Two-point ray tracing and controlled initial-value ray tracing in 3-D heterogeneous block structures

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Summary

It is well known, that even in quite simple seismic models we need some kind of the two–point ray tracer to find all existing two–point rays for all the receivers. One solution is a sophisticated shooting program, based on an accurate searching for the subdomains of ray take-off parameters, where the two–point rays may occur. The subdomains may be found by a special triangularization of the 2-D domain of ray take–off parameters. The two–point rays may be then identified inside the triangles whose all the three vertices are formed by rays arriving into a receiver area.

Once the domain of ray take-off parameters is triangularized, we can use the triangles mentioned above to construct the ray cells through the model volume, and to interpolate travel times and other quantities inside the cells, like in the wavefront tracing method. The described controlled initial–value ray tracing method is a useful extension of two–point ray tracing programs based on triangularization, providing interpolated values of ray–theory travel times and other quantities at dense grids located in the seismic model.

Method

The two–point ray tracing by means of shooting is based on iterative application of an *initial–value ray tracing* algorithm. In each computation, only the rays of a single *elementary wave*, i.e. the rays whose behavior at structural interfaces is prescribed by a priori information called the *ray code*, are considered. The rays are thus specified uniquely by the *ray take–off parameters*. For each given ray code, the principal problem of two–point ray tracing in heterogeneous block models then consists in a sufficiently accurate determination of the boundaries between rays with different *ray histories*, i.e. passing through different geological blocks, across different structural interfaces, terminated in different areas for different reasons, or passing through a different number of caustics. The demarcation belts between the different ray histories, corresponding to the numerical uncertainty of the boundaries between the ray histories, have to be kept reasonably narrow, because all two–point rays, situated inside the demarcation belts, will be lost.

The demarcation of the boundaries and the decomposition of the 2-D domain of ray take–off parameters, *ray domain,* into *homogeneous subdomains* corresponding to rays of equal ray histories is done by means of sophisticated triangularization of the ray domain. The individual homogeneous subdomains are then represented as sets of *homogeneous triangles*, each homogeneous triangle in the ray domain being created by three rays with the same ray history.

Once the ray domain is triangularized, the two–point rays can be found (using, e.g., the paraxial ray approximation) within the individual homogeneous triangles corresponding to *successful rays* (rays passing through the *reference surface* at which the receivers are situated).

The shooting algorithm, above all based on the procedures of demarcating the boundaries between different ray histories, is thus closely related to *controlled initial–value ray tracing*, whose aim is not to hit given receivers situated at the reference surface, but to cover the model with a sufficiently dense

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system of rays. The triangularization of homogeneous subdomains corresponding to rays of equal ray histories play the principal role in controlled initial–value ray tracing. The space between two consecutive wavefronts is then decomposed into *ray cells*, as in the wavefront tracing method (Vinje *et al.* 1996a,b), and the travel times and all other calculated quantities may routinely be interpolated within the individual ray cells.

The corresponding Fortran 77 computer code for 3-D two–point ray tracing was developed on the basis of package `MODEL` to specify smooth block models of geological structures, and package `CRT` for initial–value ray tracing in such models (Červený *et al.* 1988). The code for the controlled initial–value ray tracing is recently being developed as its extension.

Example

The numerical example of two-point ray tracing in 3-D is the computation of a diffracted elementary wave in a model with a lens. The two-layer model with a smooth curved Earth surface and first–order structural interface has a homogeneous lenticular inclusion in the upper constant velocity gradient layer. The lenticular inclusion has a higher velocity than its neighborhood, and the velocity is highest in the lower layer. The dimensions of the model are: length 60 km, width 40 km, and height 30 km, including the free space. The model is fully artificial, it is the demo model of the `MODEL` package.

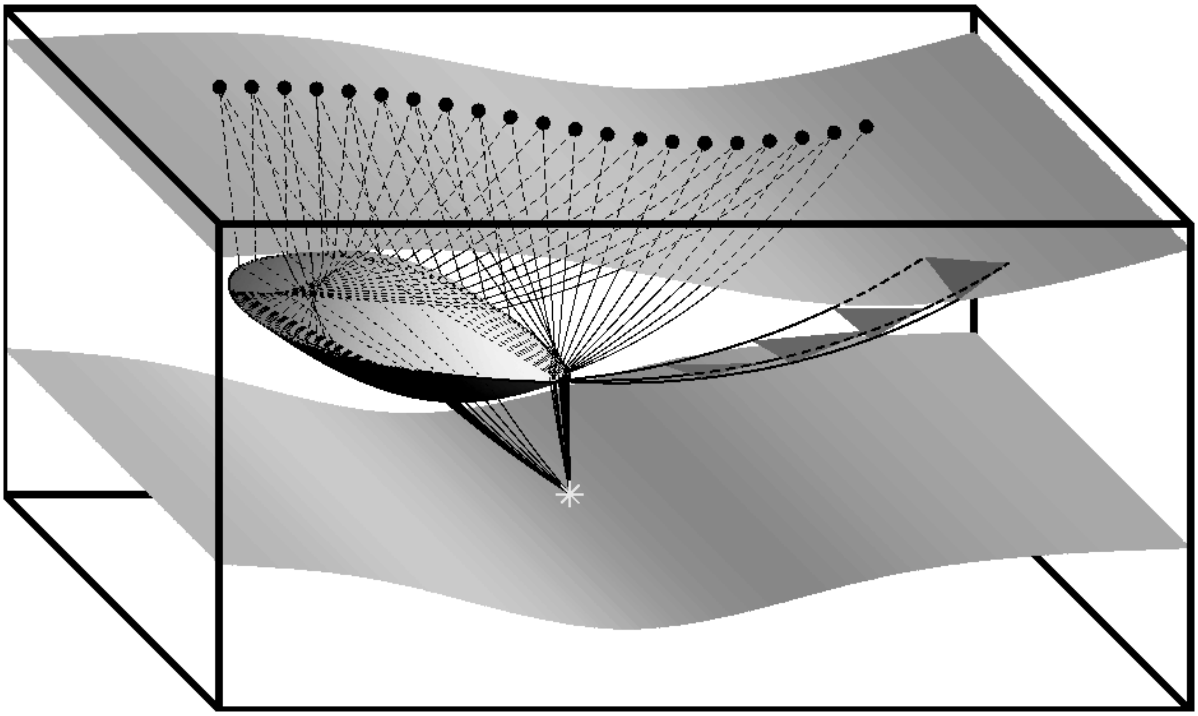

Figure 1. Edge-diffracted rays in a simple artificial model with a homogeneous lenticular inclusion embedded in a layer of a constant velocity gradient. The receivers are specified along the curved Earth surface, and found two–point rays are plotted as thiner lines.
Surface of the model is covered by diffracted rays with prescribed density, which enables interpolation of, e.g., travel times anywhere near the model surface using the controlled initial–value ray tracing. One ray tube, formed by three rays (thicker lines), and its ray cells are plotted for illustration.

Two-point ray tracing

Two-point ray tracing of the elementary wave diffracted at the edge of the lenticular inclusion is performed from the point source, receivers form a linear profile on the curved Earth surface. The computation of the diffracted wave is stepwise.

In the first step, the upper structural interface bounding the lens is understood to be the reference surface, and receivers are placed along the edge with a regular step. Two–point rays are then computed from the point source to the receivers (to the edge). Only the rays corresponding to the refracted elementary wave are taken into account in this numerical example.

The rays of one single history computed in the first step are chosen in the second step. These are rays, which propagated through the lens, but they might be also the direct rays. The edge diffractor is now understood to be a linear source, the initial travel time is interpolated using the travel times of the chosen two–point rays computed in the first step. Note that the ray domain of the linear source is again 2-dimensional, the first ray take–off parameter is a parameter along the edge, and the second is the shooting angle. One selects the elementary wave to be computed (in the presented example this is the refracted wave), and the two-point rays from the edge to the receiver profile are traced. The travel times of the rays leaving the point source, diffracted at the edge, and reaching the receivers are now available, as well as a set of homogeneous triangles being ready for interpolation of the computed quantities through the model volume.

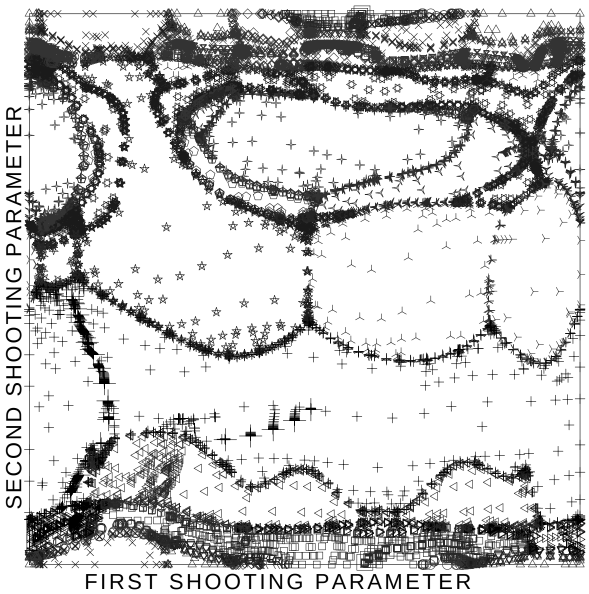

Figure 2. Take-off parameters of the rays diffracted at the edge of a lenticular inclusion. The horizontal axe is a length along the edge, the vertical one is proportional to a take-off angle. Rays of different ray histories are denoted by different marks. The two–point rays are emphasized by considerably enlarged marks.

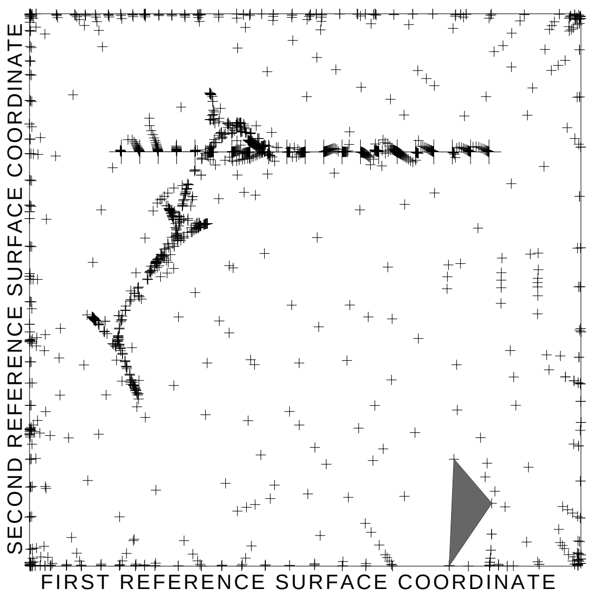

Figure 3. Endpoints of the rays on the surface of the model. Only the rays of the history marked by crosses on figure 2 are plotted. The triangle selected for visualization of the ray tube on figure 1 is here emphasized by shading. The two–point rays are denoted by large crosses.

Two-point ray tracing

Conclusions

The algorithm for two-point ray tracing in a 3-D heterogeneous medium has been developed. Special emphasis was put on the sufficiently accurate determination of the boundaries between rays with different ray histories, and on finding all the homogeneous subdomains of the ray domain. Consequently, the algorithm is very reliable in computing all two-point rays of the given elementary waves to the given receivers.

The two-point ray tracing algorithm described is relatively demanding with regard to the number of rays to be traced. The number of rays, which have to be computed, is not significantly affected by the number of receivers, but it is mostly influenced by number of auxiliary rays required to divide the domain of the ray take–off parameters into regions with the same value of the ray history. The method is highly reliable in finding two–point rays in very thin regions of the ray domain. This might be of interest for, e.g., 3-D travel time tomography, or for interpretation and model verification purposes.

The designed shooting algorithm is closely related to controlled initial–value ray tracing, which seems to be a promising tool for the calculation of multivalued ray–theory travel times, e.g., for ray-theory based migration techniques, or the nonlinear determination of seismic hypocentres.

The controlled initial–value ray tracing is very similar to the wavefront tracing method. It requires to compute a lot of auxiliary rays, which are not necessary in wavefront tracing, where they are used only in case when a ray cell is crossing an interface. On the other hand, in wavefront tracing the rays are traced by parts, which may slow down tracing. Thus the performance of the final program will probably depend more on the proper and efficient coding than on the selected method.

Acknowledgments

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