# VELOCITY MACRO MODELS

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**Summary**

Various aspects of macro models are briefly mentioned, with emphasis on their relation to geological structures and on their role in inversion of seismic data.

## 1. Introduction

A geological structure contains heterogeneities on all scales. A seismic wavefield is sensitive to heterogeneities of different wavelength in different ways. Smooth large–scale (long–wavelength, low–wavenumber) velocity heterogeneities affect forward wave propagation, whereas small–scale (short–wavelength, high–wavenumber) velocity or density heterogeneities cause wide–angle scattering. A seismic wavefield is nearly insensitive to large–scale density heterogeneities.

Klimeš (2003) demonstrated that the forward propagation of seismic waves is not very sensitive to short–wavelength velocity or density heterogeneities. It is thus possible to approximately recover large–scale velocity heterogeneities without knowing small–scale heterogeneities. On the other hand, small–scale heterogeneities cannot be correctly recovered without large–scale velocity heterogeneities. It is thus natural and reasonable to recover the heterogeneities iteratively, gradually proceeding from large–scale heterogeneities to smaller scales (shorter wavelengths, higher wavenumbers), as suggested by Pratt et al. (1996).

A *velocity macro model* approximates, above all, the large–scale velocity heterogeneities. A vague smooth macro model is created at the early stage of the iterative inversion of seismic data and is gradually being improved, making large–scale heterogeneities more precise and adding heterogeneities of smaller scales. There are, however, various reasons why to terminate incorporation of small heterogeneities into the macro model on a certain scale (wavelength, wavenumber) and leave the macro model considerably smooth.

## 2. Smoothness of velocity macro models

In the first place, the improvement of a macro model by smaller details may be limited by the inaccuracy of the macro model itself on large scales: Wave propagation in the macro model may better approximate real propagation if a small–scale heterogeneity is rather ignored than considered in an incorrect location.

Constraints upon the macro model smoothness may also be imposed by computational methods, particularly by the ray methods if being in use. Incorporation of small heterogeneities into a macro model should be avoided if the respective growth of the error of the computational method due to the roughness of the macro model exceeds the respective improvement of the accuracy of the macro model, or if the application of the computational method is getting too expensive.

In order to meet both the accuracy and efficiency criteria, the smoothness of a velocity macro model may conveniently be controlled via minimization of the respective Sobolev norm of the macro model (Bulant, 2002; Žáček, 2002). Another kind of the Sobolev norm, approximating the second–order perturbations of travel time, allow to control the size of the iterations of the linearized tomographic inversion of travel times, making the inversion stable and fast.

## 3. Constraints imposed upon a velocity macro model by the ray methods

### *3.1. Accuracy*

If using ray methods, heterogeneities causing other than forward scattering should be avoided in the macro model, with a possible exception of known and pronounced structural interfaces. The structural wavenumbers presenting problems for a ray method at the frequencies under consideration can be estimated by comparing the Born approximation of scattered waves (Klimeš 2003) with the perturbation of the ray method.

For example, the relative error of approximation of scattering from infinitesimally small Gaussian velocity perturbation $\varepsilon \exp(-\frac{1}{2}\,\Delta\mathbf{x}^{\mathrm{T}}\mathbf{K}\,\Delta\mathbf{x})$ by the perturbation of travel time is approximately

$$\rho = \omega^{-1}\sqrt{\tfrac{1}{8}\mathrm{tr}[(\mathbf{F}^{-1}\mathbf{K}^{(q)})^2] + \tfrac{1}{16}[\mathrm{tr}(\mathbf{F}^{-1}\mathbf{K}^{(q)})]^2} \quad .$$

Here $\mathbf{K}^{(q)}$ is the $2 \times 2$ projection of the $3 \times 3$ matrix $\mathbf{K}$ onto the wavefront tangent plane, transformed into the ray–centred coordinates $\mathbf{q} = (q_1, q_2)$ parametrizing the wavefront tangent plane. Matrix $\mathbf{F}$ is the frequency–independent $2 \times 2$ Fresnel zone matrix in the wavefront tangent plane (Červený, 2001, eq. 4.11.8), which is expressed in the ray–centred coordinates. The first Fresnel zone is defined by inequality

$$\omega\,|\mathbf{q}^{\mathrm{T}}\mathbf{F}\,\mathbf{q}| \leq 2\pi$$

(Červený, 2001, eq. 4.11.7).

### *3.2. Efficiency*

If a velocity macro model becomes too complex, the rays get exhibiting chaotic behaviour and the number of two–point rays grows exponentially with the distance between points, see Figure 1. The exponential growth is quantified by the Lyapunov exponent. The approximate quantitative relations between the Lyapunov exponent and the second velocity derivatives in the macro model (Klimeš 2002a) enable the computational costs to be limited by controlling the smoothness of the velocity macro model.

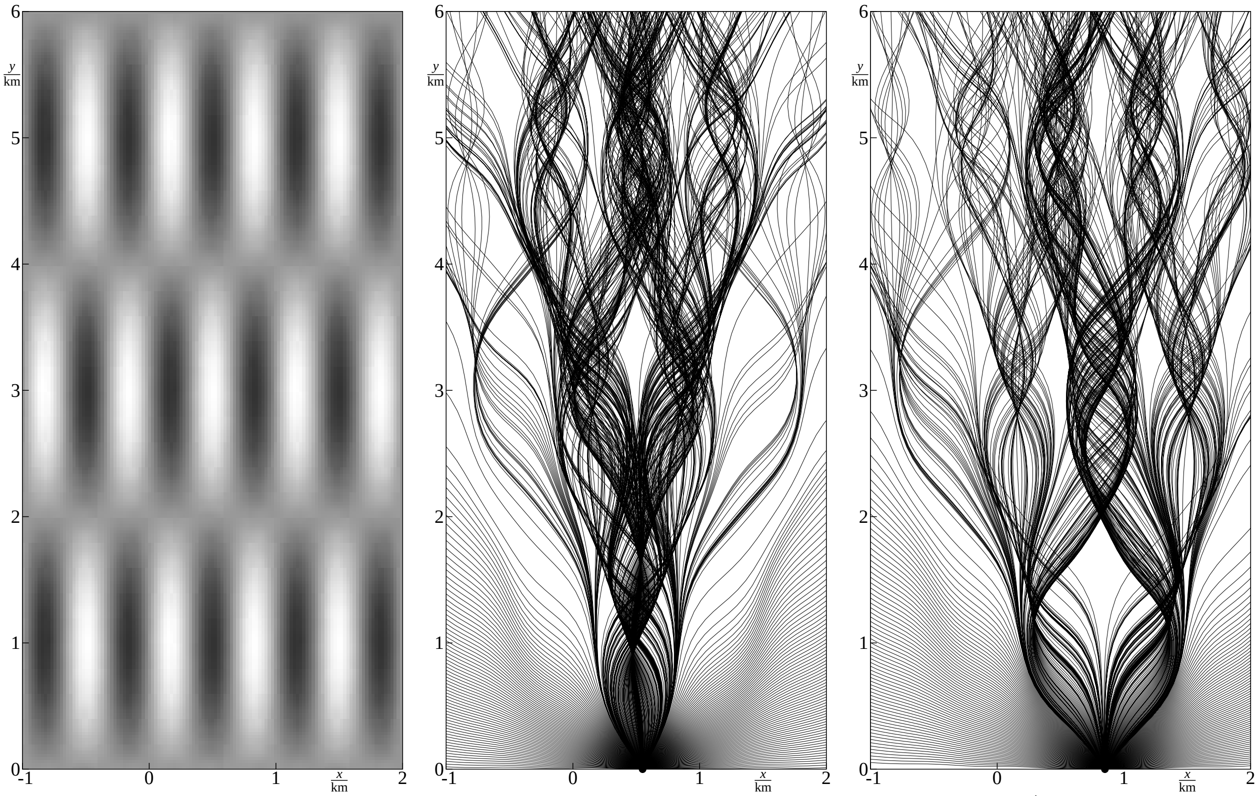


**Figure 1.** A model formed by the homogeneous background of velocity $1.0\,\mathrm{km\,s^{-1}}$, perturbed by a stretched bi–sine egg–box of amplitude $0.2\,\mathrm{km\,s^{-1}}$, with rays corresponding to point sources situated 1.55 km and 1.85 km from the left–hand corner. The maximum number of found two–point rays is 59.

## 4. Inaccuracy of velocity macro models

Inaccuracy of macro models is caused not only by the above mentioned smoothness requirements, but especially by poor coverage of the geological structure by rays corresponding to clearly identifiable waves. Missing information on large–scale heterogeneities can be replaced by statistically most probable values in terms of the medium correlation functions (Tarantola, 1987; Klimeš, 2002b) in order to complete the macro model, but at the cost of very poor accuracy of the resulting macro model.

The accuracy of a velocity macro model can be estimated using the medium correlation functions. The medium correlation functions can be estimated from seismic data, especially from travel times.

## 5. Impacts of inaccurate macro models

In the inversion of seismic data, inaccurate macro models not only mean inaccurate information on the large–scale heterogeneities, but also cause mispositioning and fuzziness of small–scale heterogeneities due to backprojection of a time section (set of seismograms corresponding to a common source) to wrong spatial locations. It is thus essential to be able to quantitatively estimate the inaccuracy of the macro model, and to be able to estimate mispositioning and fuzziness of structural details in migrated images due to inaccuracies in the velocity macro model.

## 6. Computer representation of macro models

Macro models may be represented and parametrized in many ways. Various computer representations of macro models are more or less suitable for various applications. A very general and flexible representation of smooth, isotropic or anisotropic, 3-D models and 3-D models composed of smoothly heterogeneous blocks separated by general smooth interfaces has been designed by Červený, Klimeš & Pšenčík (1988). For an example of two–point rays in a velocity macro model composed of smoothly heterogeneous blocks refer to Figure 2. The model parametrization is very suitable for both forward modelling and travel–time inversion, possibly constrained by non–seismic information.

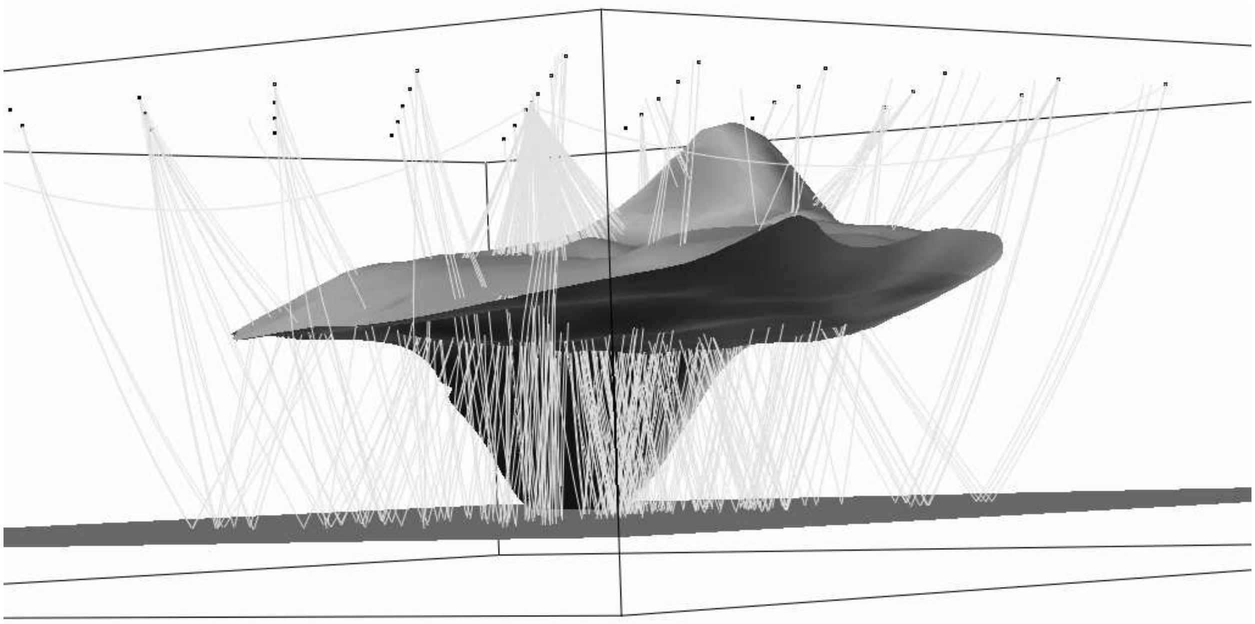

**Figure 2.** Velocity macro model of the SEG/EAGE Salt Model suitable for ray tracing.


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