Numerical comparison of the isotropic-common-ray and anisotropic-common-ray approximations of the coupling ray theory

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Summary

In the common–ray approximation of the coupling ray theory, the S–wave travel times in the coupling equations are approximated by the first–order perturbation expansion from the common reference ray. The common–ray approximation eliminates problems with ray tracing through S–wave singularities and also considerably simplifies the numerical algorithm of the coupling ray theory for S waves, but may introduce errors in travel times due to the perturbation from the common reference ray. The errors of S–wave travel times due to the common–ray approximation may be approximated by the second–order terms in the perturbation expansion. The first–order and second–order terms in the perturbation expansion can be calculated by simple numerical quadratures along the common reference ray.

The common reference ray may be represented by the isotropic common ray or by the anisotropic common ray. The anisotropic–common–ray approximation of the coupling ray theory is more accurate than the isotropic–common–ray approximation. The equations for the first–order and second–order perturbation expansions of travel time from both the isotropic and anisotropic common rays to anisotropic–ray–theory rays have already been derived. We numerically test the equations in three smooth 1–D velocity models of differing degree of anisotropy.

We first compare the second–order perturbation expansions of travel time from the isotropic common rays to the anisotropic common rays and vice versa. This comparison not only demonstrates the accuracy of the perturbation expansions, but also allows us to estimate the order of magnitude of the third–order, fourth–order and fifth–order terms in the perturbation expansions.

Then the first–order and second–order perturbation expansions of travel time from the isotropic common rays and from the anisotropic common rays to anisotropic–ray–theory rays are compared with the correct anisotropic–ray–theory travel times in order to illustrate the accuracy of the perturbation expansions and reliability of the error estimates.

Finally, the synthetic seismograms obtained by the isotropic–common–ray and anisotropic–common–ray approximations of the coupling ray theory are compared with the more accurate coupling–ray–theory synthetic seismograms simulated by the second–order perturbation expansion of travel time from the anisotropic common rays. In the numerical example, the errors of the anisotropic–common–ray approximation of the coupling ray theory are considerably smaller than the errors of the isotropic–common–ray approximation.

In: Seismic Waves in Complex 3–D Structures, Report 16 (Department of Geophysics, Charles University, Prague, 2006), pp. 155–178
[submitted to the Geophysical Journal International]

Keywords

Coupling ray theory, common–ray approximation, travel time, perturbation theory, seismic anisotropy, heterogeneous media.

1. Introduction

There are two different high–frequency asymptotic ray theories: the *isotropic ray theory* based on the assumption of equal velocities of both S waves, and the *anisotropic ray theory* assuming both S waves strictly decoupled. In the isotropic ray theory, the S–wave polarization vectors do not rotate about the ray, whereas in the anisotropic ray theory they coincide with the eigenvectors of the Christoffel matrix which may rotate rapidly about the ray. Thomson, Kendall & Guest (1992) demonstrated analytically that the high–frequency asymptotic error of the anisotropic–ray–theory wavefield is inversely proportional to the second or higher root of the frequency if a ray passes through the point of equal S–wave eigenvalues of the Christoffel matrix even in otherwise strongly anisotropic media.

In "weakly anisotropic" models, at moderate frequencies, the S–wave polarization tends to remain unrotated round the ray but is partly attracted by the rotation of the eigenvectors of the Christoffel matrix. The intensity of the attraction increases with frequency. This behaviour of the S–wave polarization is described by the *coupling ray theory* proposed by Coates & Chapman (1990). The frequency–dependent coupling ray theory is the generalization of both the zero–order isotropic and anisotropic ray theories and provides continuous transition between them. The coupling ray theory is applicable to S waves at all degrees of anisotropy, from isotropic to considerably anisotropic velocity models. The numerical algorithm for calculating the frequency–dependent complex–valued S–wave polarization vectors of the coupling ray theory has been designed by Bulant & Klimeš (2002).

There are many commonly used *quasi–isotropic approximations* of the coupling ray theory (e.g., Pšenčík, 1998a), which diminish the accuracy of the coupling ray theory both with increasing frequency and increasing degree of anisotropy. Refer to Bulant & Klimeš (2002; 2004) and to Klimeš & Bulant (2004) for the description of individual quasi–isotropic approximations and for the examples of their impact on synthetic seismograms. Most of these quasi–isotropic approximations can be avoided with minimal effort except for the common–ray approximation for S waves.

In the *common–ray approximation*, only one reference ray is traced for both anisotropic–ray–theory S waves, and both S–wave anisotropic–ray–theory travel times in the coupling equations are approximated by the first–order perturbation expansion from the common reference ray. Whereas tracing the continuous system of anisotropic–ray–theory rays may be very difficult in the vicinity of an S–wave singularity at which the S–wave slowness surfaces coincide (Vavryčuk, 2003), this problem does not occur in the common–ray approximation. The common–ray approximation thus eliminates problems with ray tracing through S–wave singularities and also considerably simplifies coding of the coupling ray theory and numerical calculations, but may introduce errors in travel times due to the perturbation. These travel–time errors can deteriorate the coupling–ray–theory solution at high frequencies. It is thus of principal importance for numerical applications to estimate the travel–time errors due to the common–ray approximation, and then the related error of the wavefield.

The common reference rays have routinely been represented by the *isotropic common rays* calculated in the isotropic reference model, but they would better be represented by the *anisotropic common rays.* The anisotropic common rays are traced in the anisotropic model, using the averaged Hamiltonian of both anisotropic–ray–theory S waves (Bakker, 2002). The algorithm of the dynamic ray tracing corresponding to the anisotropic common rays was proposed by Klimeš (2006b). The numerical results of Klimeš & Bulant (2004) suggest that the *anisotropic–common–ray approximation* may be considerably more accurate than the *isotropic–common–ray approximation*, which we demonstrate on the numerical examples in this paper. For a more detailed description of the common–ray approximations of the coupling ray theory refer to Section 2.1.

In the common–ray approximation, the S–wave travel times in the coupling equations are approximated by the first–order perturbation expansion from the common reference ray, see Section 2.2. The common reference ray, represented by the isotropic common ray or by the anisotropic common ray, is traced using the reference Hamiltonian, see Section 2.3. The errors of S–wave travel times may then be approximated by the second–order terms in the perturbation expansion as explained in Section 2.4. Note that the perturbation expansion is the Taylor expansion with respect to the perturbation parameters, see (9), which parametrize the Hamiltonian, see (6). We refer here to the partial derivatives with respect to the perturbation parameters as perturbation derivatives, in order to distinguish them from the partial derivatives with respect to the spatial coordinates.

Klimeš & Bulant (2004) derived the equations for estimating the travel–time errors due to the isotropic–common–ray approximation (and roughly also due to the anisotropic–common–ray approximation) by numerical quadrature along the *isotropic common rays* in smooth velocity models without interfaces.

Klimeš & Bulant (2006) derived the equations for estimating the travel–time errors due to the anisotropic–common–ray approximation (and roughly also due to the isotropic–common–ray approximation) by numerical quadrature along the *anisotropic common rays* in smooth velocity models without interfaces.

The derivations by Klimeš & Bulant (2004; 2006) are based on the general equations for the second–order perturbations of travel time by Klimeš (2002). The equations for estimating the travel–time errors due to the common–ray approximations of the coupling ray theory for S waves are derived in a form suitable for dynamic ray tracing in ray–centred coordinates attached to the common reference rays. The accuracy of both the isotropic–common–ray approximation and the anisotropic–common–ray approximation can thus be checked routinely.

In this paper, we numerically test the derived equations in three smooth 1–D velocity models of differing degree of anisotropy. The models are described in Sections 3.1 and 3.2.

In Section 3.4, we compare the second–order perturbation expansions of travel time from the isotropic common rays to the anisotropic common rays and vice versa. This comparison not only demonstrates the accuracy of the perturbation expansions, but also allows us to estimate the order of magnitude of the third–order, fourth–order and fifth–order terms in the perturbation expansions.

In Section 3.5, the first–order and second–order perturbation expansions of travel time from the isotropic common rays and from the anisotropic common rays to anisotropic–ray–theory rays are compared with the correct anisotropic–ray–theory travel times in order to illustrate the accuracy of the perturbation expansions and reliability of the error estimates. We observe that the travel–time errors due to the anisotropic–common–ray approximation of the coupling ray theory are considerably smaller than the errors due to the isotropic–common–ray approximation.

Since the second–order perturbation expansion of travel time from the anisotropic common rays to anisotropic–ray–theory rays is sufficiently accurate in our three velocity models, we may use this expansion for simulating the accurate coupling–ray–theory synthetic seismograms. These seismograms are used in Section 3.5 for comparison with the synthetic seismograms obtained by the isotropic–common–ray and anisotropic–common–ray approximations of the coupling ray theory to demonstrate the errors of the two common–ray approximations.

2. Common–ray approximations

For the derivation of the coupling ray theory refer to Coates & Chapman (1990) and to Červený (2001). For the description of the numerical algorithm of the coupling ray theory refer to Bulant & Klimeš (2002). The numerical algorithm of the coupling ray theory by Bulant & Klimeš (2002) is independent of the reference ray. The algorithm is thus applicable to isotropic common rays, to anisotropic common rays, and also to anisotropic–ray–theory reference rays.

The algorithms of anisotropic common S–wave ray tracing in smooth anisotropic models without interfaces and of the corresponding dynamic ray tracing in ray–centred coordinates are described by Klimeš (2006a; 2006b). We thus have all equations necessary for the anisotropic–common–ray approximation of the coupling ray theory in smooth anisotropic models without interfaces.

Klimeš & Bulant (2004) derived the equations for estimating the travel–time errors due to the isotropic–common–ray approximation (and roughly also due to the anisotropic–common–ray approximation) by numerical quadrature along the isotropic common rays (ICR). Klimeš & Bulant (2006) derived the equations for estimating the travel–time errors due to the anisotropic–common–ray approximation (and roughly also due to the isotropic–common–ray approximation) by numerical quadrature along the anisotropic common rays (ACR).

2.1. Selection of the reference ray

The isotropic ray theory is always the limiting case of the coupling ray theory for decreasing anisotropy at a fixed frequency. On the other hand, the high–frequency limit of the coupling ray theory at a fixed anisotropy depends on the choice of the reference ray, and even on the choice of the *system* of reference rays, because the amplitudes are determined by the paraxial reference rays. The reference ray should be "close to the ray of the coupled S wave under study". Unfortunately, this is but a rough statement, because the ray of the coupled S wave has not been defined yet.

From the point of view of the high–frequency asymptotic validity, the frequency–independent reference ray is best represented by the *anisotropic–ray–theory reference ray*, provided we choose the initial condition for the polarization vector in the coupling equation given by the eigenvector of the Christoffel matrix corresponding to the reference ray. The anisotropic–ray–theory travel time corresponding to the selected polarization is then exact, and only the difference between the two anisotropic–ray–theory S–wave travel times is approximate. The coupling ray theory may then also be used at high frequencies because the approximate travel–time difference influences only the coupling due to low–frequency scattering. The coupling ray theory then correctly converges to the anisotropic ray theory for high frequencies. For other choices of reference rays, the high–frequency limit of the coupling ray theory at a fixed anisotropy is incorrect, although the differences may be small at the finite frequencies under consideration. Note that the anisotropic–ray–theory reference ray can be traced only if the eigenvectors of the Christoffel matrix vary continuously along the whole ray (Vavryčuk, 2001).

In the *anisotropic–common–ray approximation*, the common reference ray is traced using the averaged Hamiltonian of both anisotropic–ray–theory S waves (Bakker, 2002; Klimeš, 2006b). This is probably the best common–ray approximation. In the numerical examples presented in this paper, the errors due to the anisotropic–common–ray approximation of the coupling ray theory are considerably smaller than the errors due to the routinely used isotropic–common–ray approximation.

In the less accurate *isotropic–common–ray approximation*, the reference ray is traced in the reference isotropic model. Moreover, the reference isotropic model may be selected in different ways, yielding quasi–isotropic approximations of differing accuracies.

2.2. Travel times in the coupling ray theory, anisotropic–ray–theory Hamiltonians

Let $a_{ijkl} = a_{ijkl}(x_m)$ be the density–normalized elastic moduli describing a *smooth anisotropic model*, in which the *anisotropic common rays* are traced using the averaged Hamiltonian of both anisotropic–ray–theory S waves. We also introduce S–wave velocity $v_0 = v_0(x_m)$ in the smooth reference isotropic model in which the isotropic common rays are traced.

Assume a phase–space *reference ray*, parametrized by reference travel time τ, with reference slowness vectors $p_i(\tau)$ known at all its points $x_j(\tau)$. In this Section 2.2, the reference ray is general, including the anisotropic common ray and isotropic common ray as special cases. Using the reference slowness vectors, we can calculate the reference Christoffel matrix

$$\Gamma_{jk}(\tau) = p_i(\tau)\, a_{ijkl}(x_m(\tau))\, p_l(\tau) \tag{1}$$

and its eigenvectors $g_{i\alpha}(\tau)$, $\alpha = 1, 2, 3$ along the reference ray. Whereas the Einstein summation over the pairs of identical Roman indices (both subscripts and superscripts) $i, j, k, ... = 1, 2, 3$ or $I, J, K, ... = 1, 2$ is used throughout this paper, no implicit summation applies to Greek subscripts $\alpha, \beta, ...$ indexing the eigenvectors of the Christoffel matrix or the perturbation parameters. Assume that eigenvectors $g_{i1}(\tau)$ and $g_{i2}(\tau)$ correspond to the S waves. For application of the coupling ray theory, the eigenvectors should vary continuously along the reference ray (Bulant & Klimeš, 2002). This condition is not required in regions where the two S–wave eigenvalues of the Christoffel matrix are approximately equal.

Let us denote by $\tau_\alpha(\tau)$ the anisotropic–ray–theory travel time corresponding to the selected eigenvector $g_{i\alpha}(\tau)$ of the Christoffel matrix. It may be approximated by a quadrature along the unperturbed reference ray,

$$\frac{\mathrm{d}\tau_\alpha}{\mathrm{d}\tau} = [\Gamma_{jk} g_{j\alpha} g_{k\alpha}]^{-\frac{1}{2}} \quad . \tag{2}$$

Travel–time approximation (2), suggested for the coupling ray theory by Bulant & Klimeš (2002), would become exact for a reference ray following the path of the respective anisotropic–ray–theory ray. Travel–time approximation (2) can be derived as the first–order part of perturbation expansion (9), corresponding to Hamiltonian

$$H_\alpha(x_m,p_n) = -[G_\alpha(x_m,p_n)]^{-\frac{1}{2}} \quad , \qquad \alpha=1,2 \quad , \tag{3}$$

where $G_\alpha(x_m,p_n)$ is the eigenvalue of Christoffel matrix (1), corresponding to eigenvector $g_{i\alpha}$. Hamiltonian (3) is a homogeneous function of the minus first degree with respect to the slowness vector.

2.3. Reference Hamiltonian, isotropic–common–ray Hamiltonian, anisotropic–common–ray Hamiltonian

We denote by $H_0(x_m,p_n)$ the *reference Hamiltonian* used to trace the reference ray. Since the equations for the perturbation derivatives of travel time were derived by Klimeš (2002) on the condition that the Hamiltonian has an equal value for all orthonomic systems of rays under consideration, reference Hamiltonian $H_0(x_m,p_n)$ must be a homogeneous function of the same degree with respect to the slowness vector as anisotropic–ray–theory Hamiltonians (3). We thus select the reference Hamiltonian so that it is also a homogeneous function of the minus first degree with respect to the slowness vector.

For the isotropic common ray, the reference Hamiltonian equals Hamiltonian

$$H^{\mathrm{ICR}}(x_m,p_n) = -[v_0(x_m)]^{-1}(p_i p_i)^{-\frac{1}{2}} \tag{4}$$

for the reference isotropic medium.

For the anisotropic common ray, the reference Hamiltonian is equal to the averaged Hamiltonian

$$H^{\mathrm{ACR}}(x_m,p_n) = -\left\{\tfrac{1}{2}\{[G_1(x_m,p_n)]^{\frac{N}{2}} + [G_2(x_m,p_n)]^{\frac{N}{2}}\}\right\}^{-\frac{1}{N}} \tag{5}$$

of both S waves (Klimeš, 2006b, eqs. 28 and 36), where constant N specifies the way of averaging. Note that Bakker (2002) considered $N=2$, Klimeš & Bulant (2004) considered $N=-1$, whereas Klimeš (2006b) and Klimeš & Bulant (2006) considered general N. Hamiltonian (5) is a homogeneous function of the minus first degree with respect to the slowness vector. The value of reference Hamiltonian (4) or (5) along the respective reference ray is $H_0(x_m,p_n)=-1$.

2.4. Parametric system of the Hamiltonians and errors due to the common–ray approximations

We consider the parametric set $H=H(x_m,p_n,f_\alpha)$ of Hamiltonians parametrized by f_α, $\alpha=1,2,3$

$$H(x_m,p_n,f_\alpha) = H_0(x_m,p_n) + \sum_{\beta=1}^{3}[H_\beta(x_m,p_n) - H_0(x_m,p_n)]\, f_\beta \quad , \tag{6}$$

where $H_1(x_m,p_n)$ and $H_2(x_m,p_n)$ are the anisotropic–ray–theory Hamiltonians (3). Parameters f_α are called *perturbation parameters* (Baumgärtel, 1985; Murdock, 1999; Klimeš & Bulant, 2004; 2006) or model parameters (Tarantola, 1987; Klimeš, 2002).

Parametric system (6) of the Hamiltonians then generates parametric system $\tau(x_m, f_\alpha)$ of the travel–time fields corresponding to the individual Hamiltonians $H(x_m, p_n, f_\alpha)$.

We obtain the reference Hamiltonian for $(f_1, f_2, f_3) = (0, 0, 0)$, the Hamiltonian corresponding to the first anisotropic–ray–theory S wave for $(f_1, f_2, f_3) = (1, 0, 0)$, the Hamiltonian corresponding to the second anisotropic–ray–theory S wave for $(f_1, f_2, f_3) = (0, 1, 0)$.

For the isotropic common ray (ICR), we choose

$$H_0(x_m, p_n) = H^{\mathrm{ICR}}(x_m, p_n) \quad , \qquad H_3(x_m, p_n) = H^{\mathrm{ACR}}(x_m, p_n) \quad . \tag{7}$$

In this case, we obtain the Hamiltonian corresponding to the anisotropic common ray for $(f_1, f_2, f_3) = (0, 0, 1)$.

For the anisotropic common ray (ACR), we choose

$$H_0(x_m, p_n) = H^{\mathrm{ACR}}(x_m, p_n) \quad , \qquad H_3(x_m, p_n) = H^{\mathrm{ICR}}(x_m, p_n) \quad . \tag{8}$$

In this case, we obtain the Hamiltonian corresponding to the isotropic–ray–theory ray traced in the reference isotropic model for $(f_1, f_2, f_3) = (0, 0, 1)$.

The Taylor expansion with respect to perturbation parameters f_α is called here the *perturbation expansion*. The second–order perturbation expansion of travel time is

$$\tau(x_m, f_\gamma) \approx \tau(x_m) + \sum_{\alpha=1}^{3} \tau_{,\alpha}(x_m) f_\alpha + \tfrac{1}{2} \sum_{\alpha=1}^{3} \sum_{\beta=1}^{3} \tau_{,\alpha\beta}(x_m) f_\alpha f_\beta \quad , \tag{9}$$

where the Greek subscripts following a comma denote partial derivatives with respect to perturbation parameters f_α, hereinafter called *perturbation derivatives*. Note that Klimeš (2002) refers to the perturbation derivatives briefly as the "perturbations". Here, travel time $\tau(x_m)$ and its perturbation derivatives $\tau_{,\alpha}(x_m)$, $\tau_{,\alpha\beta}(x_m)$, with arguments f_α omitted, correspond to the reference system of rays $(f_1, f_2, f_3) = (0, 0, 0)$.

Travel–time approximation (2) corresponds to the first–order part of perturbation expansion (9). The error of travel–time approximation (2) may thus be approximated by the quadratic term in perturbation expansion (9) with Hamiltonians (6).

The errors of travel–time approximation (2) from reference ray $(f_1, f_2, f_3) = (0, 0, 0)$ to anisotropic–ray–theory ray $(f_1, f_2, f_3) = (1, 0, 0)$ and to anisotropic–ray–theory ray $(f_1, f_2, f_3) = (0, 1, 0)$ are approximately

$$\delta\tau_\alpha^{\mathrm{CR}} = \frac{\tau_{,\alpha\alpha}}{2} \quad , \qquad \alpha = 1, 2 \quad . \tag{10}$$

If the reference ray is the isotropic common ray (ICR), errors (10) represent errors $\delta\tau_\alpha^{\mathrm{ICR}}$ of the isotropic–common–ray approximation. If the reference ray is the anisotropic common ray (ACR), errors (10) represent errors $\delta\tau_\alpha^{\mathrm{ACR}}$ of the anisotropic–common–ray approximation.

We may also use the second–order perturbation derivatives of travel time, calculated along the isotropic common ray, to estimate the accuracy of the anisotropic–common–ray approximation, and vice versa. The estimates of errors of travel–time approximation (2) from the common ray $(f_1, f_2, f_3) = (0, 0, 1)$ other than the reference ray to anisotropic–ray–theory ray $(f_1, f_2, f_3) = (1, 0, 0)$ and to anisotropic–ray–theory ray $(f_1, f_2, f_3) = (0, 1, 0)$ are

$$\delta\tau_\alpha^{\mathrm{OCR}} = \tfrac{1}{2}\tau_{,\alpha\alpha} - \tau_{,\alpha 3} + \tfrac{1}{2}\tau_{,33} \quad , \qquad \alpha = 1, 2 \quad . \tag{11}$$

If the reference ray is the isotropic common ray (ICR), errors (11) represent rough estimates of errors $\delta\tau_\alpha^{\rm ACR}$ of the anisotropic–common–ray approximation. If the reference ray is the anisotropic common ray (ACR), errors (11) represent rough estimates of errors $\delta\tau_\alpha^{\rm ICR}$ of the isotropic–common–ray approximation.

To calculate the first–order and second–order perturbation derivatives $\tau_{,\alpha}$ and $\tau_{,\alpha\beta}$ of travel time, we need the first–order and second–order phase–space and perturbation derivatives of Hamiltonian (6) at the reference ray. These derivatives are simply composed of the first–order phase–space derivatives of Hamiltonians (3), (4) and (5), and of the second–order phase–space derivatives of the reference Hamiltonian (Klimeš & Bulant, 2004; 2006).

3. Numerical example

Anisotropic common S–wave ray tracing in smooth anisotropic models without interfaces and corresponding dynamic ray tracing in ray–centred coordinates according to Klimeš (2006a; 2006b) have been coded, added to the Fortran 77 package CRT, debugged and partly numerically tested (Bucha & Bulant, 2004). The numerical algorithm of the coupling ray theory by Bulant & Klimeš (2002), coded in the Fortran 77 package CRT, can be applied not only to isotropic common rays but also to anisotropic common rays (Bucha & Bulant, 2004). The calculation of the second–order travel–time perturbation expansion from the anisotropic common ray to the anisotropic–ray–theory rays in smooth velocity models without interfaces have then also been coded and added to the Fortran 77 package CRT (Bucha & Bulant, 2005).

Two–point anisotropic common rays are traced in four 1–D models QI, QI2, QI4 and QI8 of differing degree of anisotropy. The first–order and second–order anisotropic–common–ray approximations of the anisotropic–ray–theory travel times are compared with the correct anisotropic–ray–theory travel times in order to demonstrate the errors of the anisotropic–common–ray approximation and the applicability of the perturbation expansion. The coupling–ray–theory synthetic seismograms calculated using the isotropic–common–ray approximation and anisotropic–common–ray approximation in models QI, QI2 and QI4 are then compared with the synthetic seismograms calculated by the second–order approximation from the anisotropic common rays.

The data for models QI, QI2, QI4 and QI8 have been released on the compact disk of Bucha & Bulant (2002). Numerical examples in this paper have been calculated using the software and data by Bucha & Bulant (2005). Note that both anisotropic common rays and anisotropic–ray–theory rays in the anisotropic 1–D models are three–dimensional because of anisotropy.

3.1. Model QI

A vertically heterogeneous 1–D anisotropic model QI was provided by Pšenčík & Dellinger (2001, model WA rotated by 45°) who performed the coupling–ray–theory calculations using the programs of package ANRAY (Pšenčík, 1998b) and compared the results with the reflectivity method. The same model has been used by Bulant & Klimeš (2002) and Klimeš & Bulant (2004) to demonstrate the coupling ray theory and its quasi–isotropic approximations. For comparison with the isotropic and anisotropic ray theory seismograms in model QI and for a more detailed discussion and description of this model refer to Pšenčík & Dellinger (2001).

The density–normalized elastic moduli a_{ijkl} in $\mathrm{km}^2\mathrm{s}^{-2}$ at the surface (zero depth) are

$$
\begin{array}{c}
\\ 11 \\ 22 \\ 33 \\ 23 \\ 13 \\ 12
\end{array}
\begin{array}{c}
\begin{array}{cccccc} \;\;\;\;11\;\;\;\; & \;\;\;\;22\;\;\;\; & \;\;\;\;33\;\;\;\; & \;\;\;\;23\;\;\;\; & \;\;\;\;13\;\;\;\; & \;\;\;\;12\;\;\;\; \end{array} \\
\begin{pmatrix}
14.48500 & 4.52500 & 4.75000 & 0.00000 & 0.00000 & -0.58000 \\
 & 14.48500 & 4.75000 & 0.00000 & 0.00000 & -0.58000 \\
 & & 15.71000 & 0.00000 & 0.00000 & -0.29000 \\
 & & & 5.15500 & -0.17500 & 0.00000 \\
 & & & & 5.15500 & 0.00000 \\
 & & & & & 5.04500
\end{pmatrix}
\end{array} \quad , \tag{12}
$$

and at the depth of 1 km they are

$$
\begin{array}{c}
\\ 11 \\ 22 \\ 33 \\ 23 \\ 13 \\ 12
\end{array}
\begin{array}{c}
\begin{array}{cccccc} \;\;\;\;11\;\;\;\; & \;\;\;\;22\;\;\;\; & \;\;\;\;33\;\;\;\; & \;\;\;\;23\;\;\;\; & \;\;\;\;13\;\;\;\; & \;\;\;\;12\;\;\;\; \end{array} \\
\begin{pmatrix}
22.08963 & 6.90063 & 7.24375 & 0.00000 & 0.00000 & -0.88450 \\
 & 22.08963 & 7.24375 & 0.00000 & 0.00000 & -0.88450 \\
 & & 23.95775 & 0.00000 & 0.00000 & -0.44225 \\
 & & & 7.86138 & -0.26688 & 0.00000 \\
 & & & & 7.86138 & 0.00000 \\
 & & & & & 7.69363
\end{pmatrix}
\end{array} \quad . \tag{13}
$$

Here the rows correspond to the first couple of indices of a_{ijkl}, the columns correspond to the second couple of indices. For the vertical sections through the S–wave phase–velocity surfaces at the depths of 0 km and 0.6 km refer to Pšenčík & Dellinger (2001, fig. 7). The reference isotropic model is given by

$$
v_{\mathrm{P}}^2 = 15.00\,\mathrm{km}^2\mathrm{s}^{-2} \quad , \qquad v_{\mathrm{S}}^2 = 5.10\,\mathrm{km}^2\mathrm{s}^{-2} \tag{14}
$$

at the surface, and

$$
v_{\mathrm{P}}^2 = 23.00\,\mathrm{km}^2\mathrm{s}^{-2} \quad , \qquad v_{\mathrm{S}}^2 = 7.79\,\mathrm{km}^2\mathrm{s}^{-2} \tag{15}
$$

at the depth of 1 km. All the above values are interpolated linearly with depth. The density is constant.

The synthetic seismograms, corresponding to vertical force $\mathbf{F} = (0,0,100)^{\mathrm{T}}$ at position $(50,50,0)^{\mathrm{T}}$, are calculated at 29 receivers $(51,50,0.010)^{\mathrm{T}}$, $(51,50,0.030)^{\mathrm{T}}$, $(51,50,0.050)^{\mathrm{T}}$, ..., $(51,50,0.570)^{\mathrm{T}}$ located in a vertical well (distances in km), see Figure 1. The source time function is the Gabor signal $\cos(2\pi ft)\exp[-(2\pi ft/4)^2]$ with reference frequency $f = 50\,\mathrm{Hz}$, band–pass filtered by a cosine filter given by frequencies 0 Hz, 5 Hz, 60 Hz and 100 Hz.

For comparison with the isotropic–ray–theory and anisotropic–ray–theory seismograms in model QI and for a more detailed discussion and description of this model refer to Pšenčík & Dellinger (2001).

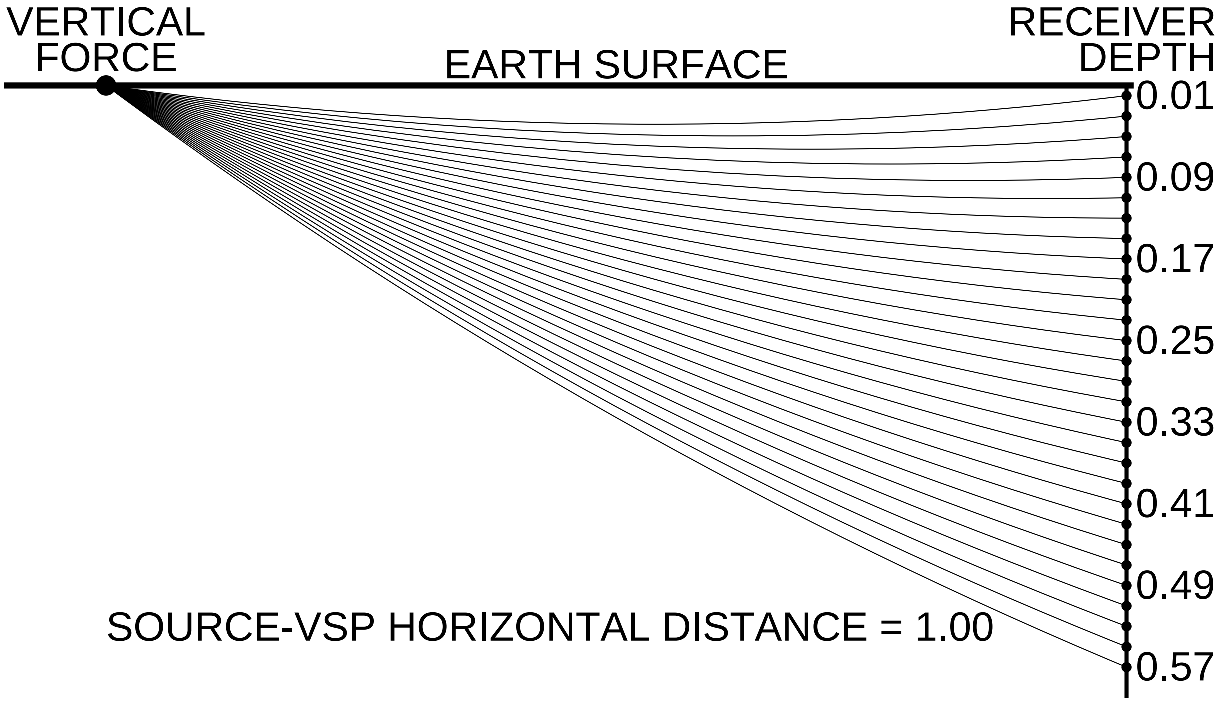

Figure 1. The source and receiver configuration in models QI, QI2, QI4 and QI8.

3.2. Models QI2, QI4 and QI8

To emphasize the effects of perturbations of travel time, new models with an increased degree of anisotropy have been derived from the QI model.

The differences of the elastic moduli of model QI2 from the elastic moduli of the reference isotropic model (14), (15) are exactly twice larger than the differences of model QI. The density–normalized elastic moduli a_{ijkl} of model QI2 in $\mathrm{km}^2\mathrm{s}^{-2}$ at the surface (zero depth) are

$$
\begin{array}{c}
 \\ 11 \\ 22 \\ 33 \\ 23 \\ 13 \\ 12
\end{array}
\begin{array}{c}
\begin{array}{cccccc} 11 & 22 & 33 & 23 & 13 & 12 \end{array} \\
\begin{pmatrix}
13.97000 & 4.25000 & 4.70000 & 0.00000 & 0.00000 & -1.16000 \\
 & 13.97000 & 4.70000 & 0.00000 & 0.00000 & -1.16000 \\
 & & 16.42000 & 0.00000 & 0.00000 & -0.58000 \\
 & & & 5.21000 & -0.35000 & 0.00000 \\
 & & & & 5.21000 & 0.00000 \\
 & & & & & 4.99000
\end{pmatrix}
\end{array} , \qquad (16)
$$

and at the depth of 1 km they are

$$
\begin{array}{c}
 \\ 11 \\ 22 \\ 33 \\ 23 \\ 13 \\ 12
\end{array}
\begin{array}{c}
\begin{array}{cccccc} 11 & 22 & 33 & 23 & 13 & 12 \end{array} \\
\begin{pmatrix}
21.17926 & 6.38126 & 7.06750 & 0.00000 & 0.00000 & -1.76900 \\
 & 21.17926 & 7.06750 & 0.00000 & 0.00000 & -1.76900 \\
 & & 24.91550 & 0.00000 & 0.00000 & -0.88450 \\
 & & & 7.93276 & -0.53376 & 0.00000 \\
 & & & & 7.93276 & 0.00000 \\
 & & & & & 7.59726
\end{pmatrix}
\end{array} . \qquad (17)
$$

Analogously, the differences of the elastic moduli of model QI4 from the elastic moduli of the reference isotropic model (14), (15) are exactly 4 times larger than the differences of model QI. The density–normalized elastic moduli a_{ijkl} of model QI4 in

$\mathrm{km^2 s^{-2}}$ at the surface (zero depth) are

$$\begin{array}{c} \\ 11 \\ 22 \\ 33 \\ 23 \\ 13 \\ 12 \end{array}\begin{array}{c} \begin{array}{cccccc} \;\;11\;\; & \;\;22\;\; & \;\;33\;\; & \;\;23\;\; & \;\;13\;\; & \;\;12\;\; \end{array} \\ \begin{pmatrix} 12.94000 & 3.70000 & 4.60000 & 0.00000 & 0.00000 & -2.32000 \\ & 12.94000 & 4.60000 & 0.00000 & 0.00000 & -2.32000 \\ & & 17.84000 & 0.00000 & 0.00000 & -1.16000 \\ & & & 5.32000 & -0.70000 & 0.00000 \\ & & & & 5.32000 & 0.00000 \\ & & & & & 4.88000 \end{pmatrix} \end{array} , \tag{18}$$

and at the depth of 1 km they are

$$\begin{array}{c} \\ 11 \\ 22 \\ 33 \\ 23 \\ 13 \\ 12 \end{array}\begin{array}{c} \begin{array}{cccccc} \;\;11\;\; & \;\;22\;\; & \;\;33\;\; & \;\;23\;\; & \;\;13\;\; & \;\;12\;\; \end{array} \\ \begin{pmatrix} 19.35852 & 5.34252 & 6.71500 & 0.00000 & 0.00000 & -3.53800 \\ & 19.35852 & 6.71500 & 0.00000 & 0.00000 & -3.53800 \\ & & 26.83100 & 0.00000 & 0.00000 & -1.76900 \\ & & & 8.07552 & -1.06752 & 0.00000 \\ & & & & 8.07552 & 0.00000 \\ & & & & & 7.40452 \end{pmatrix} \end{array} . \tag{19}$$

Note that we have also traced the anisotropic common rays in model QI8, in which the differences of the elastic moduli from the elastic moduli of the reference isotropic model (14), (15) are exactly 8 times larger than the differences of model QI.

3.3. Effects of common–ray approximations

Two–point anisotropic common rays have been traced from the source to the receivers using the CRT program (Červený, Klimeš & Pšenčík, 1988), and the first–order and second–order perturbation derivatives of travel time have been calculated along these rays. Equation (10) has then been used to estimate the travel–time errors $\delta\tau_1^{\mathrm{ACR}}$ and $\delta\tau_2^{\mathrm{ACR}}$ of the anisotropic–common–ray approximation. Equation (10) has also been used to estimate the second–order term $\delta\tau_3^{\mathrm{ACR}}$ in the perturbation expansion of travel time from the anisotropic common ray to the isotropic common ray. If required, the travel–time errors $\delta\tau_1^{\mathrm{ICR}}$ and $\delta\tau_2^{\mathrm{ICR}}$ of the isotropic–common–ray approximation can also be roughly estimated using equation (11). The projection of the travel–time errors on the relative error of the coupled S wavefield depends on the polarization. For the projection of the travel–time errors on the relative errors of the time–harmonic Green tensor refer to Klimeš & Bulant (2004, sec. 3.4.).

3.4. Perturbation expansion from isotropic common rays to anisotropic common rays and vice versa

To check the convergency and accuracy of the second–order perturbation expansion (9), we first compare the perturbation expansion from the isotropic common ray to the anisotropic common ray with the perturbation expansion from the anisotropic common ray to the isotropic common ray. The results of the comparison for the anisotropic common rays traced using the reference Hamiltonian (5) averaged with degree $N = -1$ are displayed in Tables 1, 2 and 3. Only the results at the 1st, 8th, 15th, 22nd and 29th receivers (see Figure 1) are shown, because the variation of the quantities along the vertical profile in models QI, QI2 and QI4 is very moderate. Note that the neglected third–order and higher odd–order perturbation corrections map onto the

Table 1. Perturbation expansion of travel time from the isotropic common ray to the anisotropic common ray and from the anisotropic common ray to the isotropic common ray in model QI.

Rec. dep.	ICR time	ICR linear term	ICR quadratic term	ICR remaining term	ACR time	ACR linear term	ACR quadratic term	ACR remaining term
0.01	0.440993	0.000796	0.000060	0.000001	0.441850	-0.000917	0.000062	-0.000001
0.15	0.438077	0.000743	0.000058	0.000002	0.438880	-0.000862	0.000061	-0.000002
0.29	0.443550	0.000575	0.000056	0.000002	0.444183	-0.000692	0.000059	-0.000001
0.43	0.456339	0.000328	0.000054	0.000000	0.456721	-0.000440	0.000057	0.000000
0.57	0.475205	0.000037	0.000050	0.000003	0.475296	-0.000143	0.000054	-0.000003

Table 2. Perturbation expansion of travel time from the isotropic common ray to the anisotropic common ray and from the anisotropic common ray to the isotropic common ray in model QI2.

Rec. dep.	ICR time	ICR linear term	ICR quadratic term	ICR remaining term	ACR time	ACR linear term	ACR quadratic term	ACR remaining term
0.01	0.440993	0.003429	0.000250	-0.000005	0.444667	-0.003921	0.000240	0.000006
0.15	0.438077	0.003310	0.000248	-0.000002	0.441634	-0.003802	0.000241	0.000004
0.29	0.443550	0.002982	0.000252	0.000005	0.446788	-0.003485	0.000248	-0.000001
0.43	0.456339	0.002500	0.000255	-0.000002	0.459093	-0.003018	0.000258	0.000005
0.57	0.475205	0.001926	0.000257	0.000004	0.477392	-0.002453	0.000266	0.000000

Table 3. Perturbation expansion of travel time from the isotropic common ray to the anisotropic common ray and from the anisotropic common ray to the isotropic common ray in model QI4.

Rec. dep.	ICR time	ICR linear term	ICR quadratic term	ICR remaining term	ACR time	ACR linear term	ACR quadratic term	ACR remaining term
0.01	0.440993	0.016119	0.001701	-0.000241	0.458572	-0.018809	0.001016	0.000214
0.15	0.438077	0.015763	0.001735	-0.000231	0.455345	-0.018545	0.001059	0.000218
0.29	0.443550	0.014973	0.001861	-0.000212	0.460171	-0.018043	0.001196	0.000226
0.43	0.456339	0.013833	0.002024	-0.000175	0.472022	-0.017316	0.001407	0.000226
0.57	0.475205	0.012454	0.002173	-0.000117	0.489715	-0.016385	0.001661	0.000214

Tables 1, 2 and 3. *Rec. dep.* stands for the receiver depth along the vertical profile, see Figure 1. The *ICR time* is the travel time along the isotropic common ray traced in the reference isotropic model. The *ICR linear term* is the linear term in perturbation expansion (9) of travel time from the isotropic common ray to the anisotropic common ray. The *ICR quadratic term* stands for the quadratic term in the perturbation expansion from the isotropic common ray to the anisotropic common ray. The *ICR remaining term* stands for the difference between the travel time along the anisotropic common ray and the second–order perturbation expansion from the isotropic common ray (*ICR time* +*ICR linear term* +*ICR quadratic term* +*ICR remaining term* =*ACR time*). The *ACR time* is the travel time along the anisotropic common ray. The *ACR linear term* is the linear term in perturbation expansion (9) of travel time from the anisotropic common ray to the isotropic common ray. The *ACR quadratic term* stands for the quadratic term in the perturbation expansion from the anisotropic common ray to the isotropic common ray. The *ACR remaining term* stands for the difference between the travel time along the isotropic common ray and the second–order perturbation expansion from the anisotropic common ray (*ACR time* +*ACR linear term* +*ACR quadratic term* +*ACR remaining term* =*ICR time*). The ICR remaining term and ACR remaining term represent both the inaccuracy of numerical ray tracing and the third–order and higher–order terms in the perturbation expansion. The third–order and fourth–order terms are significant in model QI4 only.

differences between the ACR and ICR remaining terms in Tables 1, 2 and 3, whereas the neglected fourth–order and higher even–order perturbation corrections map onto the average of the ACR and ICR remaining terms. We may approximate the third–order terms in the perturbation expansions using the differences between the ACR and ICR quadratic terms. If we subtract these approximate third–order terms from the ACR and ICR remaining terms, the differences between the ACR and ICR remaining terms decrease to the level of numerical errors. This means that the fifth–order terms in the perturbation expansion are not detectable in Tables 1, 2 and 3. The average of the ACR and ICR remaining terms in Table 3 is mostly caused by the fourth–order terms in the perturbation expansions.

3.5. Perturbation expansions from isotropic and anisotropic common rays to anisotropic–ray–theory rays

For convenient comparison of the isotropic–common–ray and anisotropic–common–ray approximations, the perturbation expansion of travel time from the isotropic common ray to the anisotropic–ray–theory rays is compared with the correct anisotropic–ray–theory travel times in Tables 4, 5 and 6. The exact anisotropic–ray–theory travel times have been calculated by the ANRAY program (Gajewski & Pšenčík, 1990; Pšenčík, 1998b). Note that the quadratic terms in analogous Tables 1, 2 and 3 by Klimeš & Bulant (2004) slightly differ from the corresponding terms in our Tables 4, 5 and 6 because of a bug which has been fixed by Bucha & Bulant (2005).

The perturbation expansion of travel time from the anisotropic common ray to the anisotropic–ray–theory rays is then compared with the correct anisotropic–ray–theory travel times. The results of the comparison for the anisotropic common rays traced using the reference Hamiltonian (5) averaged with degree $N = -1$ are displayed in Tables 7, 8 and 9. The results of the comparison for the anisotropic common rays traced using the reference Hamiltonian (5) averaged with degree $N = 2$ (Bakker, 2002) are displayed in Tables 10, 11 and 12.

The quadratic terms in the perturbation expansions displayed in Tables 4–12 represent the estimates of the errors due to the isotropic–common–ray (ICR) and anisotropic–common–ray (ACR) approximations of the anisotropic–ray–theory travel times.

In the strongly anisotropic model QI8, the errors of the anisotropic–common–ray approximation of the anisotropic–ray–theory travel times are so large that the anisotropic–common–ray approximation of the coupling ray theory is considerably inaccurate and should not be applied. In model QI8, both the anisotropic common rays and the errors of the anisotropic–common–ray approximation considerably depend on degree N of homogeneous Hamiltonians averaged in equation (5).

3.6. Comparison of the isotropic–common–ray and anisotropic–common–ray approximations of the coupling ray theory

The anisotropic–common–ray approximation of the coupling–ray–theory synthetic seismograms in models QI, QI2 and QI4 is compared with the more accurate coupling–ray–theory synthetic seismograms simulated by the second–order perturbation expansion of travel time from the anisotropic common rays in Figures 3, 5 and 7, respectively. Refer to Tables 7 to 9 (columns ACR remaining terms) for the travel–time errors of

Table 4. Perturbation expansion of travel time from the isotropic common ray to the anisotropic–ray–theory rays in model QI.

Rec. dep.	ICR time	ICR linear terms		ICR quadratic terms		ICR remaining terms		ACR q.terms
0.01	0.440993	-0.002392	0.003983	0.000251	0.000000	-0.000001	-0.000001	0.000065
0.15	0.438077	-0.002518	0.004003	0.000248	0.000000	0.000009	-0.000001	0.000066
0.29	0.443550	-0.002967	0.004117	0.000249	0.000001	0.000009	0.000001	0.000069
0.43	0.456339	-0.003661	0.004317	0.000251	0.000001	0.000010	-0.000002	0.000072
0.57	0.475205	-0.004520	0.004595	0.000250	0.000003	0.000022	0.000006	0.000076

Table 5. Perturbation expansion of travel time from the isotropic common ray to the anisotropic–ray–theory rays in model QI2.

Rec. dep.	ICR time	ICR linear terms		ICR quadratic terms		ICR remaining terms		ACR q.terms
0.01	0.440993	-0.004746	0.011604	0.000971	0.000000	0.000008	-0.000007	0.000235
0.15	0.438077	-0.004992	0.011612	0.000959	0.000000	0.000023	0.000001	0.000231
0.29	0.443550	-0.005874	0.011837	0.000960	0.000001	0.000036	0.000005	0.000229
0.43	0.456339	-0.007234	0.012235	0.000958	0.000001	0.000054	0.000001	0.000224
0.57	0.475205	-0.008912	0.012764	0.000945	0.000002	0.000069	0.000000	0.000217

Table 6. Perturbation expansion of travel time from the isotropic common ray to the anisotropic–ray–theory rays in model QI4.

Rec. dep.	ICR time	ICR linear terms		ICR quadratic terms		ICR remaining terms		ACR q.terms
0.01	0.440993	-0.009341	0.041580	0.003643	0.000488	0.000119	-0.000137	0.000365
0.15	0.438077	-0.009816	0.041343	0.003589	0.000548	0.000164	-0.000142	0.000333
0.29	0.443550	-0.011517	0.041462	0.003554	0.000711	0.000249	-0.000167	0.000272
0.43	0.456339	-0.014129	0.041796	0.003496	0.000953	0.000352	-0.000203	0.000200
0.57	0.475205	-0.017335	0.042243	0.003394	0.001224	0.000455	-0.000224	0.000136

Tables 4, 5 and 6. *Rec. dep.* stands for the receiver depth along the vertical profile, see Figure 1. The *ICR time* is the reference travel time along the isotropic common ray traced in the reference isotropic model. The *ICR linear terms* are the linear terms in perturbation expansion (9) of travel time from the isotropic common ray to the anisotropic–ray–theory rays and represent the travel–time corrections considered in the isotropic–common–ray approximation of the coupling ray theory. The *ICR quadratic terms* stand for the quadratic terms in the perturbation expansion, and represent the estimates of the errors due to the isotropic–common–ray approximation of the anisotropic–ray–theory travel times. The *ICR remaining terms* stand for the differences between the correct anisotropic–ray–theory travel times calculated by the ANRAY program version 4.40 and the second–order perturbation expansion from the isotropic common ray, in order to illustrate the reliability of the error estimates. The ICR remaining terms represent both the inaccuracy of numerical ray tracing and the third–order and higher–order terms in the perturbation expansion. The *ACR quadratic terms* stand for the estimate of the equal quadratic terms in the perturbation expansion from the anisotropic common ray, traced using the reference Hamiltonian (5) averaged with degree $N = -1$, to the anisotropic–ray–theory rays. The quadratic terms represent the estimate of the error due to the anisotropic–common–ray approximation of the anisotropic–ray–theory travel times. Note that these estimated ACR quadratic terms are approximately calculated along the isotropic common rays, i.e. without tracing the anisotropic common rays. The estimated ACR quadratic terms may be compared with the actual errors of the anisotropic–common–ray approximation (Tables 7, 8 and 9, columns ACR quadratic terms and ACR remaining terms). The differences between the ACR quadratic terms calculated along the isotropic common rays and the ACR quadratic terms calculated along the anisotropic common rays are mostly due to the neglected third–order terms in the perturbation expansion.

Table 7. Perturbation expansion of travel time from the anisotropic common ray to the anisotropic–ray–theory rays in model QI. The homogeneous Hamiltonians of the minus first degree are averaged.

Rec. dep.	ACR time	ACR linear terms		ACR quadratic terms		ACR remaining terms	
0.01	0.441850	-0.003061	0.003061	0.000066	0.000066	-0.000003	-0.000002
0.15	0.438880	-0.003135	0.003135	0.000066	0.000066	0.000004	-0.000001
0.29	0.444183	-0.003415	0.003415	0.000070	0.000070	0.000003	0.000000
0.43	0.456721	-0.003861	0.003861	0.000075	0.000075	0.000004	-0.000001
0.57	0.475296	-0.004430	0.004430	0.000079	0.000079	0.000010	0.000004

Table 8. Perturbation expansion of travel time from the anisotropic common ray to the anisotropic–ray–theory rays in model QI2. The homogeneous Hamiltonians of the minus first degree are averaged.

Rec. dep.	ACR time	ACR linear terms		ACR quadratic terms		ACR remaining terms	
0.01	0.444667	-0.007689	0.007689	0.000245	0.000245	0.000002	-0.000011
0.15	0.441634	-0.007818	0.007818	0.000244	0.000244	0.000008	-0.000004
0.29	0.446788	-0.008366	0.008366	0.000245	0.000245	0.000006	-0.000007
0.43	0.459093	-0.009240	0.009240	0.000246	0.000246	0.000017	-0.000003
0.57	0.477392	-0.010344	0.010344	0.000244	0.000244	0.000014	-0.000009

Table 9. Perturbation expansion of travel time from the anisotropic common ray to the anisotropic–ray–theory rays in model QI4. The homogeneous Hamiltonians of the minus first degree are averaged.

Rec. dep.	ACR time	ACR linear terms		ACR quadratic terms		ACR remaining terms	
0.01	0.458572	-0.023856	0.023856	0.000588	0.000588	0.000110	-0.000090
0.15	0.455345	-0.024001	0.024001	0.000560	0.000560	0.000111	-0.000079
0.29	0.460171	-0.024947	0.024947	0.000505	0.000505	0.000108	-0.000067
0.43	0.472022	-0.026489	0.026489	0.000433	0.000433	0.000092	-0.000059
0.57	0.489715	-0.028424	0.028424	0.000355	0.000355	0.000072	-0.000047

Tables 7, 8, 9, 10, 11 and 12. *Rec. dep.* stands for the receiver depth along the vertical profile, see Figure 1. The *ACR time* is the reference travel time τ along the anisotropic common ray. The *ACR linear terms* are the linear terms $\tau_{,1}$ and $\tau_{,2}$ in perturbation expansion (9) of travel time from the anisotropic common ray to the anisotropic–ray–theory rays, and represent the travel–time corrections considered in the anisotropic–common–ray approximation of the coupling ray theory. The *ACR quadratic terms* stand for quadratic terms (10) in the perturbation expansion, and represent the estimates of the errors due to the anisotropic–common–ray approximation of the anisotropic–ray–theory travel times. The *ACR remaining terms* stand for the differences between the correct anisotropic–ray–theory travel times calculated by the ANRAY program version 4.40 and the second–order perturbation expansion from the anisotropic common ray, in order to illustrate the reliability of the error estimates. The ACR remaining terms represent both the inaccuracy of numerical ray tracing and the third–order and higher–order terms in the perturbation expansion.

Table 10. Perturbation expansion of travel time from the anisotropic common ray to the anisotropic–ray–theory rays in model QI. The homogeneous Hamiltonians of the second degree are averaged.

Rec. dep.	ACR time	ACR linear terms		ACR quadratic terms		ACR remaining terms	
0.01	0.441814	-0.003027	0.003090	0.000063	0.000068	0.000001	0.000002
0.15	0.438846	-0.003098	0.003165	0.000064	0.000069	0.000005	-0.000001
0.29	0.444146	-0.003372	0.003451	0.000067	0.000073	0.000000	-0.000003
0.43	0.456674	-0.003808	0.003906	0.000071	0.000078	0.000002	-0.000002
0.57	0.475234	-0.004363	0.004487	0.000075	0.000083	0.000010	0.000004

Table 11. Perturbation expansion of travel time from the anisotropic common ray to the anisotropic–ray–theory rays in model QI2. The homogeneous Hamiltonians of the second degree are averaged.

Rec. dep.	ACR time	ACR linear terms		ACR quadratic terms		ACR remaining terms	
0.01	0.444469	-0.007465	0.007861	0.000220	0.000271	0.000002	-0.000011
0.15	0.441425	-0.007586	0.007998	0.000218	0.000270	0.000010	-0.000002
0.29	0.446554	-0.008105	0.008572	0.000219	0.000273	0.000005	-0.000007
0.43	0.458815	-0.008932	0.009487	0.000218	0.000277	0.000016	-0.000004
0.57	0.477058	-0.009977	0.010647	0.000214	0.000277	0.000012	-0.000011

Table 12. Perturbation expansion of travel time from the anisotropic common ray to the anisotropic–ray–theory rays in model QI4. The homogeneous Hamiltonians of the second degree are averaged.

Rec. dep.	ACR time	ACR linear terms		ACR quadratic terms		ACR remaining terms	
0.01	0.456727	-0.021838	0.025497	0.000450	0.000836	0.000075	-0.000135
0.15	0.453460	-0.021953	0.025686	0.000427	0.000800	0.000080	-0.000120
0.29	0.458162	-0.022780	0.026778	0.000383	0.000730	0.000071	-0.000114
0.43	0.469812	-0.024138	0.028541	0.000325	0.000634	0.000059	-0.000103
0.57	0.487256	-0.025848	0.030746	0.000264	0.000527	0.000047	-0.000081

this simulation. The isotropic–common–ray approximation of coupling–ray–theory synthetic seismograms is compared with these more accurate coupling–ray–theory synthetic seismograms in Figures 2, 4 and 6.

Although the second–order perturbation expansion of travel time has been applied successfully to the estimation of anisotropic–ray–theory travel times in these simple 1–D models QI, QI2 and QI4 with constant gradients of the density–normalized elastic moduli, we cannot recommend the approximation of travel time using the second–order perturbation expansion in more complex models, because the second–order perturbations may be infinitely large in the vicinity of caustics. The second–order perturbation expansion of travel time should be used especially for estimating and controlling the accuracy of the common–ray approximations outside caustics.

4. Conclusions

In using any kind of the common–ray approximation of the coupling ray theory, the travel–time errors due to the applied common–ray approximation should be evaluated. The travel–time errors due to the common–ray approximations can be estimated by numerical quadrature along the isotropic common rays using the equations proposed by Klimeš & Bulant (2004), or by numerical quadrature along the anisotropic common rays using the equations proposed by Klimeš & Bulant (2006).

The numerical results demonstrate that the anisotropic–common–ray approximation by Bakker (2002) and Klimeš (2006b) is considerably more accurate than the isotropic–common–ray approximation. Whereas the first–order isotropic–common–ray approximation is considerably inaccurate in model QI4, see Figure 6 and Table 6, the first–order anisotropic–common–ray approximation performs well in all three models QI, QI2 and QI4, see Figures 3, 5, 7 and Tables 7, 8, 9.

For additional information, including electronic reprints, computer codes and data, refer to the consortium research project "Seismic Waves in Complex 3–D Structures" ("http://sw3d.mff.cuni.cz").

Acknowledgements

The research has been supported by the Grant Agency of the Czech Republic under contract 205/04/1104, by the Grant Agency of the Charles University under contract 375/2004/B-GEO/MFF, by the European Commission under contract MTKI-CT-2004-517242 (project IMAGES), and by the members of the consortium "Seismic Waves in Complex 3–D Structures" (see "http://sw3d.mff.cuni.cz").

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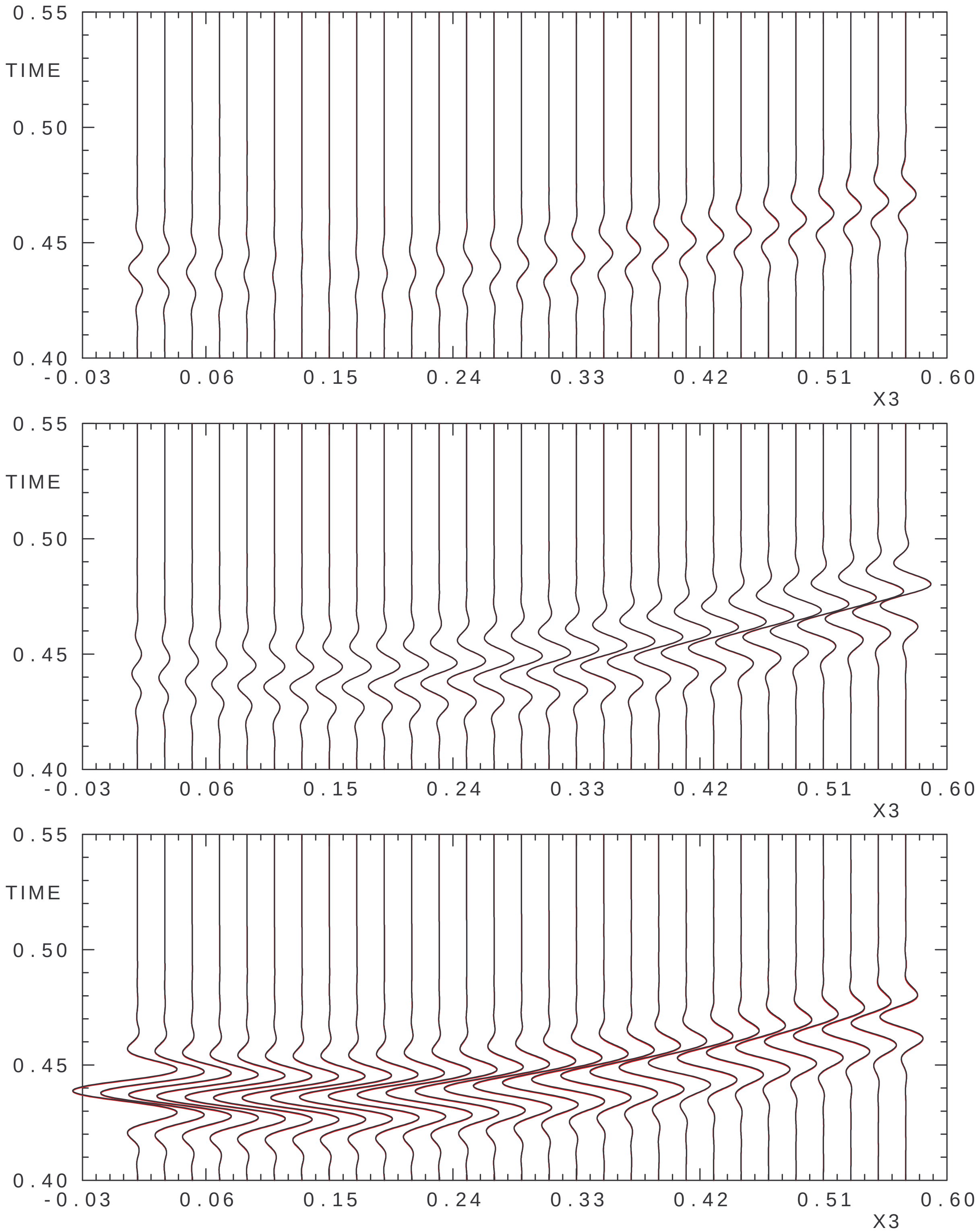

Figure 2. Inaccuracy of the isotropic–common–ray approximation of the coupling ray theory in model QI. From top to bottom: the first (radial) component, the second (transverse) component, the third (vertical) component. **Black:** Coupling–ray–theory synthetic seismograms without common ray approximation, simulated by the second–order perturbation expansion of travel time from anisotropic common rays. For the errors of this simulation refer to the *ACR remaining terms* in Table 7. **Red:** Isotropic common ray approximation. The red seismograms are mostly obscured by the black seismograms.

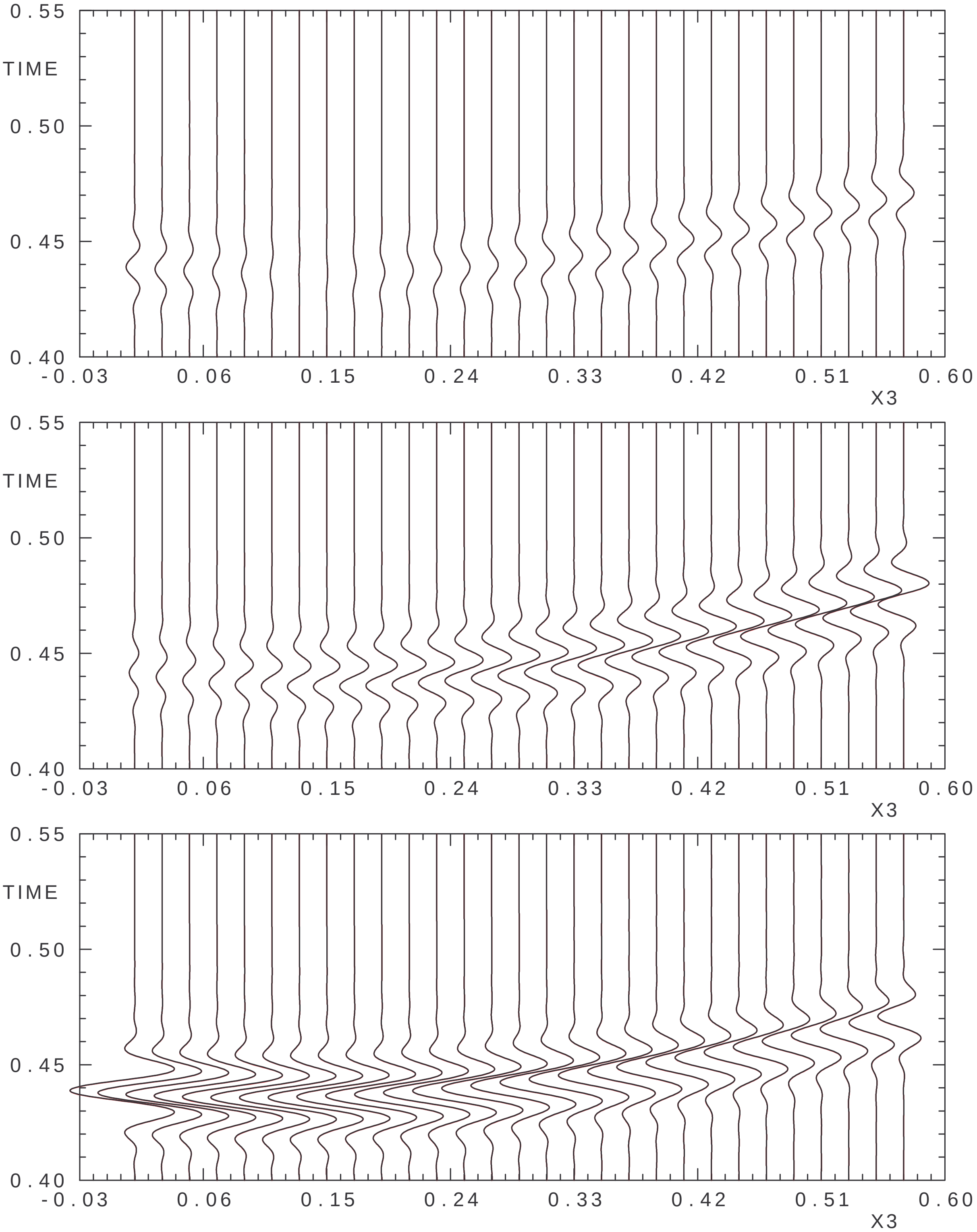

Figure 3. Inaccuracy of the anisotropic–common–ray approximation of the coupling ray theory in model QI. From top to bottom: the first (radial) component, the second (transverse) component, the third (vertical) component. **Black:** Coupling–ray–theory synthetic seismograms without common ray approximation, simulated by the second–order perturbation expansion of travel time from anisotropic common rays. For the errors of this simulation refer to the *ACR remaining terms* in Table 7. **Red:** Anisotropic common ray approximation. The red seismograms are obscured by the black seismograms.

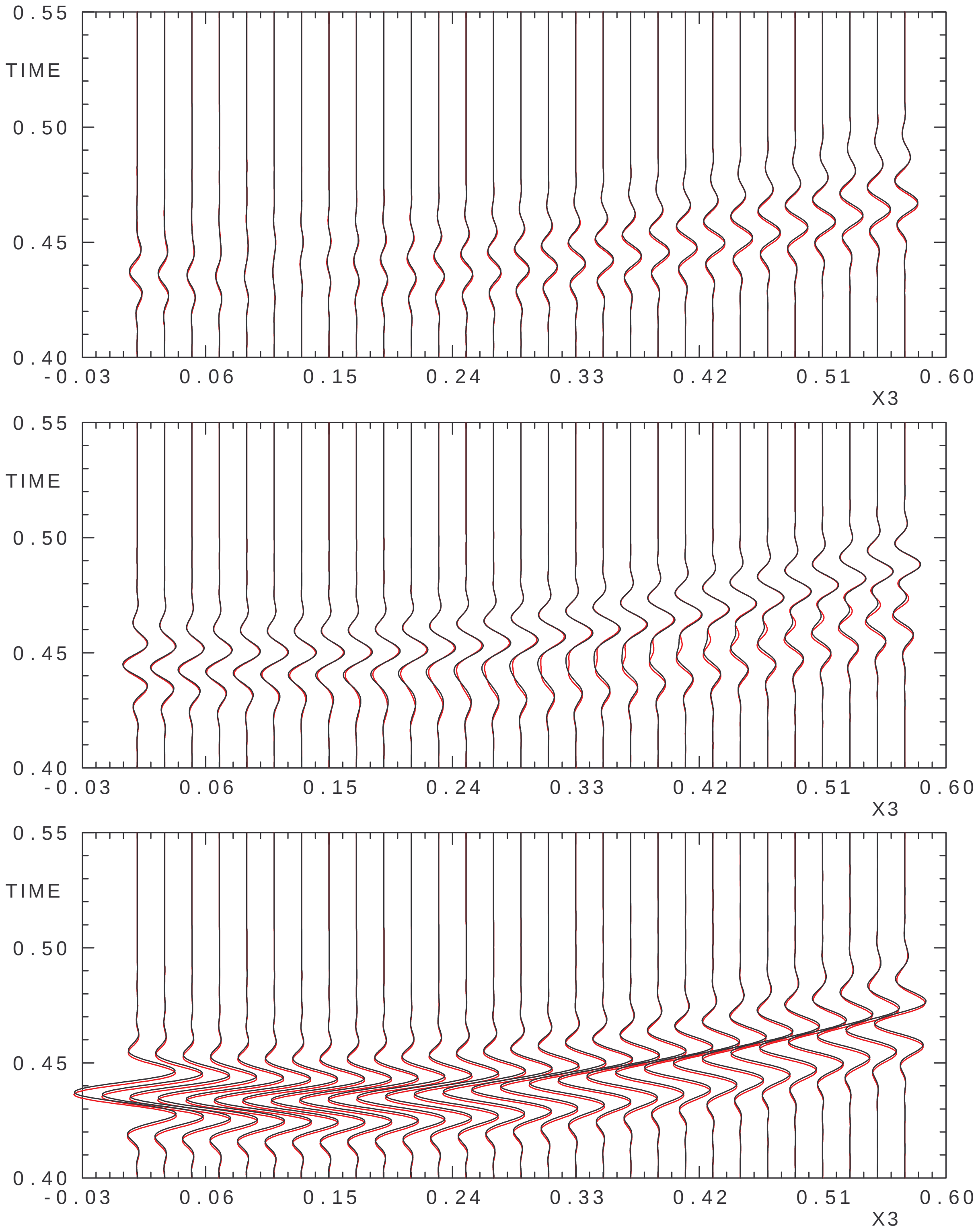

Figure 4. Inaccuracy of the isotropic–common–ray approximation of the coupling ray theory in model QI2. From top to bottom: the first (radial) component, the second (transverse) component, the third (vertical) component. **Black:** Coupling–ray–theory synthetic seismograms without common ray approximation, simulated by the second–order perturbation expansion of travel time from anisotropic common rays. For the errors of this simulation refer to the *ACR remaining terms* in Table 8. **Red:** Isotropic common ray approximation. The differences between the seismograms are small but already clearly visible.

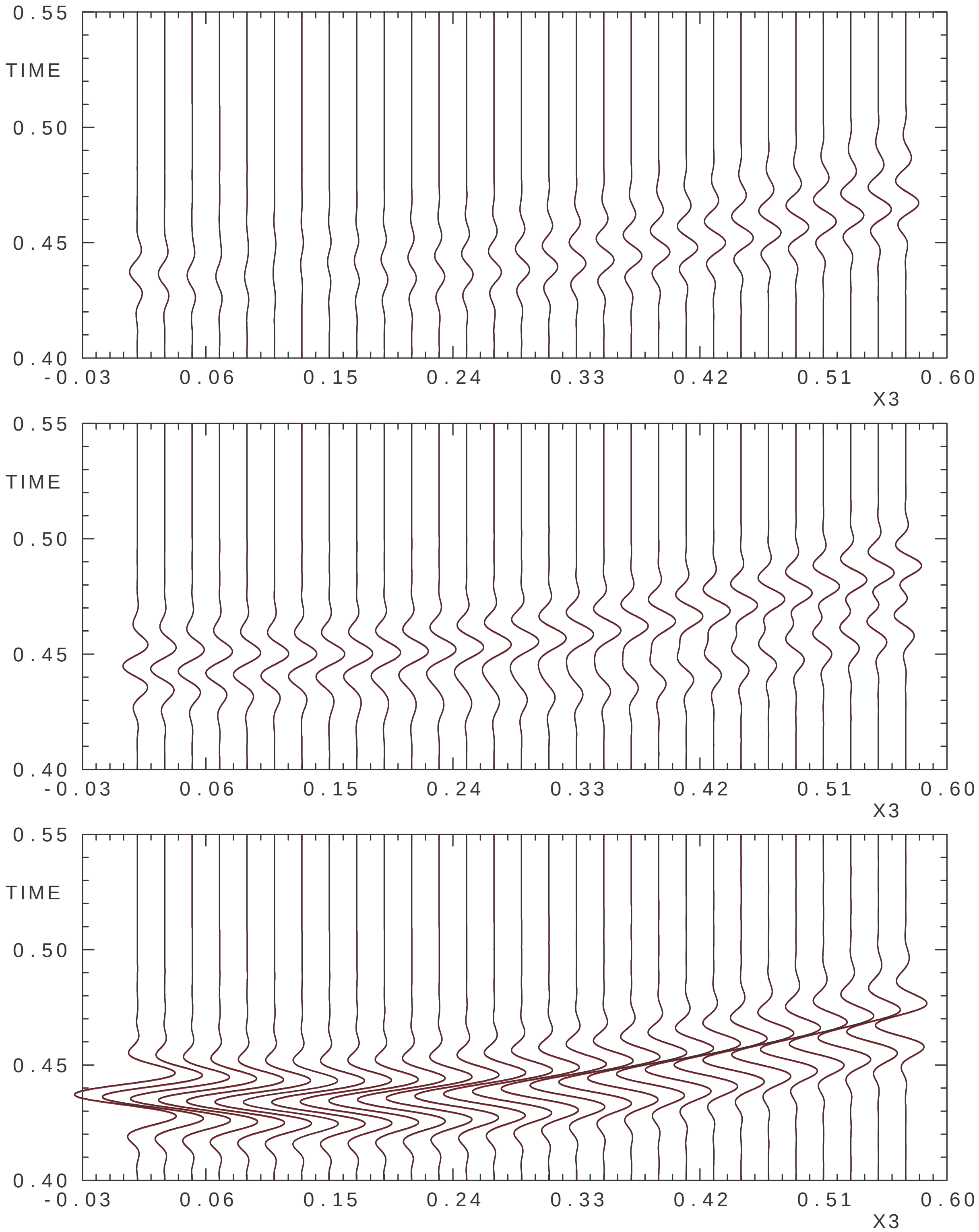

Figure 5. Inaccuracy of the anisotropic–common–ray approximation of the coupling ray theory in model QI2. From top to bottom: the first (radial) component, the second (transverse) component, the third (vertical) component. **Black:** Coupling–ray–theory synthetic seismograms without common ray approximation, simulated by the second–order perturbation expansion of travel time from anisotropic common rays. For the errors of this simulation refer to the *ACR remaining terms* in Table 8. **Red:** Anisotropic common ray approximation. The red seismograms are mostly obscured by the black seismograms.

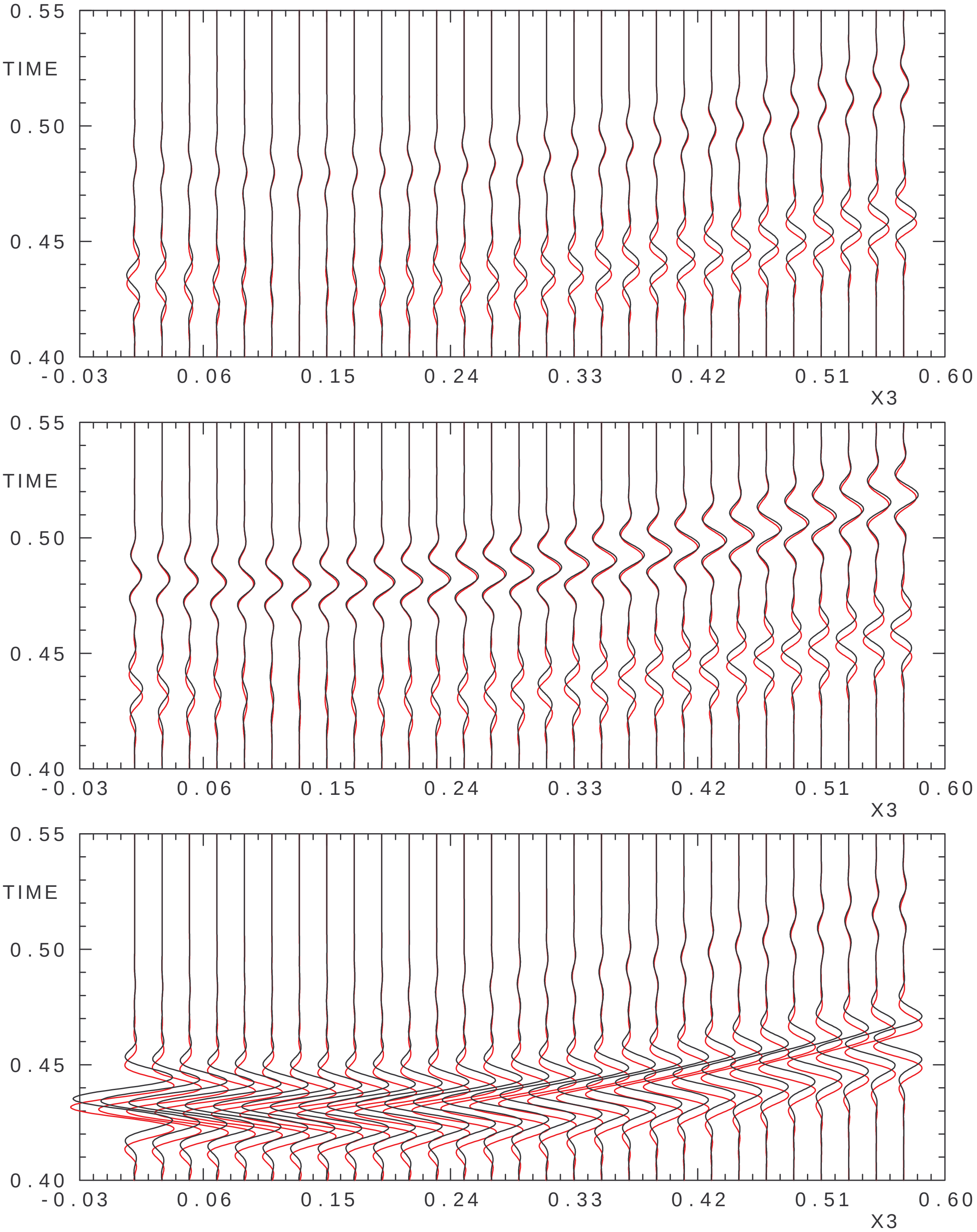

Figure 6. Inaccuracy of the isotropic–common–ray approximation of the coupling ray theory in model QI4. From top to bottom: the first (radial) component, the second (transverse) component, the third (vertical) component. **Black:** Coupling–ray–theory synthetic seismograms without common ray approximation, simulated by the second–order perturbation expansion of travel time from anisotropic common rays. For the errors of this simulation refer to the *ACR remaining terms* in Table 9. **Red:** Isotropic common ray approximation. The differences between the seismograms are considerable in this quite strongly anisotropic model QI4.

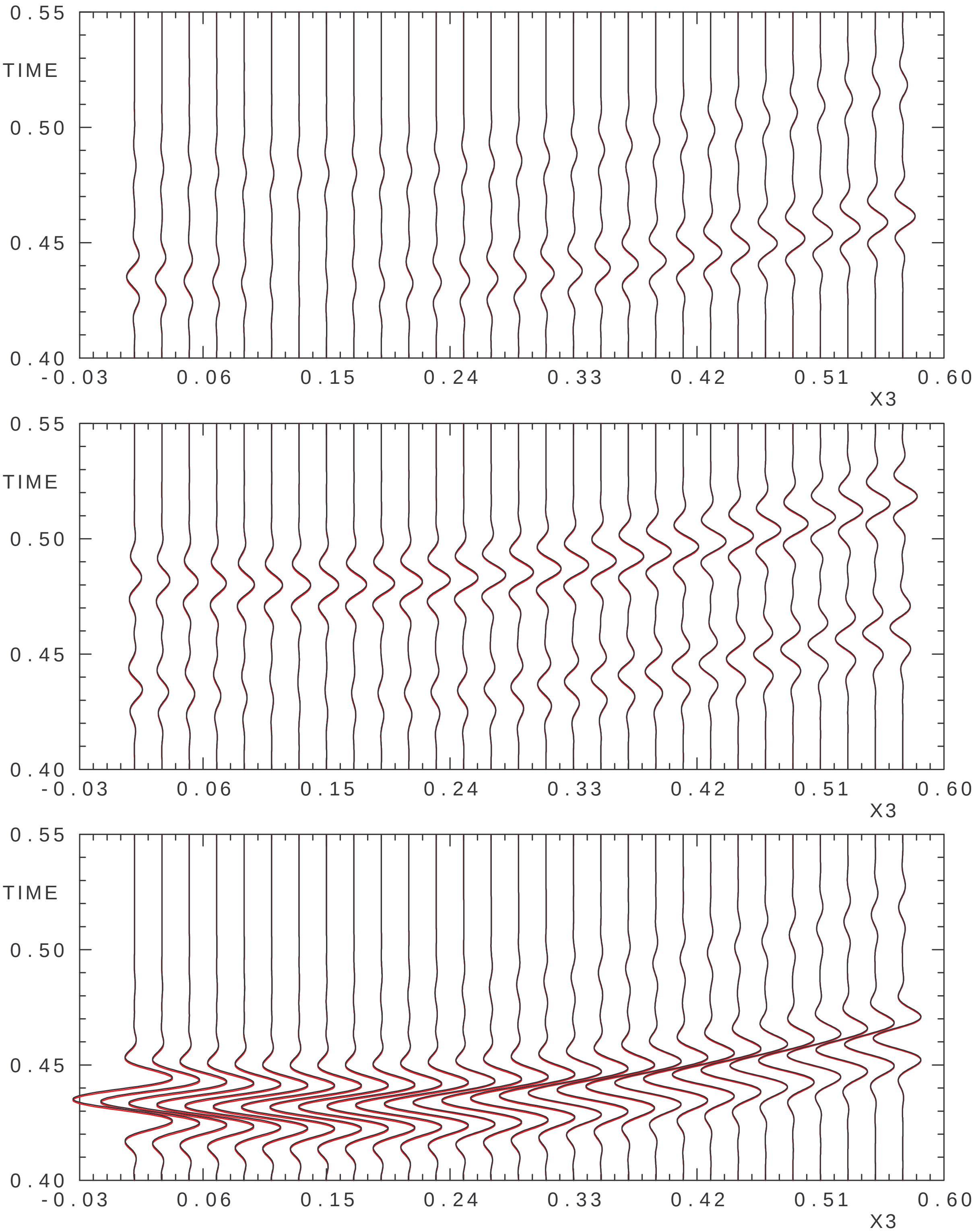

Figure 7. Inaccuracy of the anisotropic–common–ray approximation of the coupling ray theory in model QI4. From top to bottom: the first (radial) component, the second (transverse) component, the third (vertical) component. **Black:** Coupling–ray–theory synthetic seismograms without common ray approximation, simulated by the second–order perturbation expansion of travel time from anisotropic common rays. For the errors of this simulation refer to the *ACR remaining terms* in Table 9. **Red:** Anisotropic common ray approximation The anisotropic common ray approximation in model QI4 is even more accurate than the isotropic common ray approximation in model QI2.

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