

3 Ray–matrix and finite–difference calculations

The programs are executed according to the "history files", designed after the Stanford Exploration Project history files, which contain both input parameters for the programs and the instructions to run the programs. A single history file may be used to complete the whole job by running several programs. The programs may share the input parameters and, simultaneously, the same programs may be run several times with various modifications of the input parameters. This considerably simplifies and clarifies the preparation of the jobs requiring many programs to run. The history files are designed to be computer–independent job descriptions, executed by a computer–independent Perl script.

Finite differences are often limited by the memory requirements or by the computational costs. This is also the case of the grid dimensions used for the comparison: spatial grid interval 0.001 km, 601 gridpoints horizontally (centred at the source), 344 gridpoints vertically, temporal interval 0.000175 s, 1601 time levels. The number of gridpoints has been selected to fit the array dimension in the basic distribution of the software. However, the maximum number of gridpoints may be increased with regard to the computer by adjusting a single parameter in a single include file.

Since the "nonreflecting" boundary conditions of finite differences may, to some extent, reflect the waves, it is highly recommended to change the finite–difference grid roughly by a quarter of the prevailing wavelength and to run the finite differences once again. The reflections from the boundaries are then shifted roughly by half the prevailing period and are clearly visible in the common plots of the seismograms from both the computations. The reference P–wave wavelength in the homogeneous halfspace is 160 grid intervals. We thus enlarge the above mentioned grid by 40 grid intervals at the bottom and by 45 grid intervals at each side and run the finite differences once more (the increase of 40 to 45 accounts for the angle of reflection from the nonreflecting boundary, but 40 would be sufficient).

4 Comparison of the seismograms

We plot the calculated synthetic seismograms together, in the colour PostScript plots of the horizontal in–plane and vertical components. The transverse component is zero.

The amplitude scale in the plots of seismograms corresponds to a point source in 3-D. To be able to plot also the 2-D FD seismograms, corresponding to the line source, on a comparable scale, we choose the reference distance for the amplitude power scaling during the plotting equal to the hypocentral distance (in a homogeneous medium), at which the wavefields generated by the point and line sources have the same amplitudes. In plotting, we use no amplitude power scaling for the point source but the power scaling, proportional to the hypocentral distance powered to $-\frac{1}{2}$, for the line source. In this way, seismograms corresponding to the point and line sources coincide in the neighbourhood of the source point. They, of course, may considerably differ at large distances in a heterogeneous medium.

The calculated seismograms are shown in Figures 1 and 2, where the reflections from the nonreflecting boundaries can clearly be identified. The dotted–line reflections (the first FD calculation) come before the dashed–line reflections (the second FD calculation, with the enlarged grid).

The agreement between the ray–matrix (solid lines) and finite–difference seismograms is very good except for a small difference in the horizontal component around time $t = 0.15$ s, which might be worth explaining in future.

5 Conclusions

The ray–matrix calculations have been performed using programs coded for generally heterogeneous 3-D isotropic and weakly anisotropic models. The numerical results demonstrate the efficiency of the combination of the ray–methods with the matrix methods.

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Acknowledgements

The research has been supported by the consortium "Seismic Waves in Complex 3-D Structures", see "http:/seis.karlov.mff.cuni.cz/consort/main.htm".

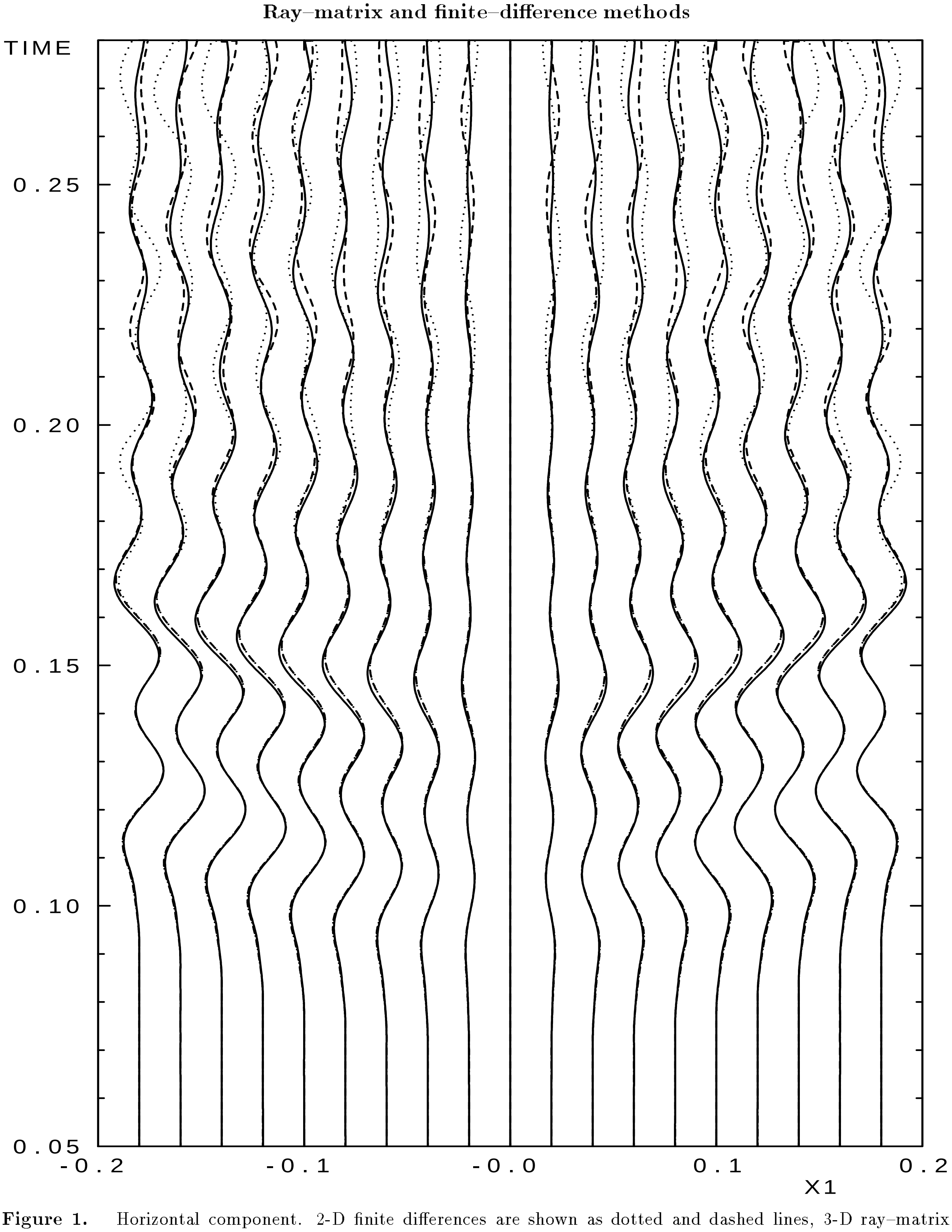

Figure 1. Horizontal component. 2-D finite differences are shown as dotted and dashed lines, 3-D ray–matrix method as solid lines.

Ray–matrix and finite–difference methods

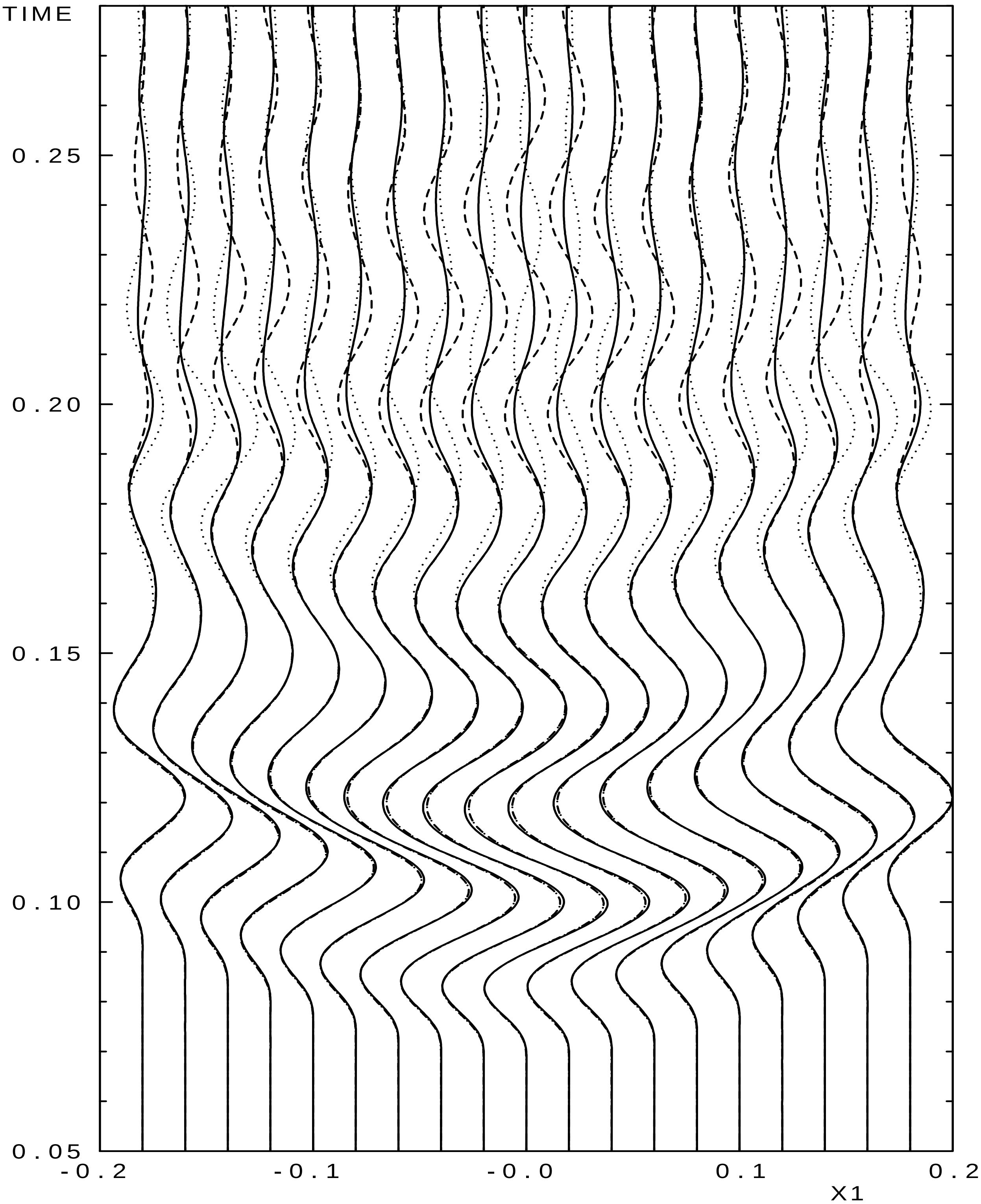

Figure 2. Vertical component. 2-D finite differences are shown as dotted and dashed lines, 3-D ray–matrix method as solid lines.