Smoothing the Marmousi model

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Summary

The only way to make a too complex velocity model suitable for the ray-based methods, such as the Gaussian beam or packet methods, is to smooth it.

We have smoothed the Marmousi model by choosing a coarser grid and by minimizing the relevant derivatives of the slowness. This has been performed by minimizing the relevant Sobolev norm of the slowness.

We show that minimization of the relevant Sobolev norm of the slowness is a suitable technique for preparing the optimum models for the asymptotic ray theory methods. However, the price for a model suitable for ray tracing is an increment of the difference between the smoothed and original model. Similarly, the assumed error of travel time due to the difference between the models increases as well. In a case of smoothing the Marmousi model, we have found the assumed error of travel times on the edge of acceptability.

Due to the low frequencies under consideration, we have found the Gaussian beams and packets on the edge of applicability even in models sufficiently smoothed for ray tracing.

Key words

Velocity model, smoothing, asymptotic ray theory, Gaussian beams, Lyapunov exponent, Sobolev norm.

1 Introduction

Computation of rays according to the asymptotic ray theory is extremely sensitive to the smoothness of the model. In rough models, the behaviour of rays becomes chaotic and the geometrical spreading and number of arrivals increase with travel time rapidly (Klimeš 1999, Matyska 1999). Similarly, a large number of two–point travel times makes calculation of two-point rays slow and expensive, or two-point rays cannot be possibly found within the numerical accuracy.

We need a reasonably smooth velocity model for a depth migration technique based on Gaussian packets. In the Gaussian packet method (Klimeš 1989), we do not need to find two-point rays, but sufficiently dense set of rays must be calculated. Thus, the desired model should be suitable for ray tracing. Since we wish to keep the width of Gaussian packets sufficiently small, and the width of a Gaussian packet also depends on frequency, the model should be sufficiently smooth for the frequencies under consideration.

A suitable way to smooth a complex velocity model, is to minimize an appropriate Sobolev norm (Klimeš 2000a) of the velocity or slowness. We show that minimizing of the Sobolev norm may be used for efficiently controlling the behaviour of rays in complex structures.

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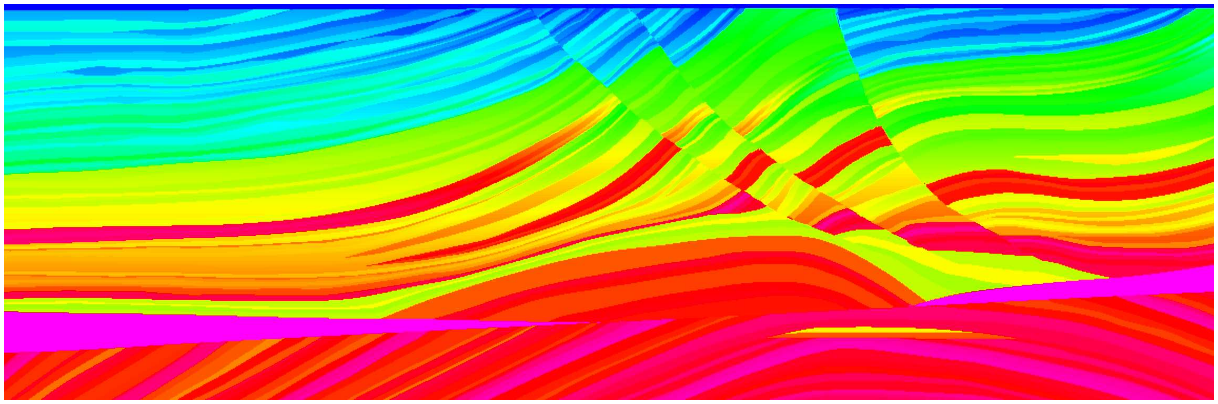


Figure 1. The Marmousi model.

2 The Marmousi model

Since we wish to use a "realistic" 2–D velocity model, we have decided to smooth the Marmousi model (Versteeg & Grau 1991, Versteeg 1991, 1993). The Marmousi model, based on real geological structure, is very complex, see Figure 1. The dimensions of the model are 9200 metres (length) by 3000 metres (depth). Values of velocity are defined at each gridpoint of the grid of cells of 4×4 metres. The grided values of velocity are from 1500 ms^{-1} to 5500 ms^{-1}.

In the Marmousi model, the synthetic seismograms are computed by the finite difference method. We wish to use these seismograms as the "real data" for the migration. The length of the seismograms is 2.9 seconds with a sampling interval 4 milliseconds. Trapezoidal frequency filter determined by the frequencies 0 Hz, 10 Hz, 35 Hz and 55 Hz has been applied to the data.

3 Basic ideas about the desired model

The desired smoothed model have to fulfil two main and, unfortunately, contradictive requirements:
(a) to be in a "good agreement" with the original Marmousi model, and
(b) to be "sufficiently smooth" for Gaussian packets.

Under the term "good agreement", we understand a small difference between the smoothed and original model, expressed in standard L2 Lebesgue norm.

Meaning of the term "sufficiently smooth" is more complicated. In a complex model, the geometrical spreading and number of arrivals exponentially increase with increasing travel time. The exponential increment is controlled by the Lyapunov exponent (Klimeš 1999, 2000c, Matyska 1999). Since the Lyapunov exponent depends on the second spatial derivatives of the velocity or slowness, the second derivatives should be minimized.

By minimizing the square of the Sobolev norm of slowness we may minimize the corresponding partial derivatives. The Sobolev scalar product is a linear combination of the L2 Lebesgue scalar products of the first, second or higher partial derivatives.

The vague terms "good agreement" and "sufficiently smooth" cannot be easily quantified before detailed study of the behaviour of rays and Gaussian packets in smoothed models.

The original Marmousi model consists of discrete values of velocity uniformly spaced on a dense grid. In obtaining a smoothed model, we

(a) choose a coarser grid to reduce the amount of data to fit,

(b) arithmetically average the densely sampled slowness of the Marmousi model over cells centred at the gridpoints of the coarser grid,

(c) choose a coarse model grid and

(d) fit the averaged slowness values by the smoothed model.

We need to interpolate the values of slowness spaced on a coarse model grid for ray tracing. As the interpolating functions we have chosen bicubic B–splines, benefiting from continuity of the second derivatives.

Let us summarize all types of grids being used in this paper. The first one is the original grid of the Marmousi model. The second one is the coarser data grid constructed from the Marmousi model as explained above, which is used to fit the smoothed model. The third one is the B–spline grid of the smoothed model.

4 Inversion

We minimize the objective function S defined by formula

$$S = \sum_{GRID} \left(\frac{u^D(x_i^{GRID}) - u^M(x_i^{GRID})}{\sigma^{GRID}} \right)^2 +$$

$$+ \left[\int \mathrm{d}^2 x \right]^{-1} \int b_{ijkl} \left(\frac{\partial^2 u^M(x_m)}{\partial x_i \partial x_j} \right) \left(\frac{\partial^2 u^M(x_n)}{\partial x_k \partial x_l} \right) \mathrm{d}^2 x \quad , \tag{1}$$

where u^D is a value of slowness in the original model, u^M is a value of slowness in model being searched, σ^{GRID} are the weighing parameters of gridpoints, b_{ijkl} are the weighing coefficients of the Sobolev scalar product. Superscript $GRID$ takes values $GRID = 1, 2, .., N$, where N is the number of gridpoints of the coarser grid with averaged values of slowness mentioned above. Subscripts take values $i, j, k, l, m, n = 1, 2$ in a 2–D model. The Einstein summation over the pairs of identical indices is used. Integration is performed over the whole model.

We can express u^M as a linear combination of bicubic B–splines $B(x_i)_\alpha$

$$u^M(x_i) = B(x_i)_\alpha u_\alpha \quad , \tag{2}$$

where u_α are the model parameters. Subscript α takes values $\alpha = 1, 2, .., P$, where P is the number of model parameters. Consequently, P is the number of B–spline gridpoints of the smoothed model.

Equation (1) now reads

$$S = \sum_{GRID} \left(\frac{u^D(x_i^{GRID}) - B(x_i^{GRID})_\alpha u_\alpha}{\sigma^{GRID}} \right)^2 + u_\alpha D_{\alpha\beta} u_\beta \quad , \tag{3}$$

where

$$D_{\alpha\beta} = \left[\int d^2 x \right]^{-1} \int b_{ijkl} \left(\frac{\partial^2 B(x_m)_\alpha}{\partial x_i \partial x_j} \right) \left(\frac{\partial^2 B(x_n)_\beta}{\partial x_k \partial x_l} \right) \mathrm{d}^2 x \quad . \tag{4}$$

Since we do not know the coefficients b_{ijkl} which lead to the optimum model, the problem is not linear. Thus, parameters u_α cannot be determined analytically. Since we do not want to solve numerically a non-linear inverse problem, we need to "linearize" formula (4) into

$$D_{\alpha\beta} = s^2 D'_{\alpha\beta} \quad , \tag{5}$$

$$D'_{\alpha\beta} = \left[\int d^2x\right]^{-1} \int b'_{ijkl} \left(\frac{\partial^2 B(x_m)_\alpha}{\partial x_i \partial x_j}\right) \left(\frac{\partial^2 B(x_n)_\beta}{\partial x_k \partial x_l}\right) \mathrm{d}^2x \quad , \tag{6}$$

where s is a free parameter and b'_{ijkl} are fixed coefficients of the Sobolev scalar product. The choice of coefficients b'_{ijkl} will be discussed in Section 6.

We can now rewrite equation (3) in form

$$S = [\mathbf{u}^D - \mathbf{B}\mathbf{u}]^T \mathbf{C}^{-1} [\mathbf{u}^D - \mathbf{B}\mathbf{u}] + s^2 \mathbf{u}^T \mathbf{D}' \mathbf{u} \quad , \tag{7}$$

where $\mathbf{C}$ is a $N \times N$ diagonal matrix, composed of $(\sigma^{GRID})^2$, see equation (1). N is the number of gridpoints.

A condition for minimum of the objective function is

$$\frac{\partial S}{\partial u_\alpha} = 0 \quad , \tag{8}$$

which yields

$$\mathbf{B}^T \mathbf{C}^{-1} [\mathbf{B}\mathbf{u} - \mathbf{u}^D] + s^2 \mathbf{D}' \mathbf{u} = 0 \quad . \tag{9}$$

The resulting vector of model parameters is

$$\mathbf{u} = [\mathbf{B}^T \mathbf{C}^{-1} \mathbf{B} + s^2 \mathbf{D}']^{-1} \mathbf{B}^T \mathbf{C}^{-1} \mathbf{u}^D \quad . \tag{10}$$

5 Criteria of acceptability

The geometrical spreading Q increases with increasing travel time τ approximately according to asymptotic formula

$$Q \sim e^{\lambda\tau} \quad , \tag{11}$$

where λ is the Lyapunov exponent corresponding to the ray.

Number of arrivals in each point of the model is an important indication, whether the behaviour of rays is regular or chaotic. We wish the number of arrivals not to exceed, let us say, 10.

In a finite model, the number of arrivals ν is proportional to the geometrical spreading. This is caused by overlaying of the ray tubes, see Figure 5. As we wish to smooth the model for migration, a sum of travel times from a source τ_S and a receiver τ_R should be substituted for the travel time τ in equation (11). Hence, we can write

$$\nu \sim e^{\lambda(\tau_S + \tau_R)} \quad . \tag{12}$$

Consequently, number of arrivals ν may be expressed as a product of the numbers of arrivals from a source ν_S and a receiver ν_R

$$\nu = \nu_S \nu_R \quad . \tag{13}$$

For $\tau = \tau_S + \tau_R = 0$, we obtain $e^{\lambda\tau} = 1$. Since this corresponds to the number of arrivals in the nearest vicinity of the source (or of the receiver for the migration), we can rewrite equation (12) as

$$\nu \approx e^{\lambda(\tau_S + \tau_R)} \quad . \tag{14}$$

We want to work with the "average Lyapunov exponent" $\widehat{\lambda}$. The "average Lyapunov exponent" $\widehat{\lambda}$ is the Lyapunov exponent averaged over the large set of rays (Klimeš 1999). The value of the "average Lyapunov exponent" may be one of the criteria of the smoothness of the model. Hence, we wish $e^{\widehat{\lambda}\tau_{MAX}}$ not to exceed 10, τ_{MAX} being the maximum sum of travel times from a source and a receiver.

The synthetic seismograms, used as the "real data" for the migration in the Marmousi model, are computed with the length of 2.9 seconds. Since the sum of travel times from a source and a receiver cannot exceed the length of the seismogram, $\tau_{MAX} = 2.9$ seconds may be used for estimation of the optimum value of $\widehat{\lambda}$. Thus, we obtain the optimum value of $\widehat{\lambda}$ close to 1 s^{-1}. This assumption of the optimum value of $\widehat{\lambda}$ will be used in the choice of parameter s.

The width of Gaussian packets should be kept small. For very wide packets, the obtained wavefield would not be the correct solution of solved equations. Thus, desired migrated section would be wrong. Since the width of a Gaussian beam is equal to the width of corresponding symmetric Gaussian packets, and the computation of the beams is easier, we study the width of Gaussian beams. The maximum halfwidth should probably not be greater then the B–spline interval.

Let us mention that the width of Gaussian beams or packets depends not only on the smoothness of a model, but also on the frequencies under consideration. From this point of view, the model is not complex for Gaussian beams or packets by itself, but in relation to the frequency.

The relative root mean square (RMS) difference of slowness between the original and the smoothed model may be the criterion of a "good agreement". The relative RMS difference of slowness corresponds approximately to the relative error of travel time. This is an asymptotic relation valuable for short rays. The relative error of travel time may be smaller for longer rays.

By the term "error of travel time" we mean the difference between the real travel time in the original structure and the computed travel time in the smoothed model. Although we cannot determine the real travel time, we can estimate the error caused by the difference between the original and the smoothed model.

6 Choice of the coefficients and of the density of the grids

We need to specify coefficients b'_{ijkl} and s, matrix $\mathbf{C}$ and the density of the grids before the computation.

As we do not have any prior information, we choose $\sigma^{GRID} = \sqrt{N}$, where N is the number of values to be fit. This makes the value of the objective function S approximately independent on the number of gridpoints.

Coefficients b'_{ijkl} may be constructed as a completely symmetric tensor (Klimeš 2000a). The 4×4 matrix $\mathbf{b}'$ is then defined by

$$b'_{ijkl} = \langle e_i e_j e_k e_l \rangle \quad , \tag{15}$$

where $\mathbf{e}$ is the unit vector and $\langle ... \rangle$ means averaging over all directions of the unit vector.

Thus, desired matrix $\mathbf{b}'$ is

$$\mathbf{b}' = \begin{array}{c} \\ 11 \\ 12 \\ 21 \\ 22 \end{array} \begin{array}{c} \begin{array}{cccc} 11 & 12 & 21 & 22 \end{array} \\ \begin{pmatrix} 3/8 & 0 & 0 & 1/8 \\ 0 & 1/8 & 1/8 & 0 \\ 0 & 1/8 & 1/8 & 0 \\ 1/8 & 0 & 0 & 3/8 \end{pmatrix} \end{array} \quad . \tag{16}$$

The original grid consists of cells of 4×4 metres, which yields $751 \times 2301 = 1728051$ gridpoints. Three B–spline grids of cells of
(a) 100×400 metres,
(b) 200×230 metres and
(c) 200×400 metres
are studied. These grids consist of $P = 744$, 656 and 384 gridpoints, respectively.

Three data grids of cells of
(a) 20×80 metres,
(b) 40×40 metres and
(c) 40×80 metres
are used in the inversion. Values at the gridpoints are calculated by averaging of the values of slowness in the Marmousi model, as described in section 3. These grids consist of $N = 17516$, 17556 and 8816 gridpoints, respectively.

For fixed Sobolev norm (4), parameter s in equation (5) is proportional to $\widehat{\lambda}^{-2}$. Thus, n-fold decrement of $\widehat{\lambda}$ requires n^4-fold increment of the square of the Sobolev norm in the objective function in equation (1). Consequently, decrement of $\widehat{\lambda}$ increases the difference between the new and the original model. From this point of view, $\widehat{\lambda}$ should not be *too* small. We should keep $\widehat{\lambda}$ close to the optimum value estimated above.

Klimeš (2000b, equation 20) suggests that parameter s may be connected with the maximum value of the Sobolev norm of slowness magnitude $\|u\|_{MAX}$, the standard slowness magnitude deviation of the model $|u - u_0|$ and the given slowness deviation σ by approximative formula

$$s \approx \frac{|u - u_0|}{\sigma} \frac{1}{\|u\|_{MAX}} \quad . \tag{17}$$

We can estimate the maximum value of the Sobolev norm of slowness by $\|u\|_{MAX} \approx (2\widehat{\lambda})^2 u_A^3$ (Klimeš 2000a, Sec. 3.5), u_A being the average slowness in the model. The standard slowness deviation of the model may be estimated by $|u - u_0| \approx \varepsilon u_A$, ε being the relative travel time error. The given slowness deviation is determined by $\sigma = \sigma^{GRID} N^{-\frac{1}{2}}$. Hence, we can rewrite equation (17) as

$$s \approx \varepsilon \left(2 u_A \widehat{\lambda}\right)^{-2} \sqrt{N} \left(\sigma^{GRID}\right)^{-1} \quad . \tag{18}$$

For $\varepsilon = 0.1$, $(u_A)^{-1} = 3000$ ms^{-1} and $\widehat{\lambda} = 1$ s^{-1} we obtain $s = 225000$ m^2. This is probably the maximum useful value of s. We study values of s of
(1) 0 m^2 (without Sobolev norm included in the inversion),
(2) 10000 m^2,
(3) 25000 m^2,
(4) 50000 m^2,
(5) 100000 m^2 and
(6) 225000 m^2.

7 Numerical examples

We have calculated the smoothed models, corresponding values of the relative RMS difference of slowness between the smoothed and original model, angular dependence of the Lyapunov exponents, values of the "average Lyapunov exponents", rays, numbers of arrivals and the halfwidths of Gaussian beams.

Figure 2 shows the models without minimized Sobolev norm, smoothed just by the use of the coarse B–spline grid.

Figure 3 shows the smoothed models with the grid of cells of 200×400 metres and with minimized Sobolev norm. The respective figures for the models with the grid of cells of 100×400 or 200×230 metres look similarly and are not shown.

Figure 4 displays the angular dependence of the Lyapunov exponents and the values of the "average Lyapunov exponents" for smoothed models with the B–spline grid of cells of 200×400 metres. We can see that the model without minimized Sobolev norm of slowness seems to be too rough, whereas models with minimized Sobolev norm have the "average Lyapunov exponent" close to or less than our initial assumption of the optimum value. Unfortunately, a strong angular dependence of the Lyapunov exponents (and consequently too great maximum value of the Lyapunov exponent) indicates that even models with $\widehat{\lambda}$ close to 1 (as the model with $s = 10000\mathrm{m}^2$) may be still too rough.

The synthetic seismograms, used for the migration in the Marmousi model, are computed with the length of 2.9 seconds. The streamer composed of hydrophone groups has been used for data acquisition for each shot. The farthest hydrophone has been 2575 metres from the watergun. Since the maximum travel time cannot exceed the length of the seismogram, let us assume a fixed travel time of 2.9 seconds. For an almost horizontal ray with the endpoint in the farthest hydrophone and for the velocity of 1500 ms^{-1} we obtain the farthest possible reflection point 4350 metres from the watergun and 900 metres from the farthest hydrophone. As 900 metres corresponds to 0.6 seconds, we may estimate the maximum useful value of the travel time as 2.3 seconds. Rays have been calculated for this value of the maximum travel time.

Figure 5 displays rays computed in the smoothed models. We can see the dependence of the behaviour of rays on parameter s. We moved the source along the whole profile and tested ray tracing. The behaviour of rays was always of the same kind as in these illustrative figures. Models with $s = 0\mathrm{m}^2$ and $s = 10000\mathrm{m}^2$ seem not to be suitable for ray methods due to shadow zones and the density of caustics. In Figure 5a, we can see rays trapped in the low velocity channels.

The number of arrivals for selected smoothed models are shown in Figure 6. The maximum number of arrivals for the models with the B–spline grid of cells of 200×400 metres is
(a) 19 for the value of $s = 0\ \mathrm{m}^2$,
(b) 18 for the value of $s = 10000\ \mathrm{m}^2$,
(c) 7 for the value of $s = 25000\ \mathrm{m}^2$,
(d) 5 for the value of $s = 50000\ \mathrm{m}^2$,
(e) 3 for the value of $s = 100000\ \mathrm{m}^2$ and
(f) 2 for the value of $s = 225000\ \mathrm{m}^2$.
Due to the requirements established above, the models with $s = 0\mathrm{m}^2$ and $s = 10000\mathrm{m}^2$ are probably not suitable for ray-based methods.

In the model without interfaces, the number of arrivals should be odd. Even numbers for $s = 10000$ m^2 and $s = 225000$ m^2 may be explained by the influence of the borders of the model. The respective figures for the models with $s = 0$ m^2 and $s = 225000$ look similarly as those for the models with $s = 10000$ m^2 and $s = 100000$, respectively, and are not shown.

The relative RMS difference of slowness between all calculated models and the original Marmousi model are in Table 1. We can see that the price for a model suitable for ray tracing is a massive increment of the relative RMS difference of slowness between the smoothed and original model, representing here the geological structure. This is caused by the complexity of the original Marmousi model. In the case of a larger value of parameter s, the relative RMS difference is the same for all studied B–spline grids. Hence, the model with the B–spline grid of cells of 200×400 metres (with only 384 B–spline gridpoints) is probably the best choice.

In general, we believe it is useless to work with a too dense B–spline grid, because the differences between the original model and smoothed models with various densities of the model grid converge with increasing weight of the Sobolev norm.

The halfwidths of Gaussian beams calculated for the models with the B–spline grid of cells of 200×400 metres and with $s = 25000$ m^2, $s = 50000$ m^2, $s = 100000$ m^2 and $s = 225000$ m^2 are shown in Figures 7, 8 and 9. The colour coded halfwidths are displayed at the respective points along central rays of the beams. We have calculated the halfwidths for three different initial conditions: we denote them as
(1) type A (input parameters for program *mtt.for* are GBY22=$0.050 \cdot 10^{-6}$, GBR22=0),
(2) type B (GBY22=$0.250 \cdot 10^{-6}$, GBR22=0) and
(3) type C (GBY22=$0.250 \cdot 10^{-6}$, GBR22=$-0.400 \cdot 10^{-6}$).
Parameter GBR22 is the real part and parameter GBY22 is the imaginary part of the complex-valued travel time of a Gaussian beam along the surface of the model.

Satisfactory values are coloured in yellow and green. The green colour corresponds to the Gaussian beam halfwidth of 202 and 378 metres for the frequencies of 35 Hz and 10 Hz, respectively.

Let us make some comments on Figure 7, 8 and 9. We can see that the width of Gaussian beams in various regions of the model depends on the initial shape of Gaussian beams and on the position of a source. In all smoothed models (except of the model with $s = 225000$ m^2), the Gaussian beams with the initial condition of type A are the best choice for the position of a source close to both ends of the profile, whereas the Gaussian beams with the initial condition of type C are the most suitable for the position of a source close to the middle of the profile.

In the model with $s = 225000$ m^2, the Gaussian beams with the initial condition of type A are still the best for the position of a source close to both ends of the profile, but the Gaussian beams with the initial condition of type B are probably better than the Gaussian beams with the initial condition of type C for the position of a source close to the middle of the profile. We can assume that for even smoother models, the Gaussian beams with the initial condition of type A would become the best choice for all positions of a source.

We can see that even in models with sufficiently small number of arrivals, the widths of Gaussian beams are on the edge of acceptability. This is caused by the low frequencies under consideration.

Let us summarize that models with parameter s equal to or greater then 25000 m^2

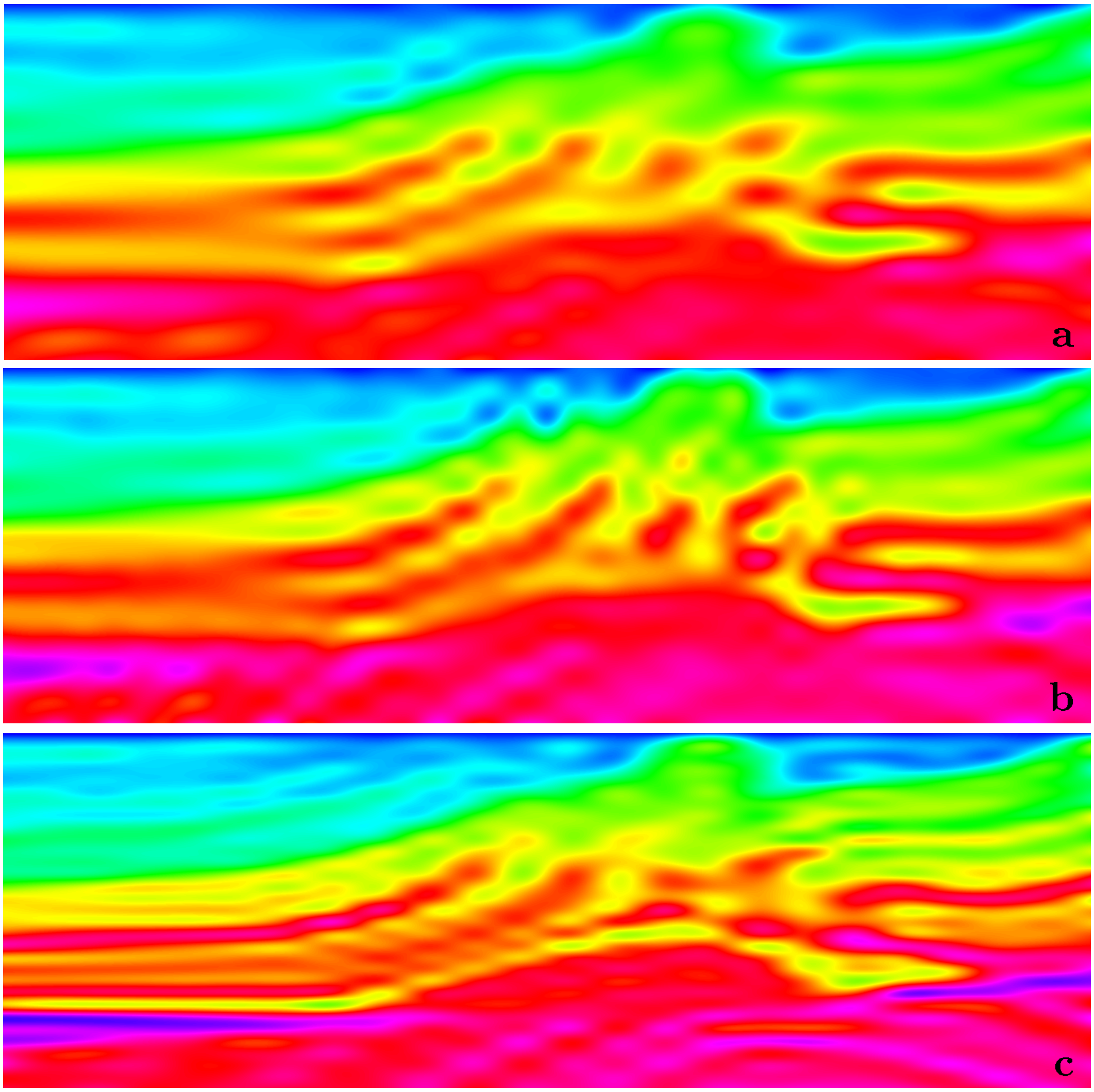
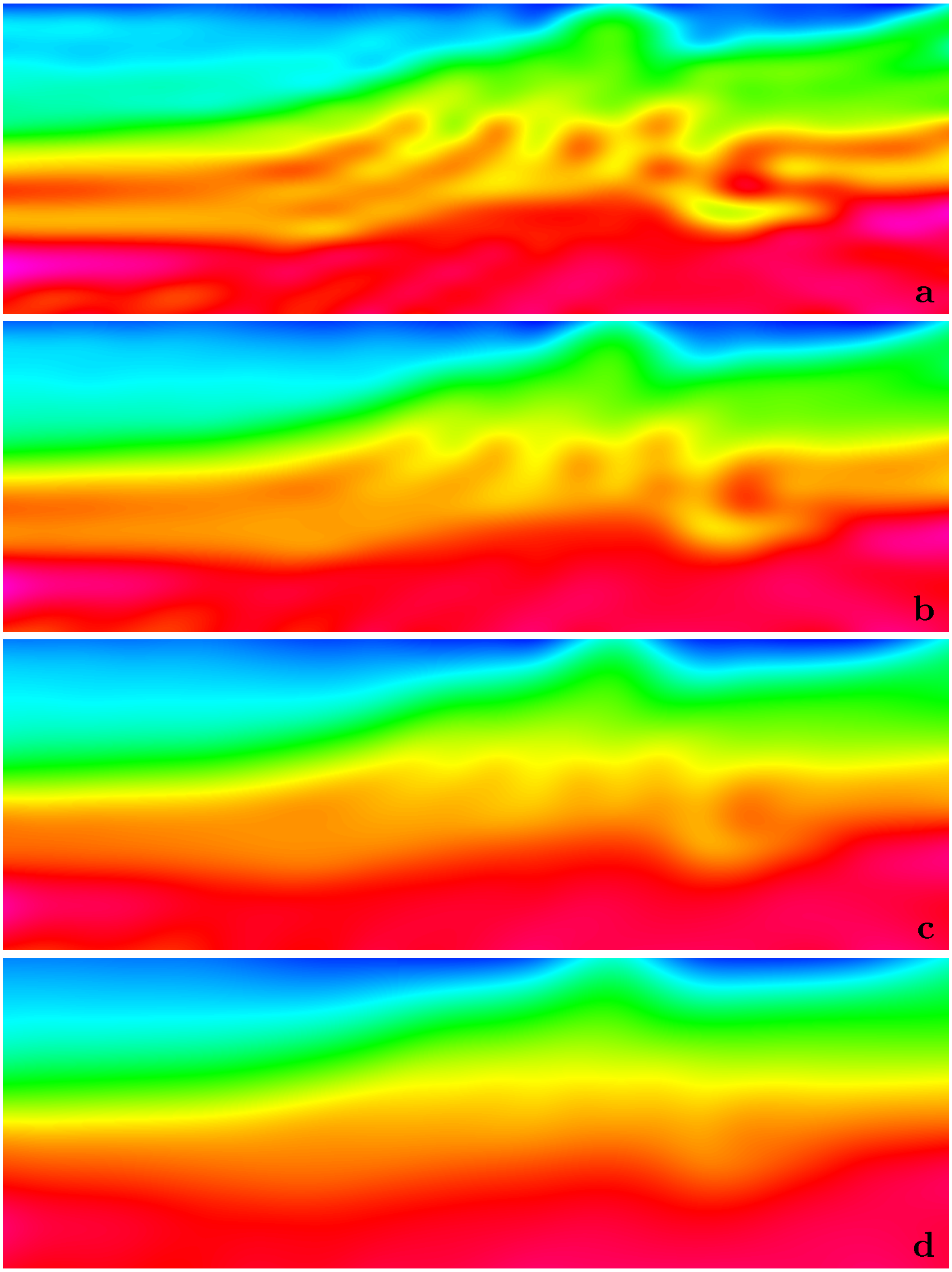


Figure 2. The smoothed models without minimized Sobolev norm of slowness ($s = 0\mathrm{m}^2$) for the B–spline grids of cells of (a) 200×400 metres, (b) 200×230 metres and (c) 100×400 metres.

seem to be suitable for ray tracing with the travel time of 2.3 seconds, see Figure 5 and 6. From this point of view, these models are sufficiently smooth. However, the low frequencies under consideration make the use of the Gaussian beam or packet method almost impossible. We should probably improve the applicability of the Gaussian packet method by using shapes of Gaussian packets optimized in dependence on the position of the source, or of the receiver in the migration.

We are still not sure, whether the asymptotic high-frequency method can be applied to the data with the central frequency of 25 Hz. This is the theme for further study.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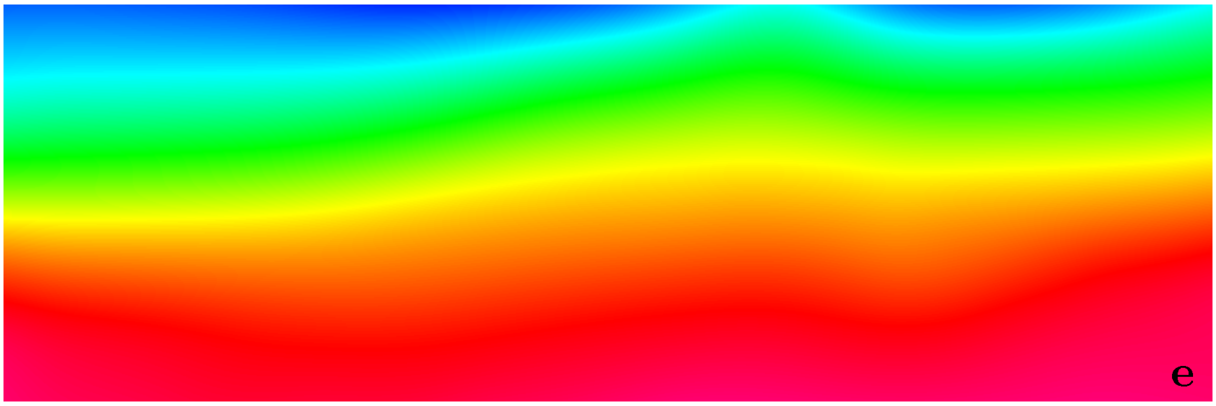


Figure 3. The smoothed models for the B–spline grid of cells of 200×400 metres and for values of parameter s of (a) 10000 m^2, (b) 25000 m^2, (c) 50000 m^2, (d) 100000 m^2 and (e) 225000 m^2.

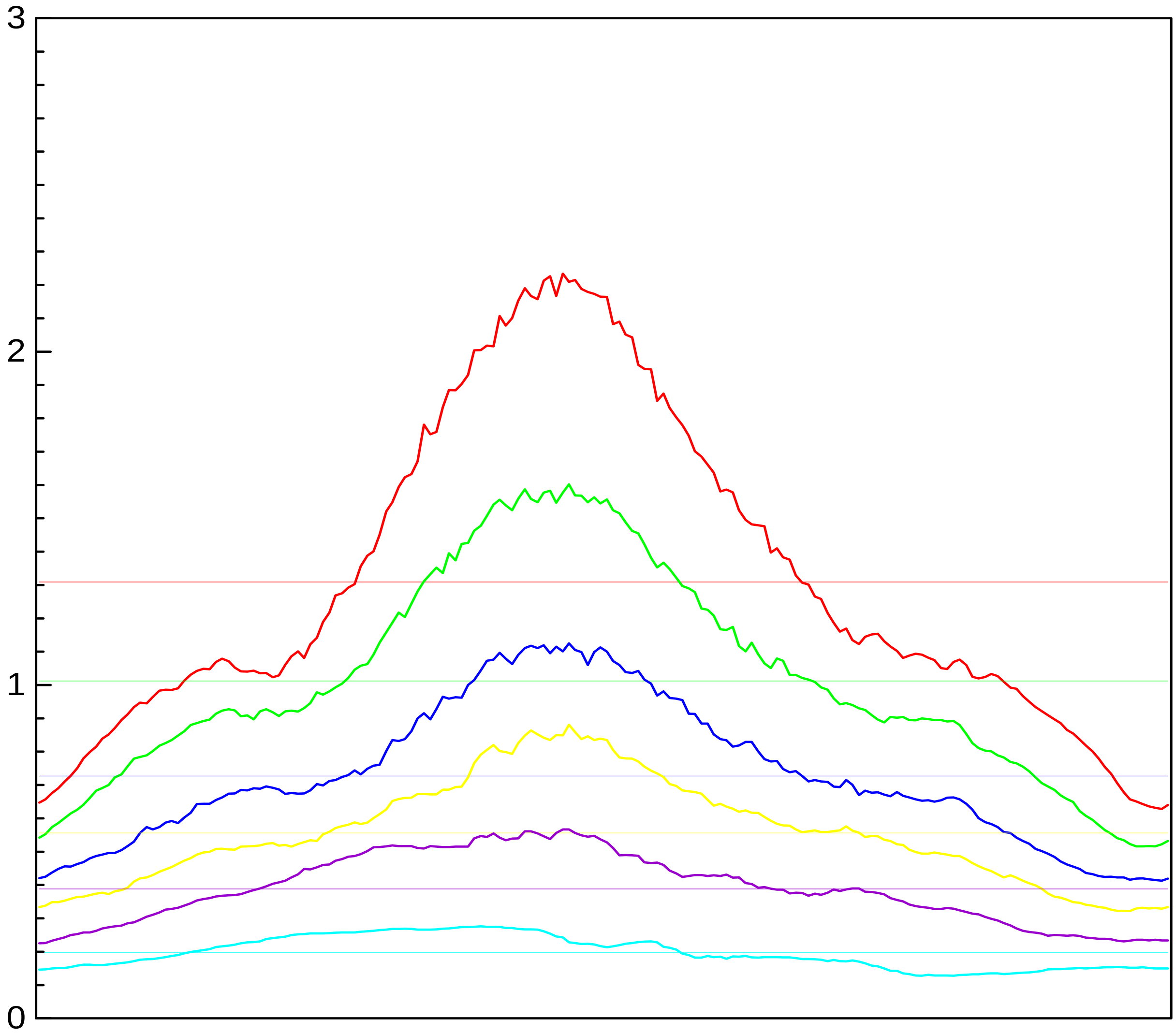
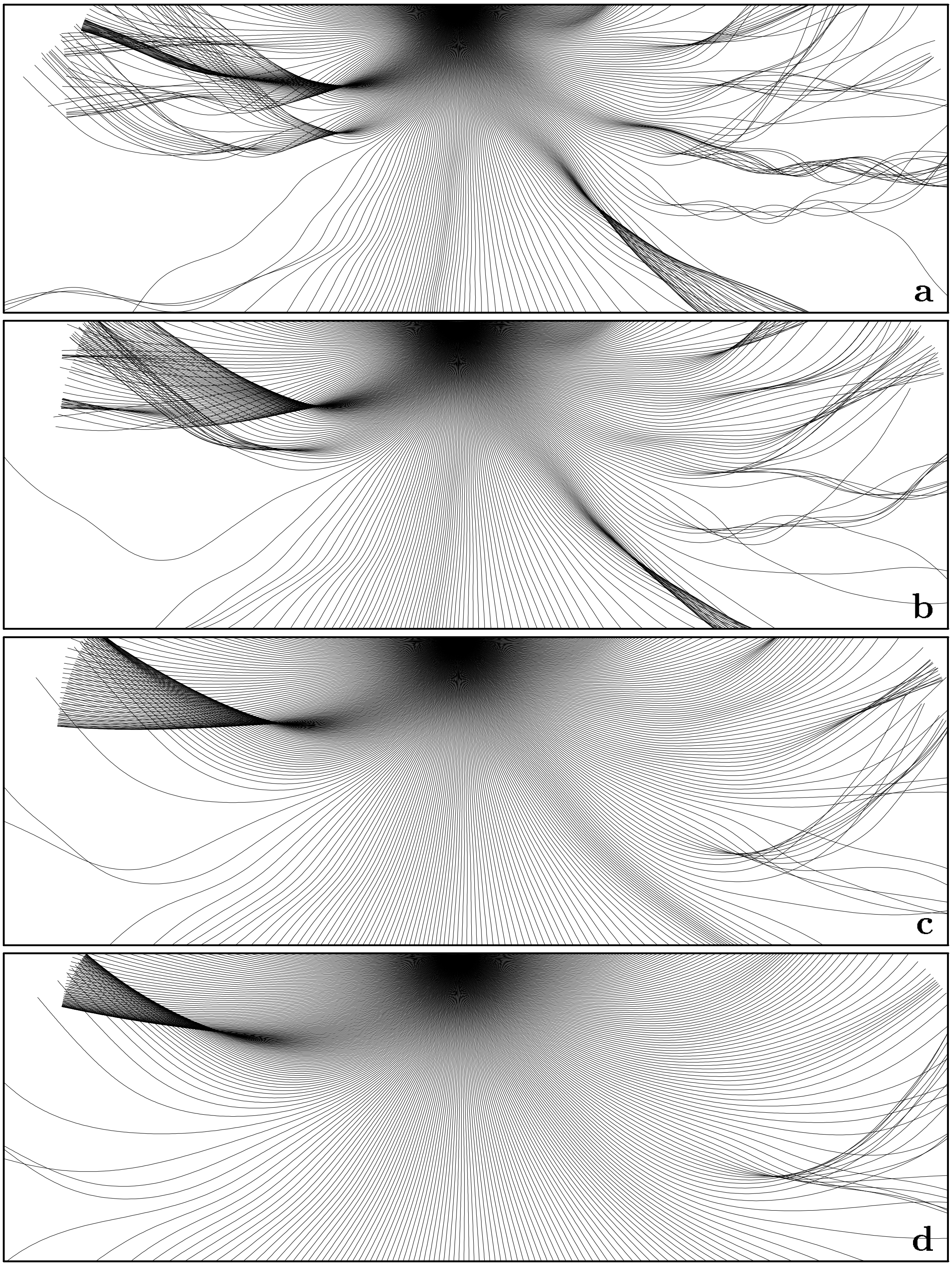


Figure 4. The angular dependence of the Lyapunov exponents for the models with the B–spline grid of cells of 200×400 metres. The left and right border corresponds to the horizontal ray, the middle corresponds to the vertical ray. The values of parameter s are (1) 0m^2 for *red*, (2) 10000m^2 for *green*, (3) 25000m^2 for *blue*, (4) 50000m^2 for *yellow*, (5) 100000m^2 for *magenta* and (6) 225000m^2 for *cyan*. The thin horizontal lines correspond to the "average Lyapunov exponents", averaged over angles with a uniform weight.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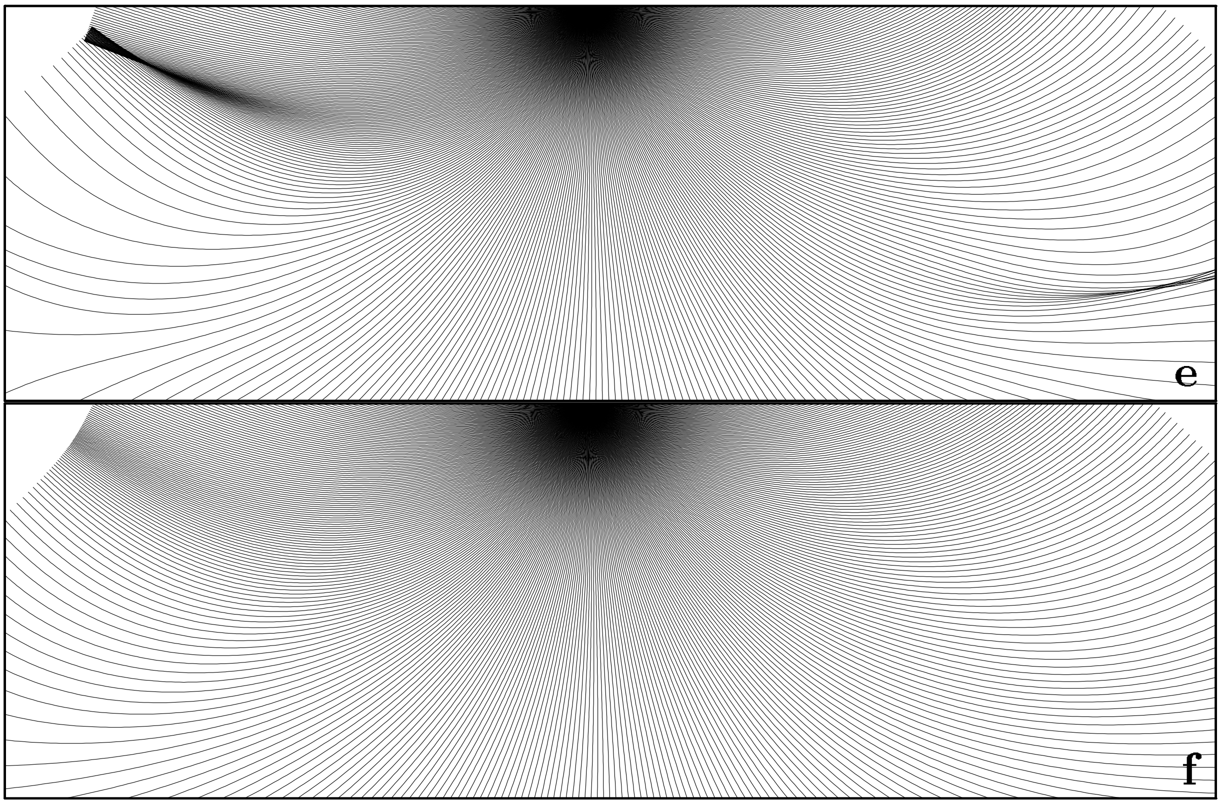


Figure 5. Rays in models with the B–spline grid of cells of 200×400 metres and with values of parameter s of (a) $0\mathrm{m}^2$, (b) $10000\mathrm{m}^2$, (c) $25000\mathrm{m}^2$, (d) $500000\mathrm{m}^2$, (e) $100000\mathrm{m}^2$ and (f) $225000\mathrm{m}^2$. The maximum travel time is of 2.3 seconds.

	100 – 400m	**200 – 230m**	**200 – 400m**
0m^2	0.083	0.108	0.111
10000m^2	0.114	0.119	0.120
25000m^2	0.132	0.132	0.133
50000m^2	0.143	0.143	0.143
100000m^2	0.151	0.151	0.151
225000m^2	0.158	0.158	0.158

Table 1. The relative RMS difference between the smoothed and the Marmousi model. Columns correspond to various B–spline grids, rows correspond to various values of parameter s.

8 Conclusions

Minimization of the relevant Sobolev norm of slowness is a powerful tool for preparing the optimum models for the asymptotic ray theory methods. As we have demonstrated in numerical examples, it can be used for the smoothing of very complex models. However, the difference of slowness between the smoothed and the original model is then increasing rapidly. Similarly, the error of travel time is increasing as well.

We believe there exists a natural relation between the complexity of the original model and the resulting difference between the sufficiently smoothed model and original model. The more complex the original model is, the more we have to change it. Thus, the judgement is up to the user, whether or not the model is too complex for smoothing. Required maximum error of travel time is then a key argument.

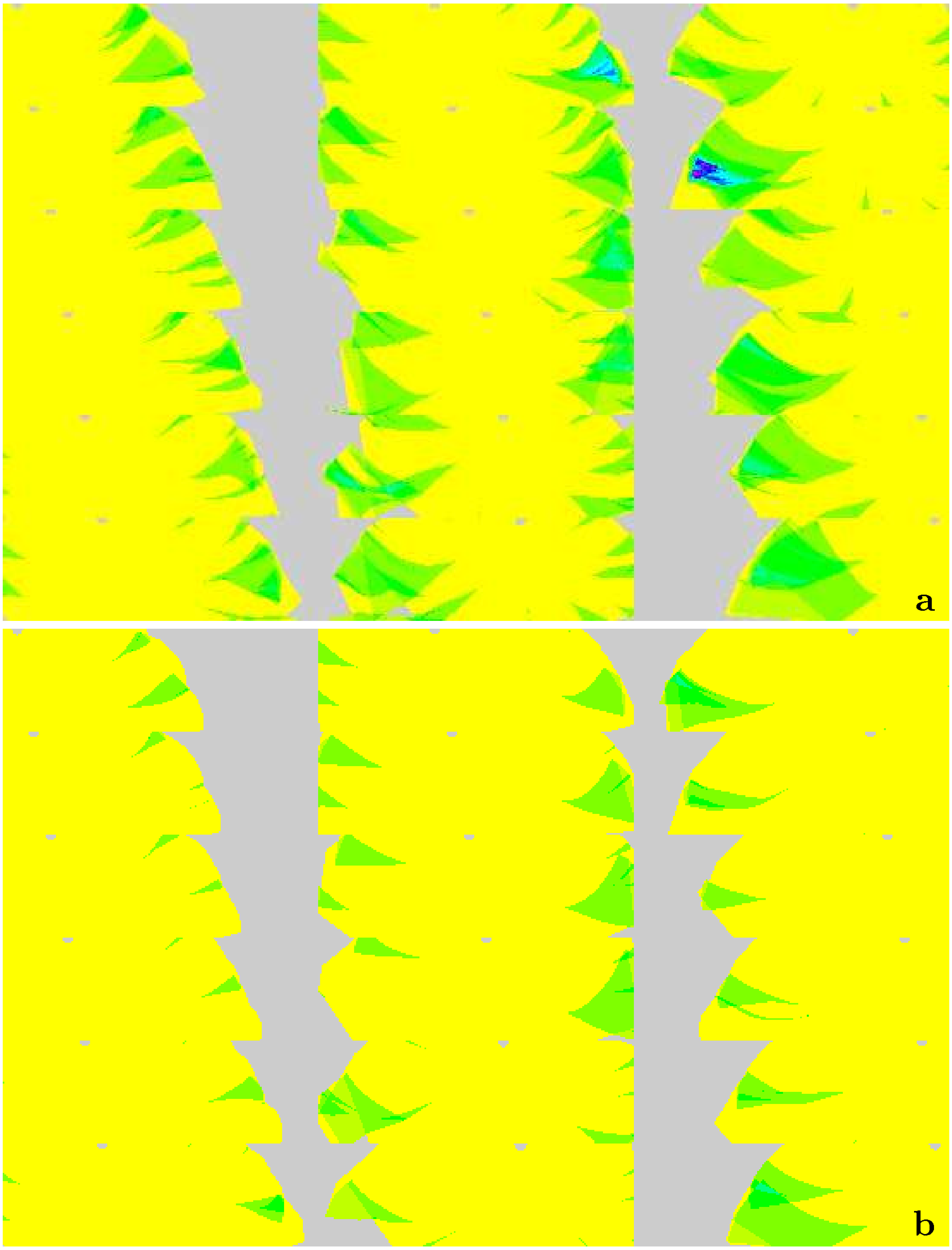
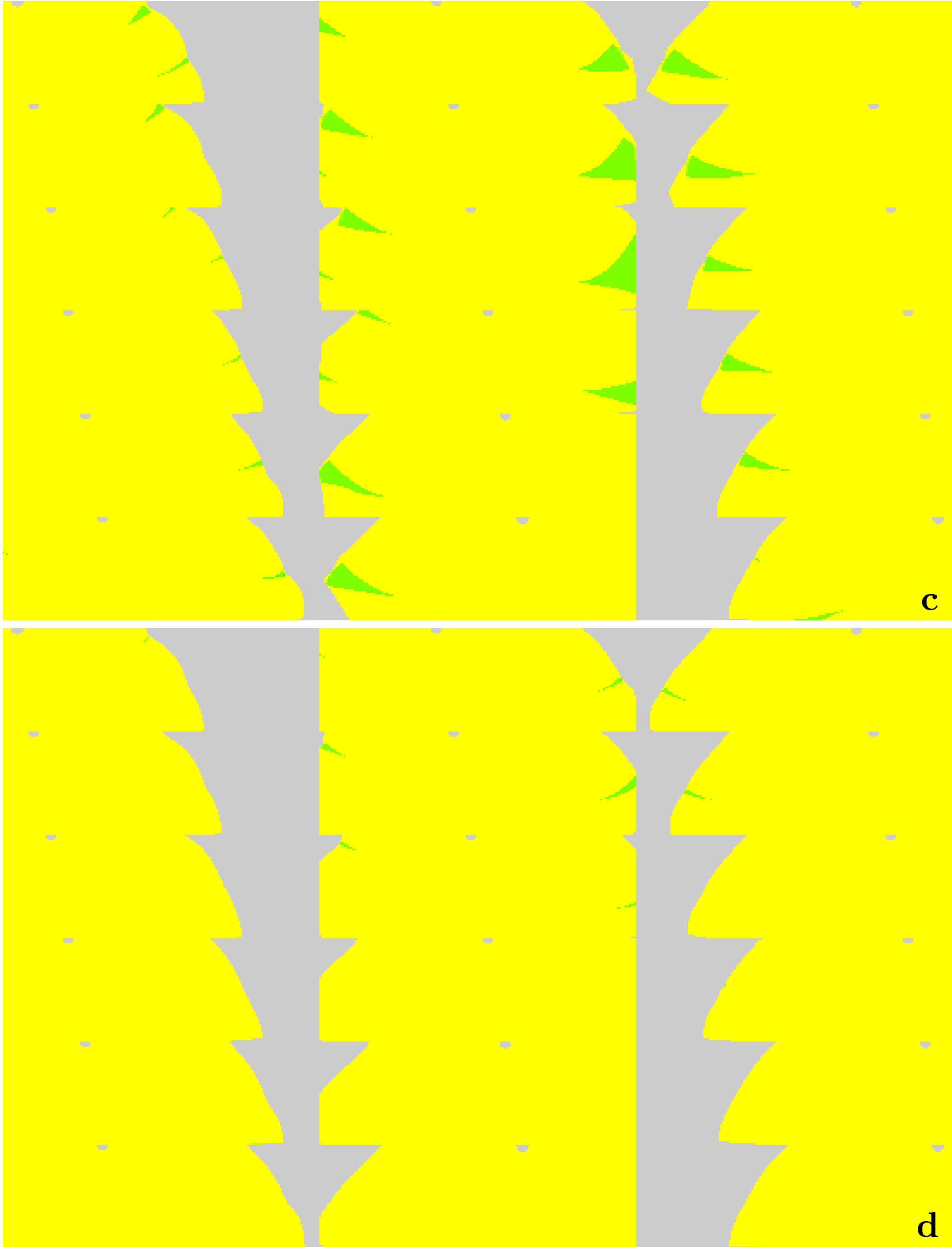


Figure 6. Numbers of arrivals in smoothed models with the B–spline grid of cells of 200×400 metres and with values of parameter s of (a) 10000m^2, (b) 25000m^2, (c) 50000m^2 and (d) 100000m^2. In the colour scale, grey corresponds to 0 arrivals, *yellow* corresponds to 1 arrival, *green* corresponds to 5 arrivals, *cyan* corresponds to 9 arrivals, *blue* corresponds to 13 arrivals and *magenta* corresponds to 17 arrivals. Consequently, other colours of this colour scale correspond to remainder numbers of arrivals.



The maximum number of arrivals is (a) 18, (b) 7, (c) 5 and (d) 3. The source has been moved over the whole profile. Numbers of arrivals are demonstrated for 18 different positions of a source in each figure.

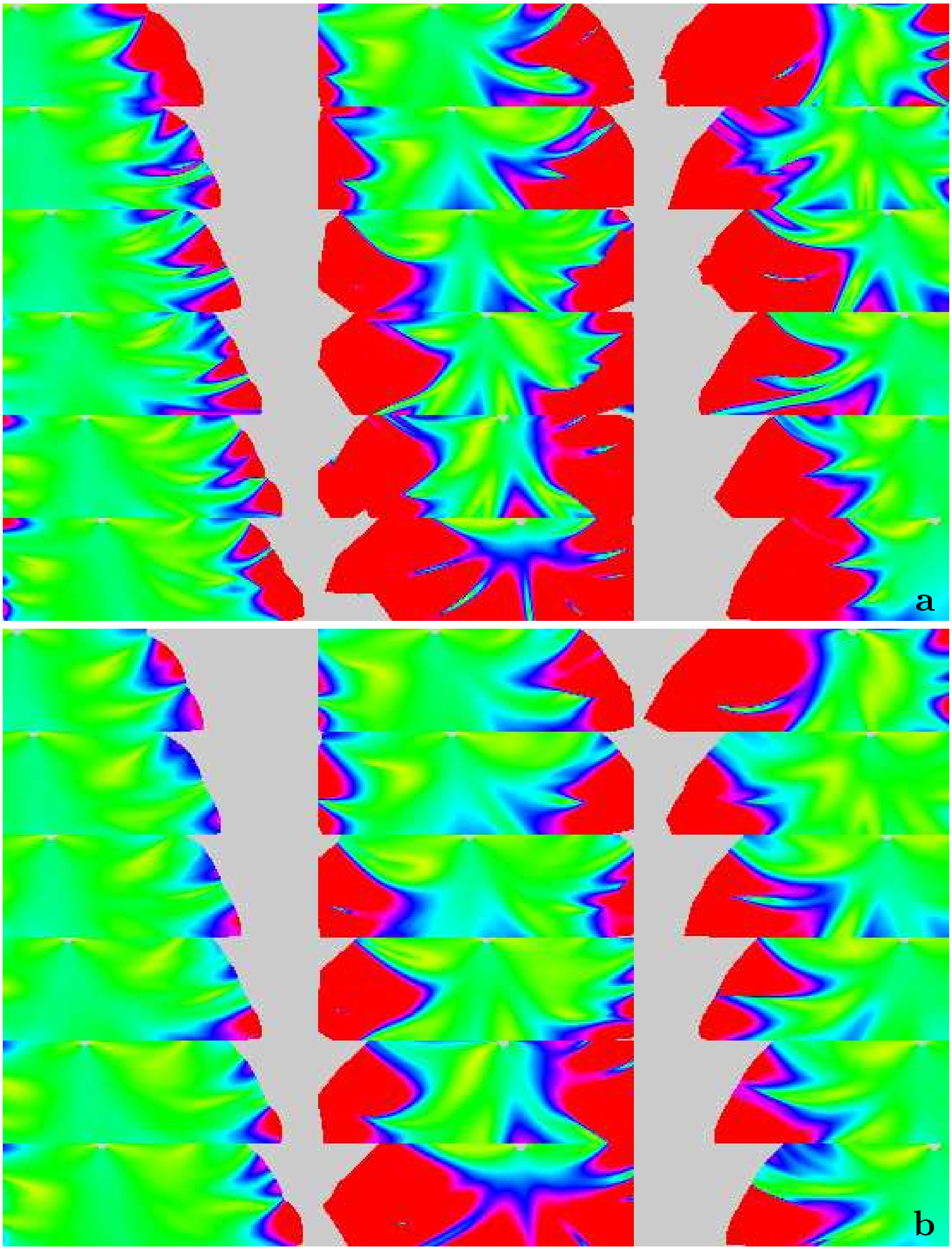
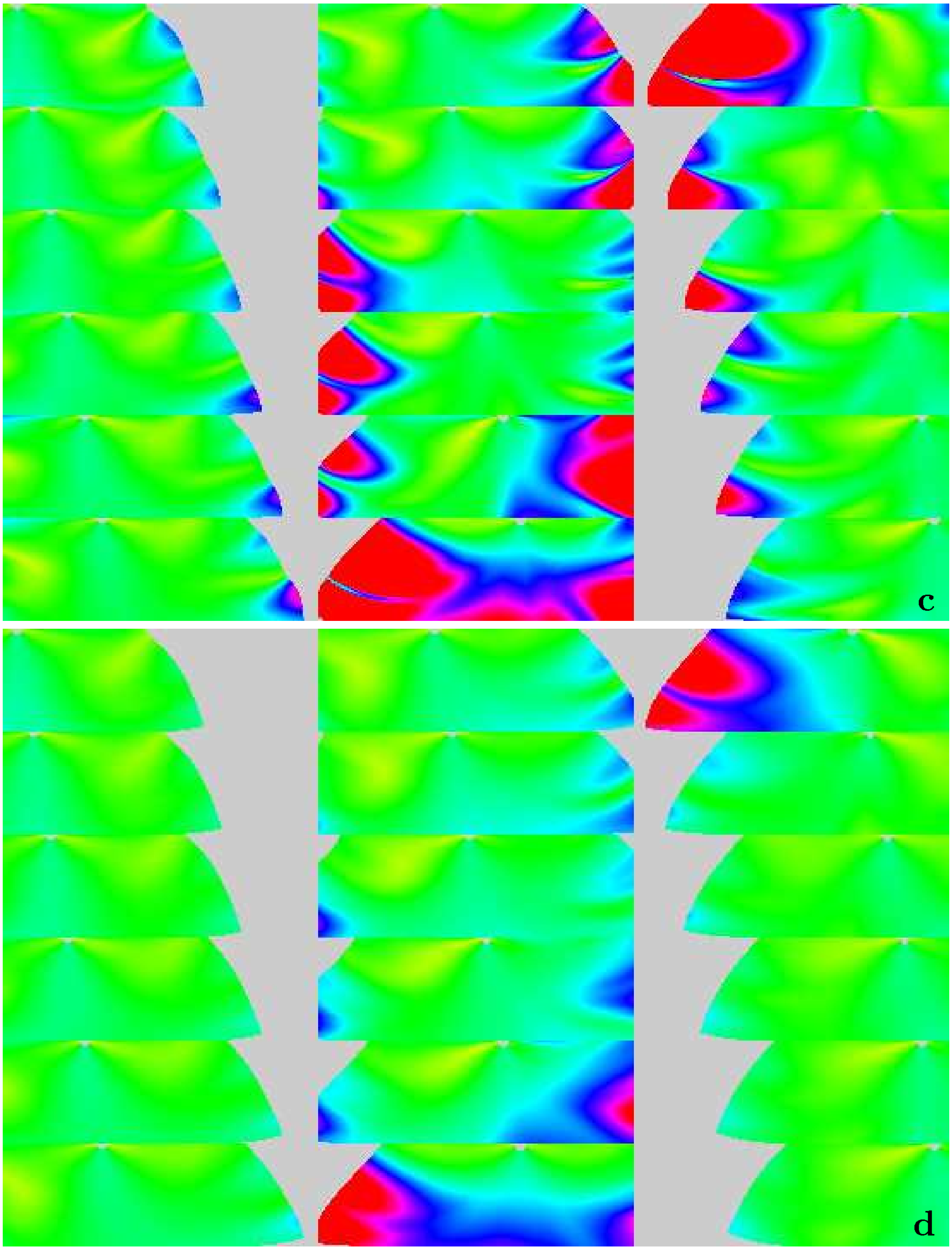


Figure 7. The halfwidths of Gaussian beams in smoothed models with the B–spline grid of cells of 200×400 metres and with values of parameter s of (a) $25000 \mathrm{m}^2$, (b) $50000 \mathrm{m}^2$, (c) $100000 \mathrm{m}^2$, (d) $225000 \mathrm{m}^2$. Acceptable values are coloured in *yellow* and *green*, totally unacceptable values are coloured in *red*. The *green* colour corresponds to the Gaussian beam halfwidth of 202 and 378 metres for the frequencies of 35 Hz and 10 Hz, respectively.



The *red* colour corresponds to the Gaussian beam halfwidth of more than 1010 and 1890 metres for the frequencies of 35 Hz and 10 Hz, respectively. The initial condition of Gaussian beams is of type A. The source has been moved over the whole profile. Halfwidths of Gaussian beams are demonstrated for 18 different positions of a source in each figure.

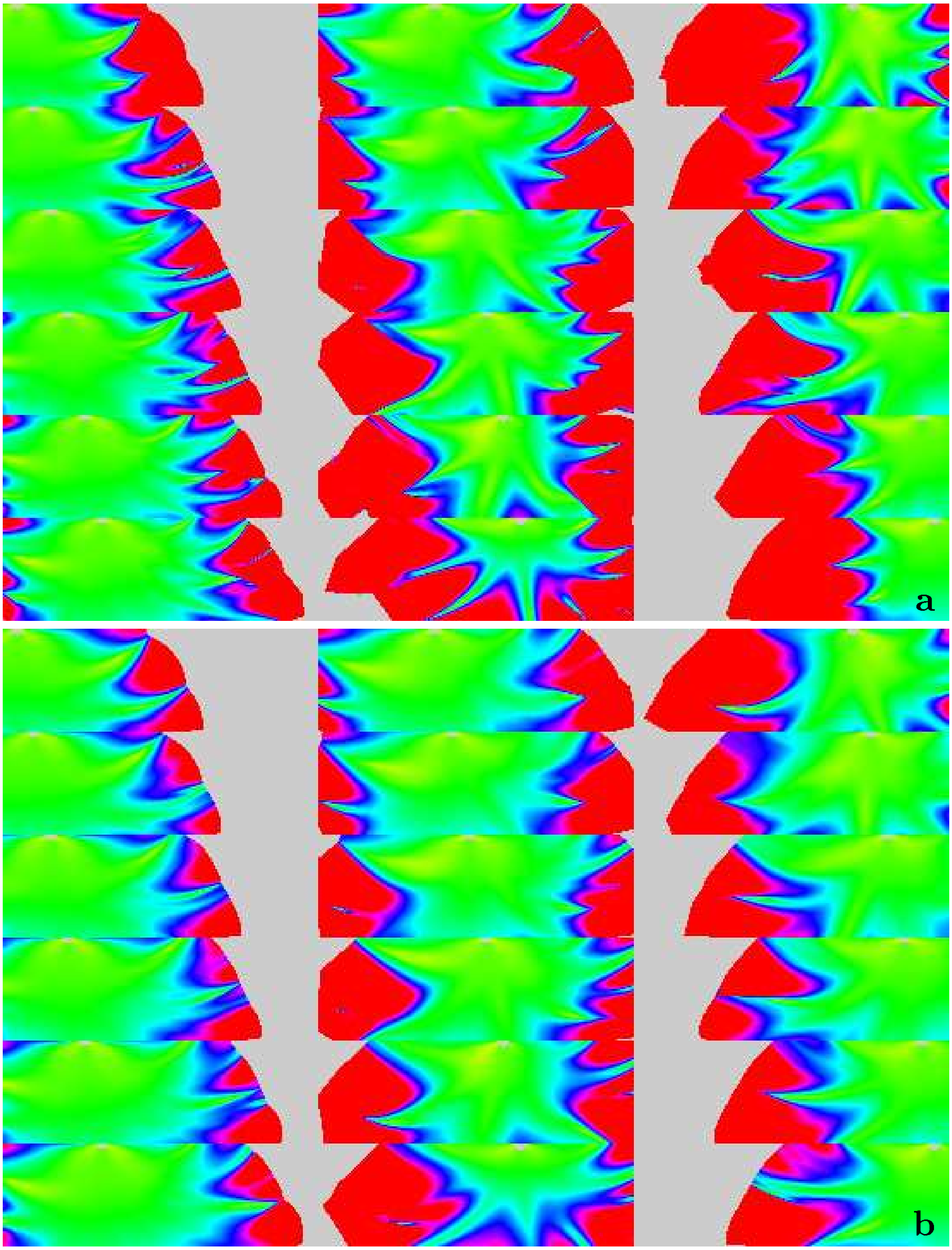
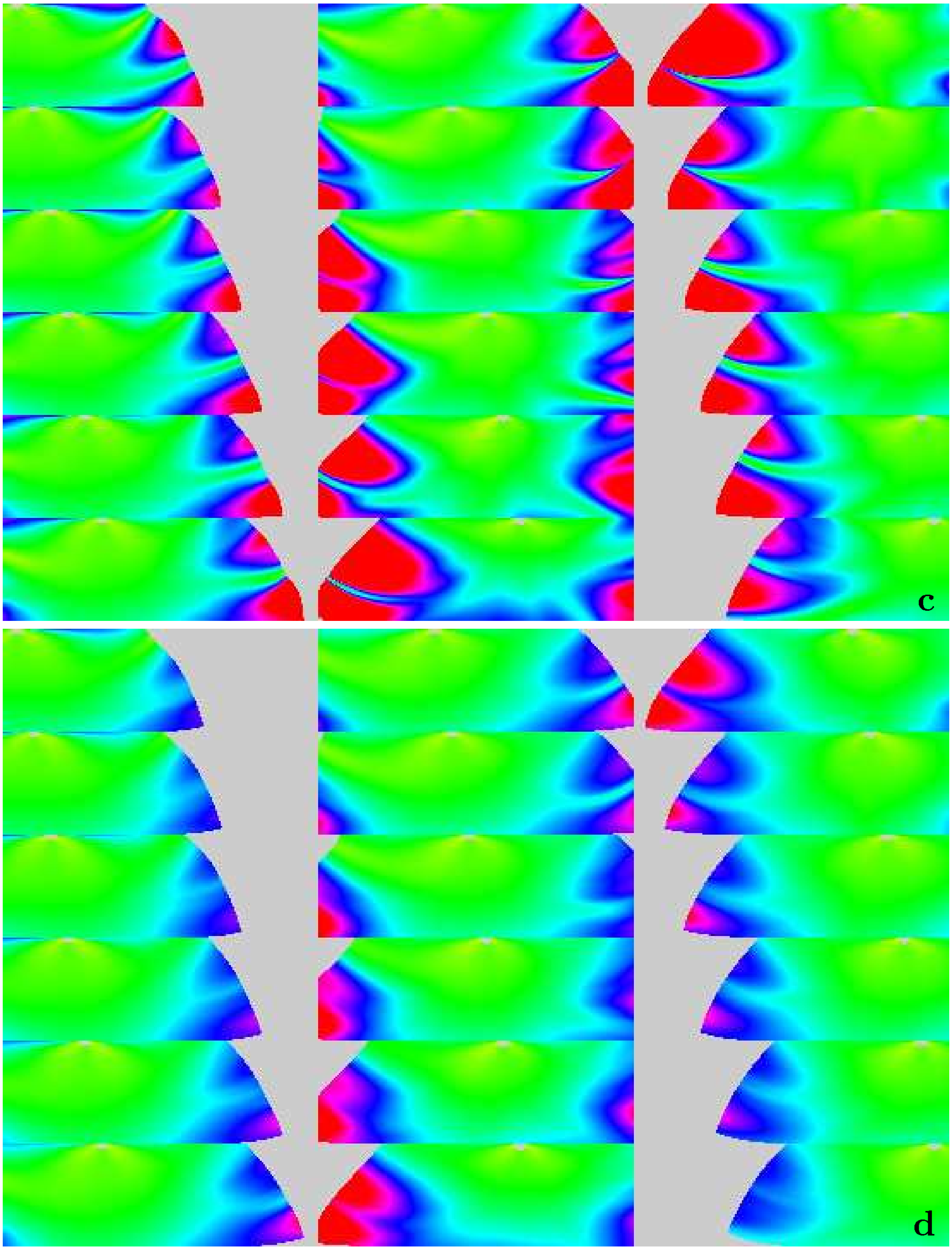


Figure 8. The halfwidths of Gaussian beams in smoothed models with the B–spline grid of cells of 200×400 metres and with values of parameter s of (a) $25000\mathrm{m}^2$, (b) $50000\mathrm{m}^2$, (c) $100000\mathrm{m}^2$, (d) $225000\mathrm{m}^2$. Acceptable values are coloured in *yellow* and *green*, totally unacceptable values are coloured in *red*. The *green* colour corresponds to the Gaussian beam halfwidth of 202 and 378 metres for the frequencies of 35 Hz and 10 Hz, respectively.



The *red* colour corresponds to the Gaussian beam halfwidth of more than 1010 and 1890 metres for the frequencies of 35 Hz and 10 Hz, respectively. The initial condition of Gaussian beams is of type B. The source has been moved over the whole profile. Halfwidths of Gaussian beams are demonstrated for 18 different positions of a source in each figure.

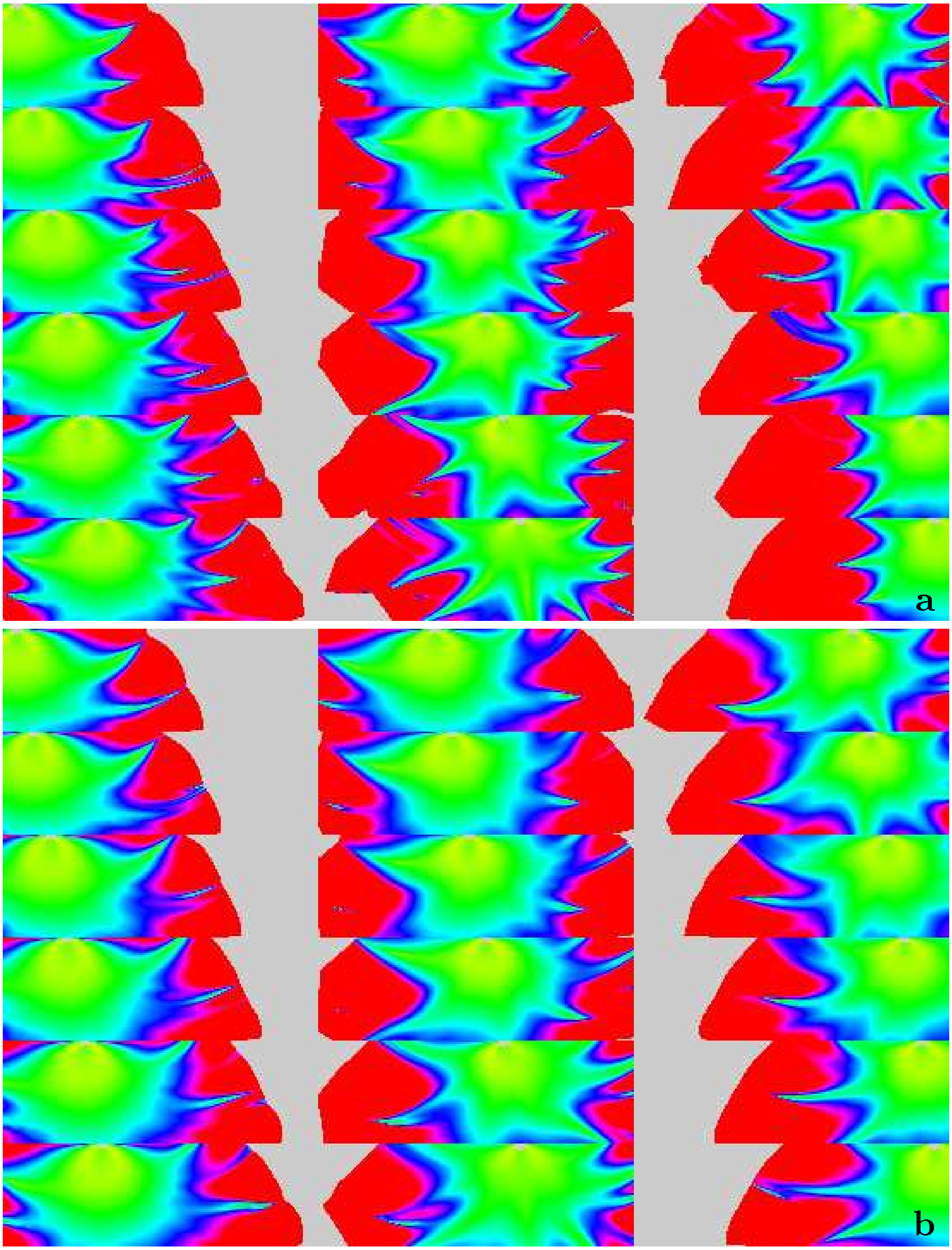
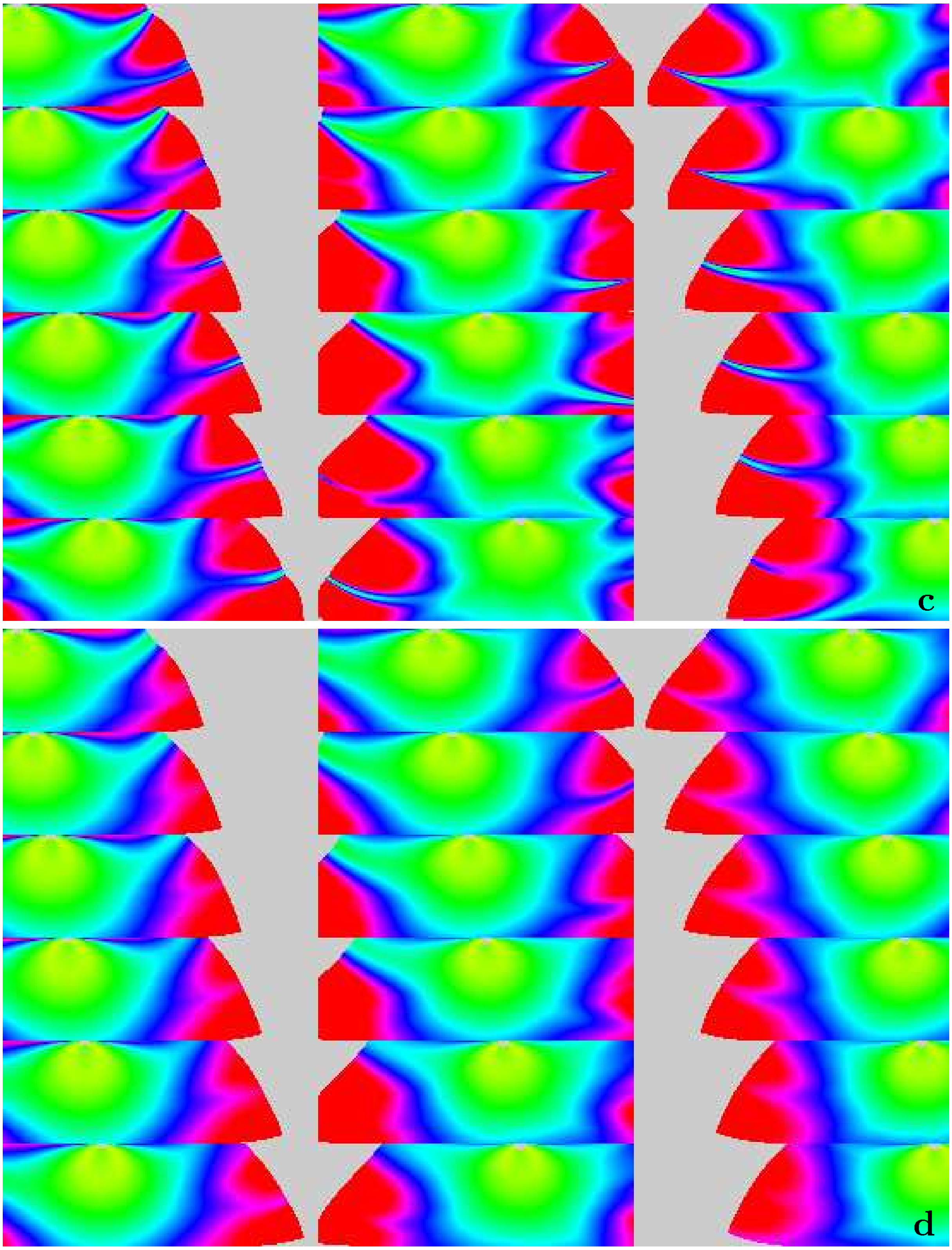


Figure 9. The halfwidths of Gaussian beams in smoothed models with the B–spline grid of cells of 200×400 metres and with values of parameter s of (a) $25000\mathrm{m}^2$, (b) $50000\mathrm{m}^2$, (c) $100000\mathrm{m}^2$, (d) $225000\mathrm{m}^2$. Acceptable values are coloured in *yellow* and *green*, totally unacceptable values are coloured in *red*. The *green* colour corresponds to the Gaussian beam halfwidth of 202 and 378 metres for the frequencies of 35 Hz and 10 Hz, respectively.



The *red* colour corresponds to the Gaussian beam halfwidth of more than 1010 and 1890 metres for the frequencies of 35 Hz and 10 Hz, respectively. The initial condition of Gaussian beams is of type C. The source has been moved over the whole profile. Halfwidths of Gaussian beams are demonstrated for 18 different positions of a source in each figure.

We have also demonstrated that even in models sufficiently smoothed for ray tracing, the Gaussian beams may be still too wide for the frequencies under consideration. In the case of preparation of a model for Gaussian beams or packets, we cannot judge solely from number of arrivals and values of the "average Lyapunov exponents", whether the model is convenient. The widths of Gaussian beams or packets should be studied as well.

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