Examples of seismic models. Part 2

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Summary

The structure of input data files specifying seismic models by means of the `MODEL` Fortran 77 package is demonstrated on three additional simple examples.

Preface

Five examples of input data for the `MODEL` Fortran 77 package, illustrating the specification of seismic macro models of geologic structures, have been presented in the former paper by Klimeš (1995). Here is the continuation of the former paper, containing three additional examples. Below, the Introduction of the former paper is reprinted for the convenience of a reader. Three sections containing the new examples are numbered further to the sections of the former paper, i.e. starting with Section 7. The figures are numbered further to the figures of the former paper, too.

1 Introduction

This contribution is devoted to the specification of seismic macro models of geologic structures by means of the Fortran 77 modeling package `MODEL`, built on the basis of the paper by Červený, Klimeš & Pšenčík (1988), and is intended as an illustration to the `MODEL` package.

The computer representation of a seismic model consists of both the data and routines. In general, either the routines of the `MODEL` package or user–defined routines may be used. The aim of this contribution is to elucidate the description of the models in terms of input data corresponding to the `MODEL` package.

The structure of input data files specifying seismic models is demonstrated on five examples. The input data are reprinted here, and interlarded with some comments and illustrations.

The presented pictures of the models have been described in terms of the PostScript language and were generated by the Fortran 77 programs in a computer–independent way.

All input data for the `MODEL` package are read by the list directed input (free format). The data may be written in an individually preferred form, data corresponding to a single `READ` operation may be to split over several lines, and blank lines may be inserted anywhere in order to make the data file well-arranged. The data corresponding to different `READ` operations have to be placed on different lines. The comments which are not a part of the input data may only be placed after the last data corresponding to a `READ` operation, on the same line, and cannot continue on the next line.

The input data for the model are printed in a `TYPEWRITER` font. The text printed in *ITALICS* contains author's remarks included within the data file in order to make the data more clear for him. The remarks are not a part of the input data and are not read by the Fortran 77 code. They are placed at the ends of the last lines corresponding to individual `READ` operations.

Seismic Waves in Complex 3-D Structures, Report 4 (Department of Geophysics, Charles University, Prague 1996)

7 Slit model

The "slit" model consists of two complex blocks separated by the interface depicted in Fig. 15. The interface may be imagined as originally smooth interface, slitted in its central region by a curved vertical fault, the nearer part of the interface being depressed and the further part raised along the fault.

```
'SLIT MODEL'
/
 -8  8  -8  8  -8  0                                  (BOUNDARIES OF THE MODEL)
```

The model is situated in the box $< -8, 8 > \times < -8, 8 > \times < -8, 0 >$, with vertical axis pointing upwards.

Upper complex block 1 is divided by the slitting fault into further simple block 1 and nearer simple block 2. Similarly, lower complex block 2 is divided by the slitting fault into further simple block 3 and nearer simple block 4.

The vertical x_3 axis has been selected positive upwards, and both the raised and depressed parts of the slitted interface are to be described by the functions of the form of $f(x_1, x_2, x_3) = w(x_1, x_2) - x_3$, negative above the corresponding surface and positive below it. The vertical slitting fault is to be described by the function of the form of $f(x_1, x_2, x_3) = w(x_1) - x_2$, negative beyond the fault (larger x_2). and positive in front of the fault (smaller x_2). The slitted interface and the fault may be covered by 3 smooth surfaces. The raised surface is indexed by 1, the depressed surface by 2, and the fault surface by 3. Then the simple and complex blocks are specified in terms of the following table:

```
3 SURFACES
4 SIMPLE BLOCKS:
   -1  -3        /
   -2   3        /
    1  -3        /
    2   3        /
2 COMPLEX BLOCKS:
    1   2        /
    3   4        /
```

The two surfaces covering the slitted interface are specified by the functions of the form of $f(x_1, x_2, x_3) = w(x_1, x_2) - x_3$, with $w(x_1, x_2)$ interpolated by natural cubic splines in the grid of 9×9 points:

```
'RAISED PART OF THE SLITTED INTERFACE' 1
    1    2   -3    0 /                            (I.E. W(X1,X2)-X3=0, TENSION=0)
    9    9                                        (NUMBERS OF GRID POINTS)
   -8   -6   -4   -2    0    2    4    6    8     (X1 GRID COORDINATES)
   -8   -6   -4   -2    0    2    4    6    8     (X2 GRID COORDINATES)
  -3.2 -3.6 -3.2 -3.7 -3.8 -4.7 -4.4 -4.8 -4.4
  -3.6 -3.2 -3.6 -3.3 -4.2 -4.3 -4.8 -4.4 -4.8
  -3.2 -3.6 -3.2 -3.7 -3.8 -4.7 -4.4 -4.8 -4.4
  -3.6 -3.2 -3.6 -3.2 -3.8 -4.2 -4.8 -4.4 -4.8
```

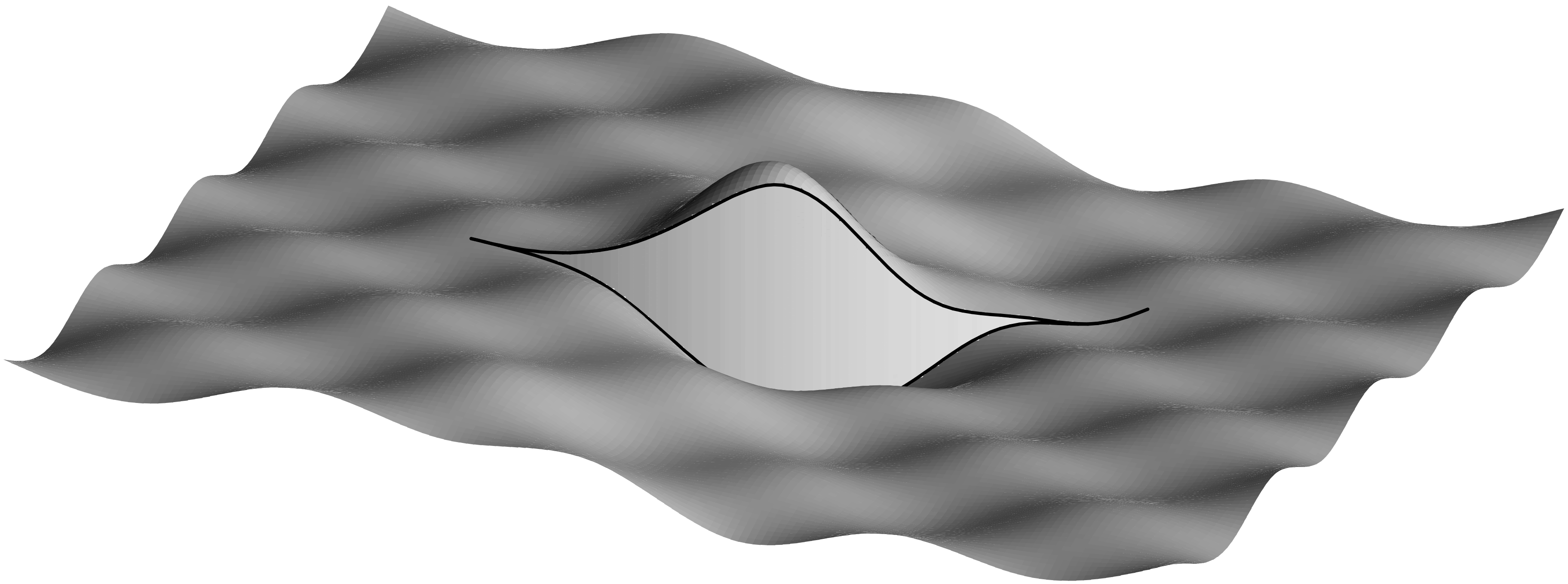

Figure 15. Interface locally slitted by a curved vertical fault. The edge of the slit is emhasized by a solid line. The horizontal dimensions are 16 km * 16 km.

```
  -3.2 -3.6 -3.2 -3.3 -2.2 -4.3 -4.4 -4.8 -4.4
  -3.6 -3.2 -3.6 -3.2 -3.8 -4.2 -4.8 -4.4 -4.8
  -3.2 -3.6 -3.2 -3.7 -3.8 -4.7 -4.4 -4.8 -4.4
  -3.6 -3.2 -3.6 -3.3 -4.2 -4.3 -4.8 -4.4 -4.8
  -3.2 -3.6 -3.2 -3.7 -3.8 -4.7 -4.4 -4.8 -4.4   (GRIDPOINT VALUES OF W)

'DEPRESSED PART OF THE SLITTED INTERFACE' 2
     1    2   -3    0 /                         (I.E. W(X1,X2)-X3=0, TENSION=0)
     9    9                                     (NUMBERS OF GRID POINTS)
    -8   -6   -4   -2    0    2    4    6    8  (X1 GRID COORDINATES)
    -8   -6   -4   -2    0    2    4    6    8  (X2 GRID COORDINATES)
  -3.2 -3.6 -3.2 -3.7 -3.8 -4.7 -4.4 -4.8 -4.4
  -3.6 -3.2 -3.6 -3.3 -4.2 -4.3 -4.8 -4.4 -4.8
  -3.2 -3.6 -3.2 -3.7 -3.8 -4.7 -4.4 -4.8 -4.4
  -3.6 -3.2 -3.6 -3.4 -4.6 -4.4 -4.8 -4.4 -4.8
  -3.2 -3.6 -3.2 -4.1 -5.4 -5.1 -4.4 -4.8 -4.4
  -3.6 -3.2 -3.6 -3.4 -4.6 -4.4 -4.8 -4.4 -4.8
  -3.2 -3.6 -3.2 -3.7 -3.8 -4.7 -4.4 -4.8 -4.4
  -3.6 -3.2 -3.6 -3.3 -4.2 -4.3 -4.8 -4.4 -4.8
  -3.2 -3.6 -3.2 -3.7 -3.8 -4.7 -4.4 -4.8 -4.4   (GRIDPOINT VALUES OF W)
```

The smooth surfaces covering the slitted interface may be arbitrarily curved, which is, to some extent, also demonstrated in this example by choosing the interface in the form of step-wise smoothly double bent "egg box" that has been subsequently slitted. To keep the slit local, the difference of the above two functions should be constrained by proper conditions if solving an inverse problem. In our case, the above values were entered manually, with the constraints taken into account.

The vertical slitting fault is described in terms of the function of the form of $f(x_1, x_2, x_3) = w(x_1) - x_2$, with $w(x_1)$ interpolated by natural cubic splines from 3 points:

```
'SLITTING FAULT' 3
     1   -2    0    0 /                         (I.E. W(X1)-X2=0, TENSION=0)
     3                                          (NUMBERS OF GRID POINTS)
    -8    0    8                                (X1 GRID COORDINATES)
   0.6 -0.4  8.0                                (GRIDPOINT VALUES OF W)

'END OF SURFACES' /
```

Both complex blocks are chosen homogeneous, with specified P-wave velocities and default S-wave velocities:

```
'UPPER COMPLEX BLOCK' 1
'VP ' 1                                         (P WAVE VELOCITY)
    0    0    0    0                            (I.E. VP=CONSTANT)
    4                                           (VELOCITY VALUE)
```

```
'LOWER COMPLEX BLOCK' 2
'VP ' 1                                         (P WAVE VELOCITY)
   0   0   0   0                                (I.E. VP=CONSTANT)
   5                                            (VELOCITY VALUE)

'END OF COMPLEX BLOCKS, END OF THE INPUT DATA FOR THE MODEL' /
```

8 Model L7

Model L7, provided by the Institut Français du Pétrole (IFP), is a model of a real geological structure. Model L7 is composed of layers separated by curved interfaces, see Figure 16. The Earth surface is flat. Present version contains only upper two of several structural interfaces.

The structural interfaces were interpolated using general bicubic splines in IFP, whereas the present version of the `MODEL` package considers natural (bi–) (tri–) cubic splines or analogic splines with tension by Cline (1974a,b, 1981). The general cubic splines have a general curvature at the outermost gridpoints used for the interpolation, whereas the natural splines have a zero normal curvature at the outermost gridpoints. The number of parameters thus equals the number of gridpoints for the natural splines, whereas the number of parameters of the general splines includes also the normal curvatures at the outermost gridpoints, i.e. it corresponds to the number of gridpoints of the grid increased by 2 additional gridpoints in each direction. It is thus very useful to have a simple algorithm for transformation between the general and natural splines. The transformation from natural to general (bi–) (tri–) cubic splines is straightforward. The transformation from general to natural (bi–) (tri–) cubic splines may be done in the way described in Section 8.1.

```
'MODEL L7 - 2 UPPER LAYERS'
/
3.13000 11.10134  1.30100 10.70100  -.00100 3.00000 / (MODEL BOUNDARIES)
```

Model is limited by a rectangular box in Cartesian coordinates. Coordinate x_1 extends from `3.130` to `11.10134` length units, probably kilometers, coordinate x_2 extends from `1.301` to `10.701`. Coordinate x_3 is pointing downwards and extends from `-0.001` to `3.000` length units. The top of the box has been selected 0.001 length units above the Earth surface of elevation $x_3 = 0.000$.

```
3 SURFACES
3 SIMPLE BLOCKS:
-1 2 /
-2 3 /
-3 /
3 COMPLEX BLOCKS:
 1 /
 2 /
 3 /
```

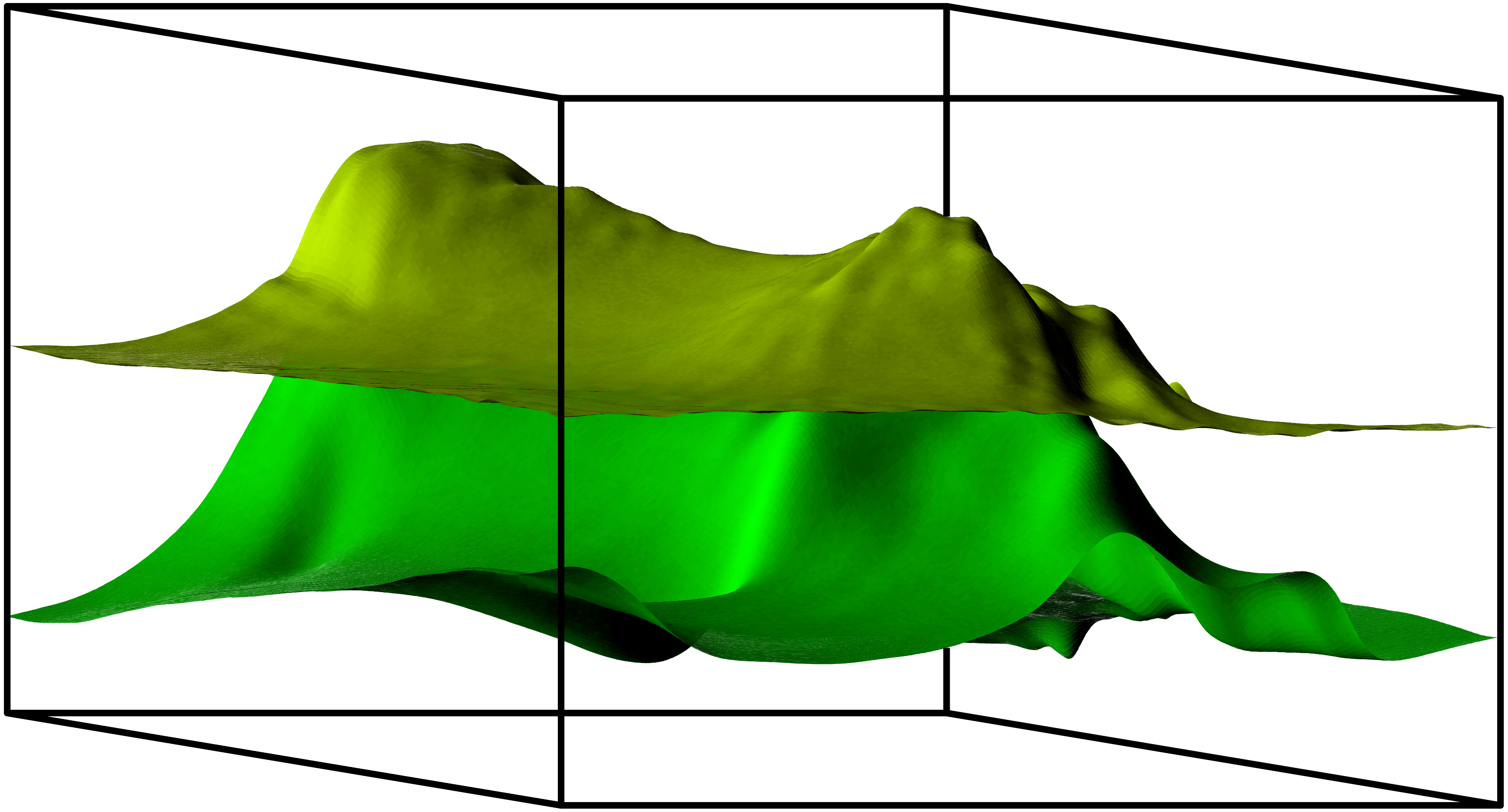

Figure 16. Structural interfaces in model L7.

The layers in the model are limited by 3 surfaces numbered 1, 2, 3 from the top to the bottom. The model is composed of the free space above the Earth surface (surface no. 1), and of 3 layers numbered 1, 2, 3 from the top to the bottom. The simple and complex blocks coincide with the layers. The functions f describing the smooth surfaces will be selected in the form of $f(x_1, x_2, x_3) = w(x_1, x_2) - x_3$, and will thus be positive above the corresponding surfaces [smaller x_3 than $w(x_1, x_2)$] and negative below them [greater x_3 than $w(x_1, x_2)$], because the x_3 axis is selected to point downwards.

```
'EARTH SURFACE' 1
   -3    0    0    0 /  (I.E. W-X3=0, TENSION=0)
    0                   (VALUE OF ELEVATION W)

```

The Earth surface is flat, $x_3 = constant$, at the elevation of 0 length units.

```
'SURFACE BTERT' 2
    1    2   -3    0 /  (I.E. W(X1,X2)-X3=0, TENSION=0)
   50   50              (NUMBERS OF GRIDPOINTS)
```

Surface 2 was interpolated in the form of $x_3 = w(x_1, x_2)$ by the general bicubic splines in the regular rectangular grid of 48×48 gridpoints in IFP. The grid has then been extended to 50×50 gridpoints according to Section 8.1 for the interpolation by the natural bicubic splines used in the MODEL package.

```
2.96040 3.13000 3.29960 3.46921 3.63881 3.80841 3.97801 4.14762 4.31722
4.48682 4.65643 4.82603 4.99563 5.16523 5.33484 5.50444 5.67404 5.84365
6.01325 6.18285 6.35245 6.52206 6.69166 6.86126 7.03087 7.20047 7.37007
7.53967 7.70928 7.87888 8.04848 8.21809 8.38769 8.55729 8.72690 8.89650
9.06610 9.23571 9.40531 9.57491 9.74452 9.91412 10.08372 10.25333
10.42293 10.59253 10.76213 10.93174 11.10134 11.27094
1.10100 1.30100 1.50100 1.70100 1.90100 2.10100 2.30100 2.50100 2.70100
2.90100 3.10100 3.30100 3.50100 3.70100 3.90100 4.10100 4.30100 4.50100
4.70100 4.90100 5.10100 5.30100 5.50100 5.70100 5.90100 6.10100 6.30100
6.50100 6.70100 6.90100 7.10100 7.30100 7.50100 7.70100 7.90100 8.10100
8.30100 8.50100 8.70100 8.90100 9.10100 9.30100 9.50100 9.70100 9.90100
10.1010 10.3010 10.5010 10.7010 10.9010

```

The above data specify 50 x_1 grid coordinates, and 50 x_2 grid coordinates, separated from the elevations (see below) by a blank line. The blank line has no other meaning than visual.

```
1.27813 1.36453 1.39120 1.37347 1.39938 1.40954 1.39391 1.38983 1.36539
1.33655 1.43668 1.39789 1.38532 1.44252 1.42058 1.35852 1.34208 1.36084
1.34453 1.34406 1.36777 1.35020 1.32895 1.30080 1.30729 1.29968 1.28615
1.26650 1.28418 1.29968 1.28073 1.32400 1.31104 1.28344 1.30819 1.34768
1.33923 1.29728 1.29001 1.35720 1.35519 1.30558 1.33044 1.34959 1.33161
1.33723 1.31190 1.28706 1.29228 1.29100
1.28868 1.38674 1.39561 1.40320 1.40916 1.40929 1.40616 1.39616 1.40613
1.40631 1.41366 1.43467 1.42453 1.42991 1.41703 1.40350 1.39452 1.40713
```

```
1.38624 1.37659 1.38452 1.35482 1.34295 1.33426 1.33858 1.32518 1.32957
1.32692 1.32675 1.32587 1.32158 1.32927 1.33052 1.33406 1.32722 1.32952
1.32837 1.32863 1.32820 1.32931 1.32765 1.33176 1.33799 1.35121 1.34121
1.33944 1.33858 1.34649 1.35078 1.34999
⋮
1.32860 1.50404 1.51110 1.48434 1.49204 1.50871 1.54056 1.50365 1.50837
1.50345 1.48003 1.48937 1.48451 1.44405 1.40823 1.38171 1.32946 1.28159
1.22340 1.15071 0.96251 0.80702 0.91986 0.72992 0.46613 0.47498 0.59161
0.57568 0.52199 0.58541 0.61414 0.67498 0.81167 1.03137 1.12229 1.17135
1.21890 1.27110 1.31330 1.34119 1.36814 1.38828 1.40994 1.42637 1.42434
1.43922 1.45363 1.45734 1.41773 1.32183
```

The first 50 elevations above correspond to $x_2 = 1.101$ length units, The second 50 elevations to $x_2 = 1.301$ length units. Three dots $\vdots$ represent subsequent $(50-3)\times 50$ elevations not reprinted here. The last 50 elevations above correspond to $x_2 = 10.901$ length units.

```
'SURFACE' 3
    1   2  -3   0 /  (I.E. W(X1,X2)-X3=0, TENSION=0)
   50  50            (NUMBERS OF GRIDPOINTS)
2.96040 3.13000 3.29960 3.46921 3.63881 3.80841 3.97801 4.14762 4.31722
4.48682 4.65643 4.82603 4.99563 5.16523 5.33484 5.50444 5.67404 5.84365
6.01325 6.18285 6.35245 6.52206 6.69166 6.86126 7.03087 7.20047 7.37007
7.53967 7.70928 7.87888 8.04848 8.21809 8.38769 8.55729 8.72690 8.89650
9.06610 9.23571 9.40531 9.57491 9.74452 9.91412 10.08372 10.25333
10.42293 10.59253 10.76213 10.93174 11.10134 11.27094
1.10100 1.30100 1.50100 1.70100 1.90100 2.10100 2.30100 2.50100 2.70100
2.90100 3.10100 3.30100 3.50100 3.70100 3.90100 4.10100 4.30100 4.50100
4.70100 4.90100 5.10100 5.30100 5.50100 5.70100 5.90100 6.10100 6.30100
6.50100 6.70100 6.90100 7.10100 7.30100 7.50100 7.70100 7.90100 8.10100
8.30100 8.50100 8.70100 8.90100 9.10100 9.30100 9.50100 9.70100 9.90100
10.1010 10.3010 10.5010 10.7010 10.9010
```

The data for surface 3 has also been transformed from the general bicubic splines used in IFP to the natural bicubic splines. Surface 3 is interpolated in the same way and same grid as surface 2.

```
2.24864 2.27367 2.29496 2.31364 2.34118 2.36801 2.38526 2.36723 2.30205
2.13468 2.01124 1.98890 2.01737 2.05856 2.08856 2.09630 2.07686 2.02523
1.95848 1.89071 1.86231 1.89371 1.96817 2.07307 2.16002 2.21398 2.25619
2.29029 2.32128 2.34934 2.36833 2.38038 2.38957 2.39981 2.40717 2.40142
2.40042 2.39864 2.40026 2.39585 2.38286 2.35715 2.30915 2.22477 2.13540
2.05837 1.99962 1.97063 1.97326 1.98821
2.25805 2.28159 2.30213 2.32105 2.34267 2.36390 2.38172 2.37990 2.30717
2.14552 2.03800 2.00825 2.01829 2.04629 2.06734 2.07027 2.04912 1.99872
1.93200 1.86812 1.83672 1.86904 1.95049 2.05419 2.14290 2.20285 2.24562
```

```
2.28013 2.30940 2.33393 2.35206 2.36519 2.37571 2.38710 2.39594 2.39851
2.39687 2.39195 2.38794 2.38396 2.37344 2.34567 2.29700 2.22780 2.14431
2.07136 2.02228 1.99332 1.99242 2.03008
⋮
2.72987 2.70116 2.69711 2.69701 2.68987 2.67797 2.67032 2.67221 2.67881
2.68131 2.68716 2.69105 2.68705 2.67252 2.65792 2.63819 2.63935 2.65130
2.41105 2.25916 1.93439 1.72731 1.47117 1.24960 1.02455 0.81609 0.67968
0.65235 0.65050 0.67623 0.74915 0.87064 0.99570 1.15632 1.48633 1.58448
1.75795 1.90516 2.05503 2.16933 2.26047 2.33436 2.39401 2.44144 2.48226
2.51631 2.53506 2.55190 2.57548 2.61613
'END OF SURFACES' /
```

The first 50 elevations above correspond to $x_2 = 1.101$ length units, The second 50 elevations to $x_2 = 1.301$ length units. Three dots ⋮ represent subsequent $(50-3)\times 50$ elevations not reprinted here. The last 50 elevations above correspond to $x_2 = 10.901$ length units.

```
'COMPLEX BLOCK' 1
'VP ' 1                (P WAVE VELOCITY)        (GRADIENT 0.549)
    3   0   0   0 /    (I.E. VP=W(X3), TENSION=0)
    2                  (NUMBERS OF GRID POINTS)
0       1              (X3 GRID COORDINATES)
1.627   2.176          (VELOCITIES AT GRID POINTS)
```

Only P-wave velocity is specified in the upper layer `1`. The velocity is dependent of only the vertical coordinate axis x_3. The velocity at the Earth surface $x_3 = 0$ is `1.627` and increases towards the depth with the gradient of $0.549\,\mathrm{s}^{-1}$.

```
'COMPLEX BLOCK' 2
'VP ' 1                (P WAVE VELOCITY)        (GRADIENT 1.056)
    3   0   0   0 /    (I.E. VP=W(X3), TENSION=0)
    2                  (NUMBERS OF GRID POINTS)
0       1              (X3 GRID COORDINATES)
2.156   3.212          (VELOCITIES AT GRID POINTS)
```

P-wave velocity in layer `2` does not vary laterally, too. The velocity increases from the value of `1.627` at $x_3 = 0$ with the gradient of $1.056\,\mathrm{s}^{-1}$.

```
'COMPLEX BLOCK' 3
'VP ' 1                (P WAVE VELOCITY)
    0   0   0   0 /    (I.E. VP=CONSTANT, TENSION=0)
4.000                  (VELOCITY)

'END OF COMPLEX BLOCKS, END OF THE INPUT DATA FOR THE MODEL' /
```

Layer `3` is homogeneous.

8.1 Transformation from general to natural splines

First we generate the values

1-D: $f,\ f_1,\ f_{11},$

2-D: $f,\ f_1,\ f_{11},\ f_2,\ f_{12},\ f_{22},\ f_{112},\ f_{122},\ f_{1122},$

3-D: $f,\ f_1,\ f_{11},\ f_2,\ f_{11},\ f_{22},\ f_{112},\ f_{122},\ f_{1122},\ f_3,\ f_{13},\ f_{23},\ f_{33},\ f_{113},\ f_{123},$
$f_{223},\ f_{133},\ f_{233},\ f_{1123},\ f_{1223},\ f_{1133}, f_{1233},\ f_{2233},\ f_{11223},\ f_{11233},\ f_{12233},\ f_{112233}$

of the function and its partial derivatives at the gridpoints of the original 1-D, 2-D or 3-D grid. We extend the original 1-D, 2-D or 3-D grid by 2 additional gridpoints in the direction of each grid axis, situated outside of the original grid, see Figure 17. The distances of the new gridpoints from the original grid may be arbitrary. We denote them h_{1-}, h_{1+}, h_{2-}, h_{2+}, h_{3-}, h_{3+}, and introduce up to 6 vectors

$$\mathbf{h}_{1\pm} = (h_{1\pm}, 0, 0)\ , \quad \mathbf{h}_{2\pm} = (0, h_{2\pm}, 0)\ , \quad \mathbf{h}_{3\pm} = (0, 0, h_{3\pm})\ .$$

If $\mathbf{x}$ is an outermost gridpoint, the values at the neighbouring additional gridpoint $\mathbf{x}+\mathbf{h}_1$, where $\mathbf{h}_1$ stands for $\mathbf{h}_{1-}$ or $\mathbf{h}_{1+}$, may be evaluated using

$$f(\mathbf{x} + \mathbf{h}_1) = f(\mathbf{x}) + f_1(\mathbf{x})h_1 + f_{11}(\mathbf{x})\tfrac{h_1 h_1}{3}$$

and analogously for $\mathbf{x} + \mathbf{h}_2$ or $\mathbf{x} + \mathbf{h}_3$.

If $\mathbf{x}$ is a corner gridpoint in 2-D or an edge gridpoint in 3-D, the values at the neighbouring additional corner or edge gridpoint $\mathbf{x} + \mathbf{h}_1 + \mathbf{h}_2$ may be evaluated using

$$\begin{aligned} f(\mathbf{x} + \mathbf{h}_1 + \mathbf{h}_2) = {} & f(\mathbf{x}) + f_1(\mathbf{x})h_1 + f_{11}(\mathbf{x})\tfrac{h_1 h_1}{3} \\ & + [f_2(\mathbf{x}) + f_{12}(\mathbf{x})h_1 + f_{112}(\mathbf{x})\tfrac{h_1 h_1}{3}]h_2 \\ & + [f_{22}(\mathbf{x}) + f_{122}(\mathbf{x})h_1 + f_{1122}(\mathbf{x})\tfrac{h_1 h_1}{3}]\tfrac{h_2 h_2}{3} \end{aligned}$$

and analogously for $\mathbf{x} + \mathbf{h}_1 + \mathbf{h}_3$ or $\mathbf{x} + \mathbf{h}_2 + \mathbf{h}_3$.

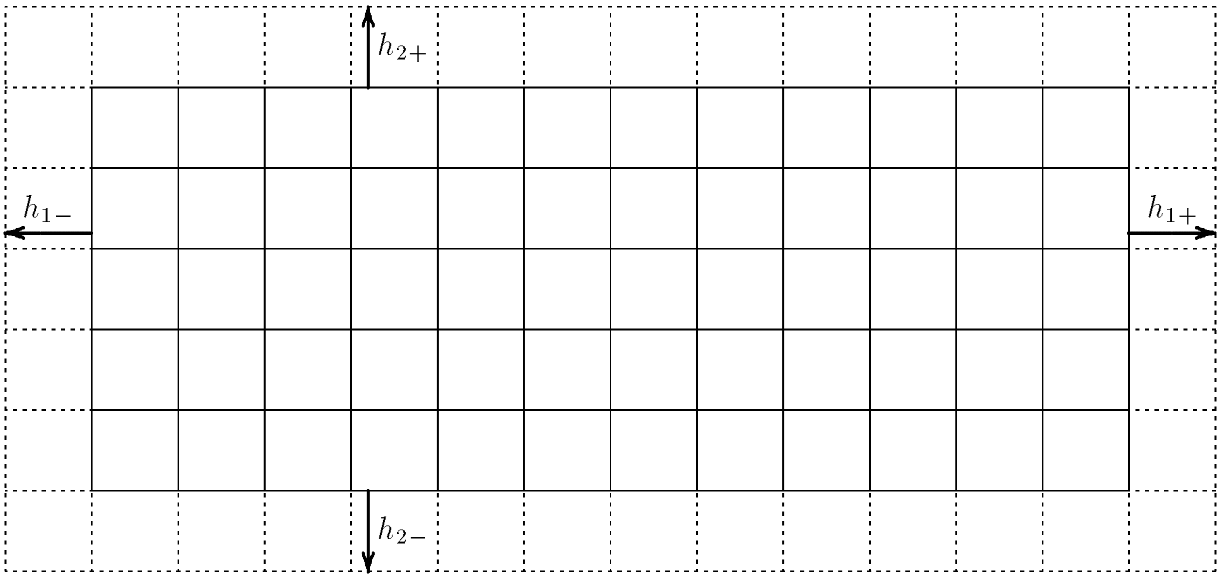

Figure 17. Extending the grid for general splines to the grid for natural splines.

If $\mathbf{x}$ is a tip gridpoint in 3-D, the values at the neighbouring additional tip gridpoint $\mathbf{x}+\mathbf{h}_1+\mathbf{h}_2+\mathbf{h}_3$ may be evaluated using

$$\begin{aligned}
f(\mathbf{x}+\mathbf{h}_1+\mathbf{h}_2+\mathbf{h}_3) = {} & f(\mathbf{x}) + f_1(\mathbf{x})h_1 + f_{11}(\mathbf{x})\tfrac{h_1h_1}{3} \\
& + [f_2(\mathbf{x}) + f_{12}(\mathbf{x})h_1 + f_{112}(\mathbf{x})\tfrac{h_1h_1}{3}]h_2 \\
& + [f_{22}(\mathbf{x}) + f_{122}(\mathbf{x})h_1 + f_{1122}(\mathbf{x})\tfrac{h_1h_1}{3}]\tfrac{h_2h_2}{3} \\
& + \{f_3(\mathbf{x}) + f_{13}(\mathbf{x})h_1 + f_{113}(\mathbf{x})\tfrac{h_1h_1}{3} \\
& + [f_{23}(\mathbf{x}) + f_{123}(\mathbf{x})h_1 + f_{1123}(\mathbf{x})\tfrac{h_1h_1}{3}]h_2 \\
& + [f_{223}(\mathbf{x}) + f_{1223}(\mathbf{x})h_1 + f_{11223}(\mathbf{x})\tfrac{h_1h_1}{3}]\tfrac{h_2h_2}{3}\}h_3 \\
& + \{f_{33}(\mathbf{x}) + f_{133}(\mathbf{x})h_1 + f_{1133}(\mathbf{x})\tfrac{h_1h_1}{3} \\
& + [f_{233}(\mathbf{x}) + f_{1233}(\mathbf{x})h_1 + f_{11233}(\mathbf{x})\tfrac{h_1h_1}{3}]h_2 \\
& + [f_{2233}(\mathbf{x}) + f_{12233}(\mathbf{x})h_1 + f_{112233}(\mathbf{x})\tfrac{h_1h_1}{3}]\tfrac{h_2h_2}{3}\}\tfrac{h_3h_3}{3} \quad .
\end{aligned}$$

The same method may be used to specify a quadratic function in terms of natural cubic splines.

9 INRIA bench–mark versions of the Marmousi model

Two smooth continuous bench–mark 2-D models without structural interfaces, based on the Marmousi model (Versteeg & Grau 1991, Versteeg 1993), have been prepared for the workshop "Computation of multi–valued traveltimes" held in INRIA Rocquencourt, France, on September 16–18, 1996. The "hard" model is globally quite rough (Figure 18) and is smooth on the scales of the order of several metres. Let us point out once again that the "hard" model is a smooth model without interfaces. The stairs in Figure 18 are due to the coarse discretization for plotting. The "smoothed" model (Figure 19) is a considerably smoothed version of the "hard" model.

The slowness in both smooth models without interfaces is interpolated by natural bicubic splines in the regular grid of 122 points vertically times 384 points horizontally. The grid spacing is 0.024km in both directions. The size of the spline grid is thus 2.904km vertically and 9.192km horizontally. The grid velocities in both models, in $\mathrm{m\,s^{-1}}$, are available at WWW server
`http://www.caam.rice.edu/benamou/marmousi.html` accessible also from
`http://www.caam.rice.edu/benamou/traveltimes.html`
or at
`http://www-rocq.inria.fr/~benamou/testproblem.html` accessible also from
`http://www-rocq.inria.fr/~benamou/traveltimes.html`
The grid velocities are stored with the inner loop from the top to the bottom and the outer loop from the left to the right.

For the `MODEL` package, kilometres have been selected for the model coordinate units, although metres could be used as well. The x_1 axis has been chosen horizontal, pointing to the right, with the origin at the left top corner of the 2-D model. The x_2 axis has been chosen vertical, pointing downwards. Let us emphasize that the order and orientation of the coordinate axes may be chosen arbitrarily. It is just on user's discretion if the vertical axis in a 2-D model is named x_2 or x_3. Moreover, interpolation subroutines of files `parm.for`, `surf.for` and `val.for` from the `MODEL` package do not

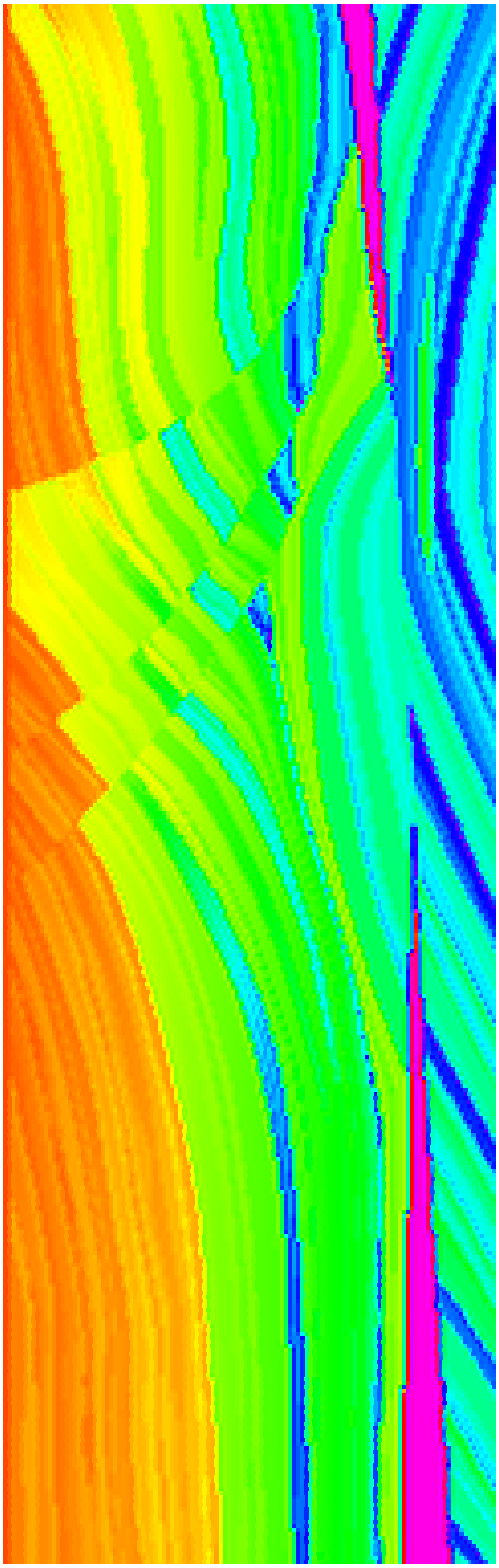

Figure 18. "Hard" INRIA bench–mark version of the Marmousi model.

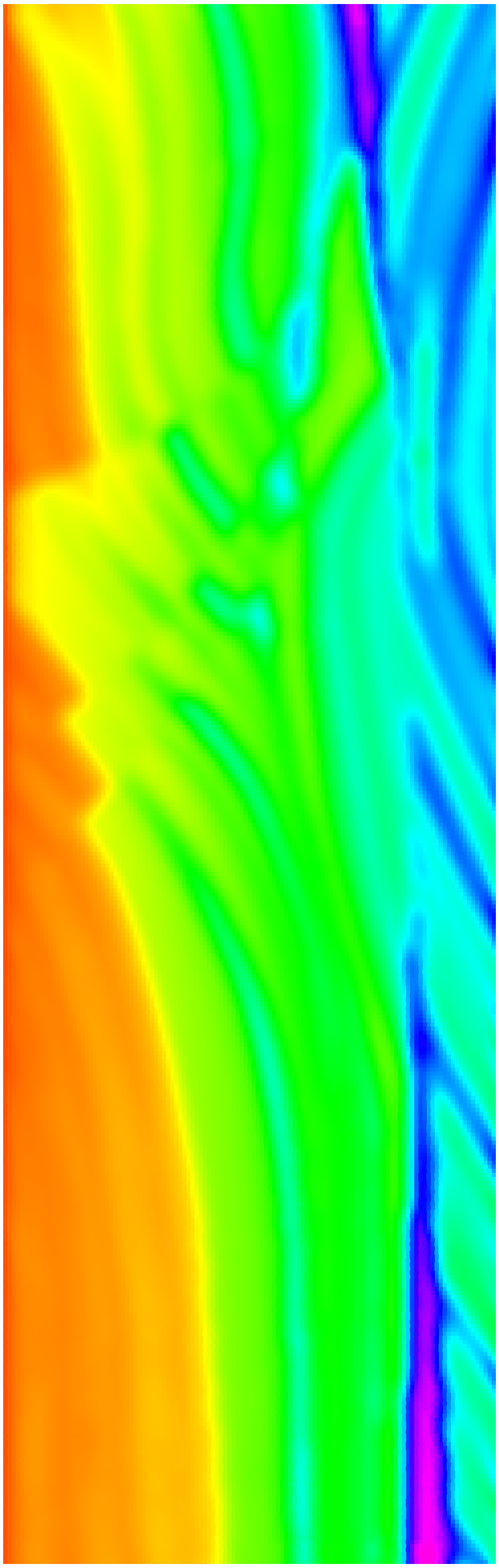

Figure 19. "Smoothed" INRIA bench–mark version of the Marmousi model.

tie the ordering of the grid values for interpolation to the order and orientation of the coordinate axes. For instance, although the grid velocities are specified from the top to the bottom, the vertical axis may point upwards without reordering the grid velocities. The input data for the models differ only in the first line,

```
'MARmousi Hard model, 384*122 gridpoints (cells 0.024*0.024)'
```

or

```
'MARmousi Smoothed model, 384*122 gridpoints (cells 0.024*0.024)'
```

and in the grid values of velocities for the spline interpolation.

```
0   1   1   /                (Cartesian coordinates, velocities, loss factors)
0.000 9.192  0.000 2.904  0 1  (boundaries of the model)
```

The horizontal x_1 coordinate extends from `0.000`km to `9.192`km, the vertical x_2 coordinate extends from the top in `0.000`km to the bottom in `2.904`km. The extent of the x_3 coordinate perpendicular to the 2-D model sections must include the corresponding coordinate of the source and receiver points, i.e. zero in most 2-D calculations. Positive extent `0 1` of the x_3 coordinate (instead of `0 0`) has been chosen especially in order not to disturb poorly debugged 3-D routines.

```
0 surfaces
1 simple block:
   /
1 complex block:
 1 /

'END of input data for smooth surfaces' /
```

The model contains no structural interfaces and the top of the model is not assumed to be the Earth surface. No smooth surface is thus specified and the model consists of a single complex block composed of a single simple block.

```
'COMPLEX BLOCK' 1
'VP ' -1                      (P wave slowness interpolated)
    2    1    0    0   -1 /   (i.e. VP=W(X2,X1), tension=0, velocity given)
  122 384                     (numbers of gridpoints)
```

The slowness is interpolated by the natural bicubic splines in this model. Grid values of velocities are ordered with the inner loop over the vertical x_2 coordinate. The fifth number in the third line specifies the power of the interpolated quantity (slowness in this case), which is specified at the spline gridpoints (velocity in this case). This feature is implemented in the `MODEL` package since version `5.00` and causes incompatibility with data for the older versions. It is necessary to insert a slash after the data at the corresponding lines of the old model data files. Vertical grid coordinates

```
0.000 0.024 0.048 0.072 0.096 0.120 0.144 0.168 0.192 0.216 0.240 0.264
0.288 0.312 0.336 0.360 0.384 0.408 0.432 0.456 0.480 0.504 0.528 0.552
⋮
```

```
2.592 2.616 2.640 2.664 2.688 2.712 2.736 2.760 2.784 2.808 2.832 2.856
2.880 2.904
```

and horizontal grid coordinates

```
0.000 0.024 0.048 0.072 0.096 0.120 0.144 0.168 0.192 0.216 0.240 0.264
0.288 0.312 0.336 0.360 0.384 0.408 0.432 0.456 0.480 0.504 0.528 0.552
⋮
8.640 8.664 8.688 8.712 8.736 8.760 8.784 8.808 8.832 8.856 8.880 8.904
8.928 8.952 8.976 9.000 9.024 9.048 9.072 9.096 9.120 9.144 9.168 9.192
```

are specified in the model coordinate units, kilometres in this data set.

The above data are followed by the grid velocities submitted by INRIA, $122 \times 384 =$ 46 848 values. Since the velocities by INRIA are specified in $\mathrm{m\,s^{-1}}$ which would result in the travel times in kiloseconds for the model coordinates in kilometres, the INRIA velocity values were converted to $\mathrm{km\,s^{-1}}$ (divided by 1000). The conversion would be unnecessary for the model coordinates in metres.

The velocity values are followed just by

```
'END of complex blocks, end of the input data for the model' /
```

to close the input data for each of the two models.

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