Ray methods in the modelling of seismic wave fields

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Abstract

Seismic ray method has broad applications in the numerical modelling of seismic wave fields and in the solution of forward and inverse problems of seismology and seismic exploration for oil. This contribution outlines basic features of the seismic ray method, and reviews its possibilities. It also shows which areas of the seismic ray method are still open for further research.

Introduction

The seismic ray method is based on the asymptotic high-frequency solution of the elastodynamic equation. It consists in the decomposition of the wavefield into independent contributions called elementary waves, which propagate along rays. These waves may be various seismic body waves propagating in isotropic or anisotropic inhomogeneous, layered and block structures (direct, multiply reflected/transmitted, etc.). Great advantage of the ray method is that individual elementary waves can be treated independently.

Individual elementary waves are described by their travel times and vectorial amplitudes. They can be obtained from the eikonal equation and the transport equation. The eikonal equation can be solved using rays. The rays are characteristics of the eikonal equation, and can be determined by solving ray tracing system (a system of ordinary differential equations for the characteristics). As soon as the rays are known, the travel times can be calculated along them. They are called the ray theory travel times in order to distinguish them from the first–arrival travel times. The transport equation can be then solved along a ray; the amplitude can be expressed analytically in terms of the geometrical spreading. The geometrical spreading can be determined along a ray using the dynamic ray tracing. The dynamic ray tracing is a very powerful procedure; it can be used to determine travel times and other relevant quantities not only along a ray, but also in its vicinity. We then speak of paraxial ray methods, paraxial rays, paraxial travel times, etc. As soon as travel times and ray amplitudes are known, calculation of the Green function, elementary seismograms and particle ground motions of the elementary wave under consideration is straightforward.

For finite frequencies, the ray method is only approximate. Its accuracy, however, is sufficient to solve many important 3-D wave propagation problems which can be hardly treated by any other means. Various extensions of the ray method, which increase its accuracy or can be applied in situations where the standard ray method is not applicable, have been proposed. Most of them use rays as a framework.

The ray method can be applied to isotropic, anisotropic and weakly anisotropic 3-D structures, with arbitrary 3-D variations of elastic moduli and density, with curved interfaces, for arbitrary source-receiver configurations, and for very general types of seismic body waves.

For more information on the seismic ray method and for additional references, see the www pages of the Consortium Project “Seismic Waves in Complex 3-D Structures” (http://seis.karlov.mff.cuni.cz/consort/main.htm) and, e.g., Červený, Molotkov and Pšenčík (1977), Červený (2001), Chapman (2001).

Seismic rays and travel times

Initial value ray tracing, in which an initial point and an initial direction of a ray are specified, is now a well-understood problem in any type of media. The rays are mostly determined numerically, by solving the ray tracing system. The solution of a boundary-value ray tracing problem, for example of a two-point ray tracing problem, is more complicated, particularly in 3-D. For simplicity, we shall consider only the two-point ray tracing problem, in which the ray passing through two specified points is sought. The methods to solve this problem can be divided into two groups: the bending methods and the shooting methods:

a) In the bending methods, the initial ray path is guessed and then perturbed iteratively so as to find the relevant two-point ray. Let us mention here methods based on the fitting ray equations, methods based on the paraxial ray approximation, methods based on minimizing travel time (also called the pseudobending methods), and methods based on the structural perturbation. The disadvantage of the bending methods is that they can miss certain rays and corresponding travel times, particularly in the case of multipathing. Only one of several arrivals of an elementary wave is usually obtained, others are lost.

b) In the shooting methods, the iterative procedure that uses the initial-value ray tracing at each step is used. The initial-value ray tracing procedure is put within an iterative loop to find the initial ray parameters corresponding to the two-point ray, i.e. the ray passing through two points. In the more sophisticated version of the shooting method, the ray parameters of the two-point ray are sought in a 2-D ray-parameter domain. The basic problem of the method resides in the accurate decomposition of the 2-D ray-parameter domain into homogeneous subdomains, containing the ray parameters of rays with the same histories, and in the determination of the demarcation belts between homogeneous subdomains. These belts can be found by means of sophisticated triangulation of the ray-parameter domain (Bulant, 1996). As soon as the ray-parameter domain is decomposed into homogeneous subdomains, the solution is easy and fast. Inside homogeneous subdomains, the two–point travel times and ray parameters of two-point rays can be found by routine interpolation. The method is suitable even for massive 3-D migration/inversion grid computations (Bulant and Klimeš, 1999).

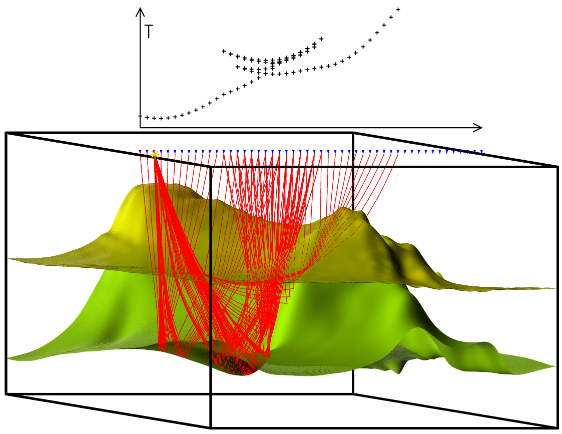

Figure 1: Two–point rays and travel times in a 3-D model.

The shooting method described above is close to the wavefront construction method, in which the current wavefront is computed from the previous wavefront by tracing short ray elements. The number of rays in the method is not fixed, but it is adjusted at each wavefront. See Vinje at al. (1996).

Recently, a large attention has been devoted to the direct computations of travel times on 2-D and 3-D grids, which do not use the ray tracing. These methods, however, do not yield the ray theory travel times, but first–arrival travel times. The first–arrival travel times are not related to the propagation of energy and should strictly be distinguished from the ray theory travel times related to energetic arrivals which have to be considered in interpretation procedures (tomography, migration, etc.). Two basic methods in the computation of first–arrival travel times are the network shortest path ray tracing and the finite–difference travel–time tracing. It should be emphasized that the ray theory travel times are safely obtained by shooting methods and standard ray tracing. A well organized shooting method, based on the decomposition of the ray parameter domain can be even faster than the finite–difference or network ray tracing method, particularly in 3-D computations.

As soon as heterogeneities in the model exceed certain limit, the rays may have a chaotic character. The chaotic rays are characterized by their extremely strong sensitivity to the initial conditions, for example to the ray parameters. The chaotic rays tend to diverge exponentially with their increasing length. The divergence is characterized by the Lyapunov exponents. An attempt to quantify the exponential divergence of rays with respect to the complexity of the model, and to formulate explicit criteria enabling to construct sufficiently smooth models, suitable for ray tracing, has already been made but this topic is still open for further research.

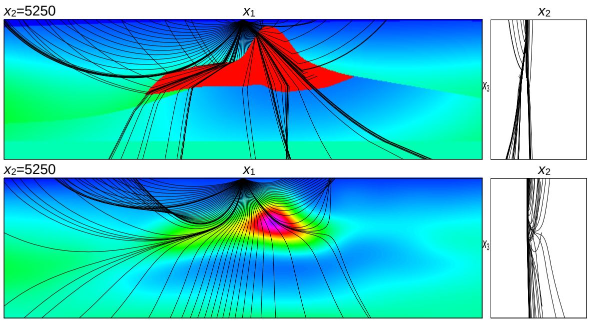

Figure 2: Vertical slice across the 3-D SEG/EAGE Salt model smoothed for ray tracing, shown with the examples of rays. Versions with smooth interfaces (top) and without interfaces (bottom).

Dynamic ray tracing, paraxial ray methods

The dynamic ray tracing consists in solving a system of ordinary differential equations along the ray (dynamic ray tracing system). Increase of computing time (with respect to the standard ray tracing) is negligible, so that it should be a part of all ray tracing packages.

Let us briefly list several important applications of dynamic ray tracing: 1) Computation of the geometrical spreading and amplitudes. 2) Computation of higher-order derivatives of the travel time field along a ray. 3) Computation of paraxial travel times and paraxial slowness vectors in a vicinity of a ray. 4) Computation of the curvature of a wavefront. 5) Fresnel volume ray tracing. 6) Computation of the KMAH index. 7) Two-point ray tracing. Boundary value problems for a ray. 8) Investigation of reciprocity relations along a ray. 9) Investigation of chaotic behaviour of rays. Computation of Lyapunov exponents. 10) Determination of the geometrical spreading from travel time measurements. 11) Computation of Gaussian beams and Gaussian wave packets. 12) Maslov-Chapman theory. 13) Applications in the ray perturbation theory (Farra, 1999). 14) Applications in migration methods. Computation of Beylkin determinant. 15) Various extensions of the ray method.

Amplitudes. Green function

As soon as ray tracing and dynamic ray tracing is performed, the computation of ray amplitudes and thus of the Green function along the ray is easy. In addition, it is only necessary to determine the R/T coefficients at points at which rays interact with structural interfaces. At present, quite general algorithms and relevant computer packages for computation of ray amplitudes of arbitrary elementary waves are available. Knowing the ray amplitude and travel time, we can determine elementary ray synthetic seismograms and particle motions. By summation of elementary ray synthetic seismograms over all rays passing through a receiver, we obtain the final ray synthetic seismogram, ray-theory Green function, and ray-theory particle motion.

Extensions of the ray method

Ray method is only approximate, it does not give exact results, and completely fails in certain situations. Many extensions of the ray method have been proposed to increase its validity. We shall briefly comment several of them.

Coupling ray theory. Quasi-isotropic ray theory: In inhomogeneous weakly anisotropic media, the two quasishear waves do not propagate independently, they are coupled (Pšenčík 1998). Neither the ray method for inhomogeneous isotropic media nor for anisotropic media can describe the coupling properly. Coupling ray theory, in which the asymptotic solution is sought for coupled waves together (not for independent elementary waves like in the standard ray methods) has to be used. Fig. 3 shows comparison of seismograms calculated by the quasi-isotropic approach and the reflectivity method.

Summation of Gaussian beams or Gaussian wave packets: For a selected ray, there is a six-parametric system of Gaussian beams and twelve-parametric system of Gaussian wave packets, connected with each ray (Červený, 2001). Individual Gaussian beams (or Gaussian wave packets) in these systems differ by width, shape, and other properties. The summation of Gaussian beams (or of Gaussian wave packets) passing in a neighbourhood of a receiver removes certain singularities of the ray method (caustics, critical regions, etc.) and removes necessity to perform two-point ray tracing. However, the problem of the choice of the parameters of Gaussian beams (or Gaussian wave packets) in the summation is open for further research.

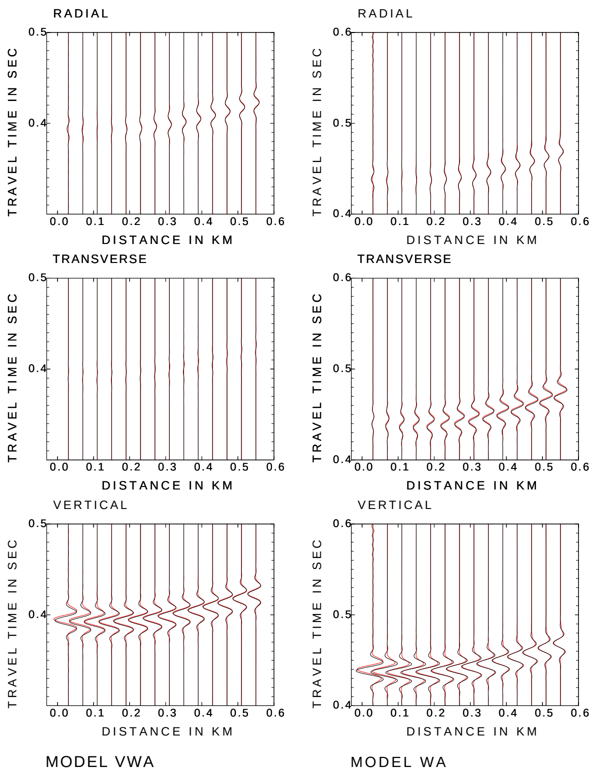

Figure 3: Synthetic seismograms calculated by the quasi-isotropic approach (black) and the reflectivity method (red) for inhomogeneous, nearly isotropic (VWA) and weakly anisotropic (WA) models.

Maslov-Chapman theory: The Maslov-Chapman method also consists in the summation over rays, but of infinitely broad beams. Particularly useful are time-domain versions of the Maslov-Chapman integrals. Some problems in the computation are caused by the so-called pseudocaustic points. A complex version of the Maslov-Chapman method (Klimeš, 1984) or the coherent state transform method (Thomson, 2001) remove these problems. The theory is open for further research.

Ray method with a complex eikonal: The application of complex rays and complex travel times in the numerical modelling of seismic wavefields would be very powerful, particularly in dissipative media. Unfortunately, complex rays have not yet been used in routine computations. In weakly dissipative media they are approximated using the ray perturbation method. This is the most important area of the seismic ray theory open for further research. See Thomson (1997).

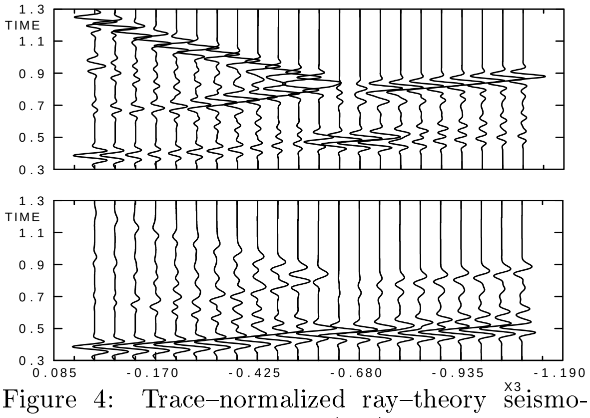

Figure 4: Trace–normalized ray–theory seismograms without attenuation (top) and with causal attenuation (bottom). The complex–valued rays are approximated using the ray perturbation method.

Hybrid methods: In hybrid methods, the seismic ray method is locally combined with some other method. Most prospective are the hybrid ray-FD method, hybrid ray-mode method and the hybrid ray-matrix method (also called thy hybrid ray-reflectivity method). Only the hybrid ray-matrix method has been implemented into routine ray-based computer codes. It can be used to compute the high-frequency waves propagating in a layered (or block) model, in which individual layers (or blocks) are mutually separated by thin transition layers. Further research would be very useful.

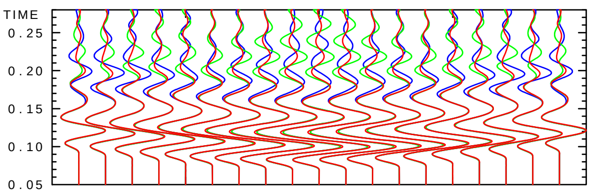

Figure 5: Vertical component of synthetic seismograms at the top of a sedimentary layer. Red: elastic seismograms calculated by the ray–matrix method are quite accurate in this case. The standard ray method would yield only the onset of the wave–train. Green and blue: finite–difference elastic seismograms are deteriorated by reflections due to imperfect non–reflecting boundary conditions.

Other extensions: There are many other useful extensions of the ray method, and many other applications of ray approaches in more sophisticated methods. Let us mention several of them: space-time ray method (Babich, Buldyrev and Molotkov, 1985), surface wave ray tracing, asymptotic diffraction theory, including geometric theory of diffraction (Hanyga, 1995), ray methods in directional (one-way) propagation, ray methods in scattering of seismic waves (Ursin and Tygel, 1997), asymptotic version of Kirchhoff integrals (Tygel, Schleicher and Hubral, 1994), ray methods for random media, for poroelastic solids, for Biot media, etc. These extensions require still a further research.

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