Coupling ray theory in weakly anisotropic media

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Summary

The paper is devoted to the numerical calculation of the frequency–dependent complex–valued vectorial amplitudes of S waves in weakly anisotropic media by the coupling ray theory.

Keywords

Ray theory, anisotropy, travel times, amplitudes, Green function.

1 Introduction

There are two different high–frequency asymptotic ray theories, the *isotropic ray theory* assuming the same velocities of both S–wave polarizations and the *anisotropic ray theory* assuming both S–wave polarizations strictly decoupled.

In the isotropic ray theory, the S–wave polarization vectors do not rotate along the ray, whereas in the anisotropic ray theory, they coincide with the eigenvectors of the Christoffel matrix which may rapidly rotate round the ray.

In "weakly anisotropic" models, at moderate frequencies, the S–wave polarization tends to stay unrotated round the ray but is partly attracted by the rotation of the eigenvectors of the Christoffel matrix. The intensity of the attraction increases with the frequency.

The isotropic and anisotropic ray theories are thus limiting cases and the gap between them has to be filled. A ray theory providing continuous transition between the isotropic and anisotropic ray theories was proposed by Coates & Chapman (1990) and is called the *coupling ray theory.*

This paper contains notes on the numerical calculation of the frequency–dependent complex–valued S–wave polarization vectors of the coupling ray theory. A method of numerical integration of the coupling equation, proposed by Červený (1998), is applied to the coupling equation derived by Coates & Chapman (1990), with emphasis on the numerical implementation. The accuracy of the method is estimated in order to control the integration step in such a way, that the relative error in the wavefield amplitudes due to the integration is kept below a given limit.

The calculation of P–wave travel–time perturbations from a reference isotropic model to an anisotropic model and the calculation of S–wave coupling ray theory travel–time and amplitude corrections in models without structural interfaces have been coded and supplemented to Fortran 77 package CRT. A simple numerical example of synthetic seismograms calculated by the coupling ray theory is presented.

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2 Coupling ray theory for S waves

Assume a reference ray, parametrized by reference travel time τ, with reference slowness vectors $p_i(\tau)$ known at all its points $x_j(\tau)$. The reference ray should be close to the ray of the wave under study. In particular, if shear wave coupling in weakly anisotropic media is investigated and the same reference ray is used for both S wave polarizations, the reference ray should be close to the high–frequency approximations of the rays of both S waves.

Using the reference slowness vectors, we can calculate Christoffel matrices

$$\Gamma_{jk}(\tau) = p_i(\tau)\, a_{ijkl}\,(x_j(\tau))\;\; p_l(\tau) \tag{1}$$

and their eigenvectors $g_{i1}(\tau)$, $g_{i2}(\tau)$, $g_{i3}(\tau)$ along the reference ray. Assume that eigenvectors $g_{i1}(\tau)$ and $g_{i2}(\tau)$ correspond to S waves and that they vary continuously along the reference ray. The continuity may be violated in the regions where the corresponding two eigenvalues are equal. Let us denote $\tau_1(\tau)$ and $\tau_2(\tau)$ the travel times corresponding to polarizations $g_{i1}(\tau)$ and $g_{i2}(\tau)$, respectively. They may be approximated by quadratures along the unperturbed reference ray,

$$\frac{\mathrm{d}\tau_1}{\mathrm{d}\tau} = [\Gamma_{jk} g_{j1} g_{k1}]^{-\frac{1}{2}} \quad , \qquad \frac{\mathrm{d}\tau_2}{\mathrm{d}\tau} = [\Gamma_{jk} g_{j2} g_{k2}]^{-\frac{1}{2}} \quad . \tag{2}$$

The coupling ray theory solution u_i of the elastodynamic equations may then be expressed, for S waves, as a linear combination of the anisotropic ray theory solutions (Coates & Chapman 1990, eq.15),

$$u_i = \sum_{M=1}^{2} g_{iM}\, A_M\; r_M \exp(\mathrm{i}\omega\tau_M) \quad , \tag{3}$$

where $A_M = A_M(\tau)$ are the complex–valued scalar amplitudes in the high–frequency approximation. In our case, both A_M are approximated by the same isotropic ray theory amplitude $A = A(\tau)$ calculated along the reference ray. The coupling ray theory equation for complex–valued factors $r_M = r_M(\tau)$ reads (Coates & Chapman 1990, eq.30)

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{pmatrix} r_1 \\ r_2 \end{pmatrix} = \frac{\mathrm{d}\varphi}{\mathrm{d}\tau}\begin{pmatrix} 0 & \exp\left(\mathrm{i}\omega[\tau_2(\tau)-\tau_1(\tau)]\right) \\ \exp\left(\mathrm{i}\omega[\tau_1(\tau)-\tau_2(\tau)]\right) & 0 \end{pmatrix}\begin{pmatrix} r_1 \\ r_2 \end{pmatrix} \quad , \tag{4}$$

where

$$\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} = g_{k2}\frac{\mathrm{d}g_{k1}}{\mathrm{d}\tau} = -g_{k1}\frac{\mathrm{d}g_{k2}}{\mathrm{d}\tau} \tag{5}$$

is the speed of the eigenvector rotation.

For high frequencies, $\exp\left(\mathrm{i}\omega[\tau_2(\tau)-\tau_1(\tau)]\right)$ in equation (4) may rapidly oscillate. These oscillations have no high–frequency impact on the solution because they cancel out after each period and the derivative of the solution is zero in average at high frequencies. However, these oscillations may either considerably reduce the accuracy or increase the costs of the numerical integration.

In addition, travel times τ_M in equation (4) depend on the initial conditions. Equation (4) is thus much less suitable for the definition of the propagator matrix than equations with purely local coefficients.

The oscillation and global character of the coefficient matrix of equation (4) is caused by different travel times τ_M with respect to which the amplitude factors r_M are defined. Let us introduce new amplitude factors a_M, both related to the same "average" travel time $\overline{\tau}$ (Červený 1998),

$$r_1 \exp(\mathrm{i}\omega\tau_1) = a_1 \exp(\mathrm{i}\omega\overline{\tau}) \quad , \qquad r_2 \exp(\mathrm{i}\omega\tau_2) = a_2 \exp(\mathrm{i}\omega\overline{\tau}) \quad . \tag{6}$$

Inserting $r_M = a_M \exp\left(\mathrm{i}\omega[\overline{\tau} - \tau_M]\right)$ into equation (4), we obtain equation

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \left[\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} - \mathrm{i}\omega\begin{pmatrix} \frac{\mathrm{d}(\overline{\tau}-\tau_1)}{\mathrm{d}\tau} & 0 \\ 0 & \frac{\mathrm{d}(\overline{\tau}-\tau_2)}{\mathrm{d}\tau} \end{pmatrix}\right]\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad . \tag{7}$$

It seems reasonable to simplify equation (7) by choosing

$$\overline{\tau}(\tau) = \tfrac{1}{2}[\tau_1(\tau) + \tau_2(\tau)] \quad . \tag{8}$$

Equation (7) than takes the form of

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \left[\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} - \begin{pmatrix} \mathrm{i} & 0 \\ 0 & -\mathrm{i} \end{pmatrix}\frac{\omega}{2}\frac{\mathrm{d}(\tau_2-\tau_1)}{\mathrm{d}\tau}\right]\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad . \tag{9}$$

3 Propagator matrix

Propagator matrix Π^g of equation (9), defined as

$$\Pi^g_{MN}(\tau, \tau_0) = \frac{\partial\, a_M(\tau)}{\partial\, a_N(\tau_0)} \quad , \tag{10}$$

is a complex–valued 2×2 matrix satisfying equation

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\Pi^g = \left[\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} - \begin{pmatrix} \mathrm{i} & 0 \\ 0 & -\mathrm{i} \end{pmatrix}\frac{\mathrm{d}\epsilon}{\mathrm{d}\tau}\right]\Pi^g \quad , \tag{11}$$

directly following from equation (9). Here

$$\epsilon(\tau) = \tfrac{1}{2}\omega\, [\tau_2(\tau) - \tau_1(\tau)] \quad . \tag{12}$$

Propagator matrix Π^g is symplectic and unitary, but we do not need to utilise these properties in this paper.

Since Π^g is a propagator matrix satisfying the chain rule, it may be numerically calculated as the product of propagator matrices Π^g corresponding to reasonably small segments of the reference ray (Červený 1998). Frequency–dependent propagator matrices along individual small ray segments may be approximated with various degrees of accuracy and efficiency.

One of the possibilities how to approximate the local solutions, derived in more detail by the method of mean coefficients by Červený (1998), is suggested in this paper.

4 Analytic solution in a special case

Assume that $\frac{d\varphi}{d\tau} = constant\ \frac{d\epsilon}{d\tau}$, i.e. that $(\varphi - \varphi_0)/(\epsilon - \epsilon_0) = constant$. Then equation (11) takes the form of

$$\frac{d}{d\tau}\Pi^g = \mathbf{A}\ \frac{d\alpha}{d\tau}\ \Pi^g \quad , \tag{13}$$

with constant matrix

$$\boxed{\mathbf{A} = \left[\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}(\varphi - \varphi_0) - \begin{pmatrix} \mathrm{i} & 0 \\ 0 & -\mathrm{i} \end{pmatrix}(\epsilon - \epsilon_0)\right]\alpha^{-1}} \tag{14}$$

and parameter $\alpha = \alpha(\tau, \tau_0)$ defined as

$$\boxed{\alpha = \sqrt{(\varphi - \varphi_0)^2 + (\epsilon - \epsilon_0)^2} \quad .} \tag{15}$$

Here we have denoted $\varphi = \varphi(\tau)$, $\varphi_0 = \varphi(\tau_0)$, $\epsilon = \epsilon(\tau)$ and $\epsilon_0 = \epsilon(\tau_0)$. Since

$$\mathbf{A}^2 = -\mathbf{1} \quad , \tag{16}$$

the solution of equation (13) with unit initial conditions may be expressed in the form of

$$\boxed{\Pi^g(\tau, \tau_0) = \exp(\mathbf{A}\alpha) = \mathbf{1}\cos(\alpha) + \mathbf{A}\sin(\alpha) \quad .} \tag{17}$$

5 Approximation for short ray segments

We now use matrix (17) with (14) and (15) as the approximation for short ray segments, propose how to calculate $\varphi - \varphi_0$ and $\epsilon - \epsilon_0$ and finally estimate the accuracy of the approximation.

Assume that vectors $g^0_{jM} = g_{jM}(\tau_0)$ are mapped onto vectors $g_{jM} = g_{jM}(\tau)$ in this way: (a) vectors are rotated in plane $g^0_{j1}g^0_{k2}$, (b) vectors are rotated round the line of intersection of planes $g^0_{j1}g^0_{k2}$ and $g_{j1}g_{k2}$, (c) vectors are rotated in plane $g_{j1}g_{k2}$. The sum of rotation angles in planes $g^0_{j1}g^0_{k2}$ and $g_{j1}g_{k2}$ is then given by

$$\Delta\varphi = \varphi - \varphi_0 = \arctan\left(\frac{g_{k1}g^0_{k2} - g_{k2}g^0_{k1}}{g_{k1}g^0_{k1} + g_{k2}g^0_{k2}}\right) \quad , \tag{18}$$

see (5).

Assume that travel times τ_M, $M = 1, 2$, can be obtained by trapezoidal quadratures of equations (2).

$$\Delta\tau_M = \tau_M(\tau) - \tau_M(\tau_0) \approx \frac{1}{2}\left[\frac{d\tau_M}{d\tau}(\tau) + \frac{d\tau_M}{d\tau}(\tau_0)\right]\Delta\tau \quad . \tag{19}$$

Then

$$\Delta\overline{\tau} = \frac{1}{2}(\Delta\tau_2 + \Delta\tau_1) \quad . \tag{20}$$

and

$$\Delta\epsilon = \epsilon - \epsilon_0 = \frac{1}{2}\,\omega\,(\Delta\tau_2 - \Delta\tau_1) \quad , \tag{21}$$

see (12).

6 Accuracy of the approximation

The derivative of equation (17) along the reference ray reads

$$\tfrac{\mathrm{d}}{\mathrm{d}\tau}\Pi^g = [-\mathbf{1}\sin(\alpha) + \mathbf{A}\cos(\alpha)]\ \tfrac{\mathrm{d}\alpha}{\mathrm{d}\tau} + \ \tfrac{\mathrm{d}\mathbf{A}}{\mathrm{d}\tau}\sin(\alpha) \quad , \tag{22}$$

the matrix inverse to (17) is

$$[\Pi^g]^{-1} = \mathbf{1}\cos(\alpha) - \mathbf{A}\sin(\alpha) \tag{23}$$

and their product is

$$\tfrac{\mathrm{d}}{\mathrm{d}\tau}\Pi^g\ [\Pi^g]^{-1} = \mathbf{A}\ \tfrac{\mathrm{d}\alpha}{\mathrm{d}\tau} + \ \tfrac{\mathrm{d}\mathbf{A}}{\mathrm{d}\tau}\cos(\alpha)\sin(\alpha) - \ \tfrac{\mathrm{d}\mathbf{A}}{\mathrm{d}\tau}\mathbf{A}\,[\sin(\alpha)]^2 \quad . \tag{24}$$

The derivatives of α and $\mathbf{A}$ along the reference ray are

$$\tfrac{\mathrm{d}\alpha}{\mathrm{d}\tau} = \left[(\varphi-\varphi_0)\ \tfrac{\mathrm{d}\varphi}{\mathrm{d}\tau} + (\epsilon-\epsilon_0)\ \tfrac{\mathrm{d}\epsilon}{\mathrm{d}\tau}\right]\alpha^{-1} \tag{25}$$

and

$$\frac{\mathrm{d}\mathbf{A}}{\mathrm{d}\tau} = \left\{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\left[\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} - (\varphi-\varphi_0)\frac{\mathrm{d}\alpha}{\mathrm{d}\tau}\alpha^{-1}\right] - \begin{pmatrix} \mathrm{i} & 0 \\ 0 & -\mathrm{i} \end{pmatrix}\left[\frac{\mathrm{d}\epsilon}{\mathrm{d}\tau} - (\epsilon-\epsilon_0)\frac{\mathrm{d}\alpha}{\mathrm{d}\tau}\alpha^{-1}\right]\right\}\alpha^{-1} \ . \tag{26}$$

Insertion of (25) and (15) into (26) yields

$$\frac{\mathrm{d}\mathbf{A}}{\mathrm{d}\tau} = \left[\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}(\epsilon-\epsilon_0) + \begin{pmatrix} \mathrm{i} & 0 \\ 0 & -\mathrm{i} \end{pmatrix}(\varphi-\varphi_0)\right]\left[(\epsilon-\epsilon_0)\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} - (\varphi-\varphi_0)\frac{\mathrm{d}\epsilon}{\mathrm{d}\tau}\right]\alpha^{-3} \quad . \tag{27}$$

The product of (27) and (14) is

$$\tfrac{\mathrm{d}\mathbf{A}}{\mathrm{d}\tau}\mathbf{A} = \begin{pmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}\left[(\epsilon-\epsilon_0)\ \tfrac{\mathrm{d}\varphi}{\mathrm{d}\tau} - (\varphi-\varphi_0)\ \tfrac{\mathrm{d}\epsilon}{\mathrm{d}\tau}\right]\alpha^{-2} \quad . \tag{28}$$

Now we subtract the matrix multiplying Π^g at the right–hand side of (11) from matrix $\mathbf{A}\ \tfrac{\mathrm{d}\alpha}{\mathrm{d}\tau}$ appearing at the right–hand side of (24). The coefficient at matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is

$$(\varphi-\varphi_0)\alpha^{-1}\ \tfrac{\mathrm{d}\alpha}{\mathrm{d}\tau} - \ \tfrac{\mathrm{d}\varphi}{\mathrm{d}\tau} = -(\epsilon-\epsilon_0)\left[(\epsilon-\epsilon_0)\ \tfrac{\mathrm{d}\varphi}{\mathrm{d}\tau} - (\varphi-\varphi_0)\ \tfrac{\mathrm{d}\epsilon}{\mathrm{d}\tau}\right]\alpha^{-2} \tag{29}$$

and the coefficient at matrix $\begin{pmatrix} \mathrm{i} & 0 \\ 0 & -\mathrm{i} \end{pmatrix}$ is

$$-(\epsilon-\epsilon_0)\alpha^{-1}\ \tfrac{\mathrm{d}\alpha}{\mathrm{d}\tau} + \ \tfrac{\mathrm{d}\epsilon}{\mathrm{d}\tau} = -(\varphi-\varphi_0)\left[(\epsilon-\epsilon_0)\ \tfrac{\mathrm{d}\varphi}{\mathrm{d}\tau} - (\varphi-\varphi_0)\ \tfrac{\mathrm{d}\epsilon}{\mathrm{d}\tau}\right]\alpha^{-2} \quad . \tag{30}$$

Now we may insert equations (27)–(30) into (24) to calculate the deviation of the approximation from equation (11),

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\Pi^g[\Pi^g]^{-1} - \left[\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\frac{\mathrm{d}\varphi}{\mathrm{d}\tau} - \begin{pmatrix} \mathrm{i} & 0 \\ 0 & -\mathrm{i} \end{pmatrix}\frac{\mathrm{d}\epsilon}{\mathrm{d}\tau}\right]$$

$$= \left\{ \left[\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} (\epsilon - \epsilon_0) + \begin{pmatrix} \mathrm{i} & 0 \\ 0 & -\mathrm{i} \end{pmatrix} (\varphi - \varphi_0) \right] \left[\cos(\alpha) \frac{\sin(\alpha)}{\alpha} - 1 \right] \right.$$

$$\left. - \begin{pmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} [\sin(\alpha)]^2 \right\} \left[(\epsilon - \epsilon_0) \frac{\mathrm{d}\varphi}{\mathrm{d}\tau} - (\varphi - \varphi_0) \frac{\mathrm{d}\epsilon}{\mathrm{d}\tau} \right] \alpha^{-2} \quad . \tag{31}$$

The relative error in Π^g per one step along a ray is then approximately matrix $\begin{pmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$ times

$$\Delta\delta \approx - \int_{\tau_0}^{\tau} \left[(\epsilon - \epsilon_0) \frac{\mathrm{d}\varphi}{\mathrm{d}\tau} - (\varphi - \varphi_0) \frac{\mathrm{d}\epsilon}{\mathrm{d}\tau} \right] \mathrm{d}\tau = - \int_{\varphi_0}^{\varphi_0 + \Delta\varphi} \left[(\epsilon - \epsilon_0) - (\varphi - \varphi_0) \frac{\mathrm{d}\epsilon}{\mathrm{d}\varphi} \right] \mathrm{d}\varphi \; .$$

(32)

Approximating $\epsilon - \epsilon_0$ by the second–order Taylor expansion with respect to $\varphi - \varphi_0$, we arrive at

$$(\epsilon - \epsilon_0) - (\varphi - \varphi_0) \tfrac{\mathrm{d}\epsilon}{\mathrm{d}\varphi} \approx -\tfrac{1}{2} \tfrac{\mathrm{d}^2\epsilon}{\mathrm{d}\varphi^2} (\varphi - \varphi_0)^2 \quad . \tag{33}$$

Within the second–order approximation (33), we can analytically integrate (32),

$$\Delta\delta \approx \frac{\mathrm{d}^2\epsilon}{\mathrm{d}\varphi^2} \frac{(\Delta\varphi)^3}{6} \approx \frac{(\Delta\varphi)^2}{6} \Delta \tfrac{\mathrm{d}\epsilon}{\mathrm{d}\varphi} \approx \frac{\Delta\varphi\Delta\tau}{6} \Delta \tfrac{\mathrm{d}\epsilon}{\mathrm{d}\tau} = \frac{\Delta\varphi\Delta\tau}{12} \omega \, \left(\Delta \tfrac{\mathrm{d}\tau_2}{\mathrm{d}\tau} - \Delta \tfrac{\mathrm{d}\tau_1}{\mathrm{d}\tau} \right) \; . \tag{34}$$

To keep the error in Π^g along the whole ray below maximum specified limit δ, we require

$$|\Delta\delta| \le \frac{\Delta\tau}{\tau} \, \delta \quad , \tag{35}$$

i.e.,

$$\boxed{ |\Delta\varphi| \left| \Delta \frac{\mathrm{d}\tau_2}{\mathrm{d}\tau} - \Delta \frac{\mathrm{d}\tau_1}{\mathrm{d}\tau} \right| \le \frac{12}{\omega\tau} \, \delta \quad , } \tag{36}$$

see (2). Let us note that, in addition to inequality (36), step $\Delta\varphi$ in the eigenvector rotation must also be sufficiently smaller than 45°,

$$|\Delta\varphi| \ll \frac{\pi}{4} \quad , \tag{37}$$

in order to reliably follow a selected eigenvector g_{kM} along the reference ray. Condition (37) may be violated in the regions where the corresponding two eigenvalues are equal, i.e., where $\Delta\epsilon = 0$ within a reasonable numerical accuracy.

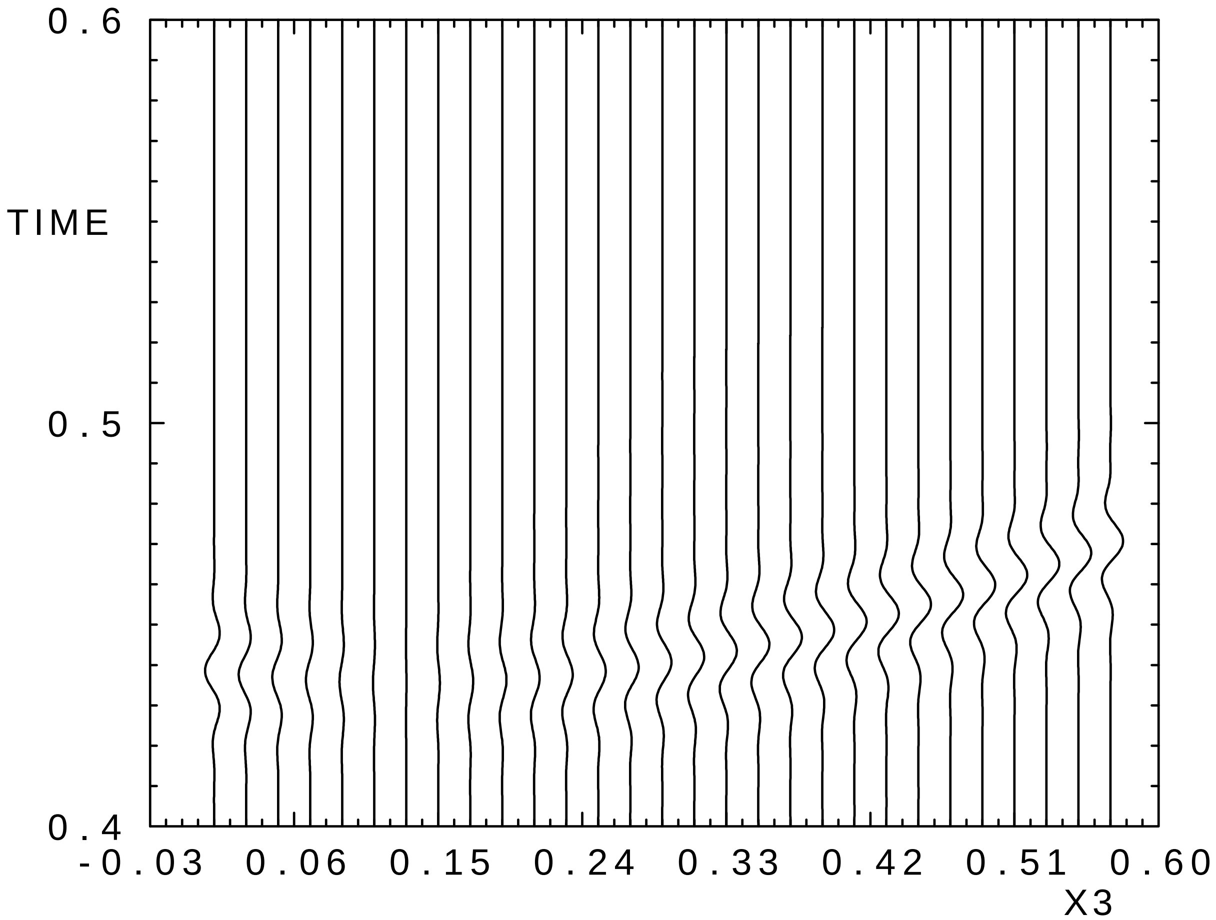

Figure 1. First (radial) component of the synthetic seismograms.

7 Numerical example

The calculation of P–wave travel–time perturbations from a reference isotropic model to an anisotropic model and the calculation of above described S–wave coupling ray theory travel–time and amplitude corrections in models without structural interfaces have been coded in subroutine file `wan.for` and supplemented to Fortran 77 package CRT. The subroutines are called by program `green.for` if the model is anisotropic. In such a case, program `green.for` generates frequency–dependent elementary Green functions (for S waves), further converted by program `greenss.for` into the response functions corresponding to a particular source radiation pattern.

A 1-D anisotropic model QI was provided by Pšenčík (1998), who performed the same coupling ray theory calculations by the programs of package ANRAY. The normalized elastic parameters a_{ijkl} at the surface (zero depth) are

$$\begin{pmatrix} 14.48500 & 4.52500 & 4.75000 & 0.00000 & 0.00000 & -0.58000 \\ & 14.48500 & 4.75000 & 0.00000 & 0.00000 & -0.58000 \\ & & 15.71000 & 0.00000 & 0.00000 & -0.29000 \\ & & & 5.15500 & -0.17500 & 0.00000 \\ & & & & 5.15500 & 0.00000 \\ & & & & & 5.04500 \end{pmatrix} \tag{38}$$

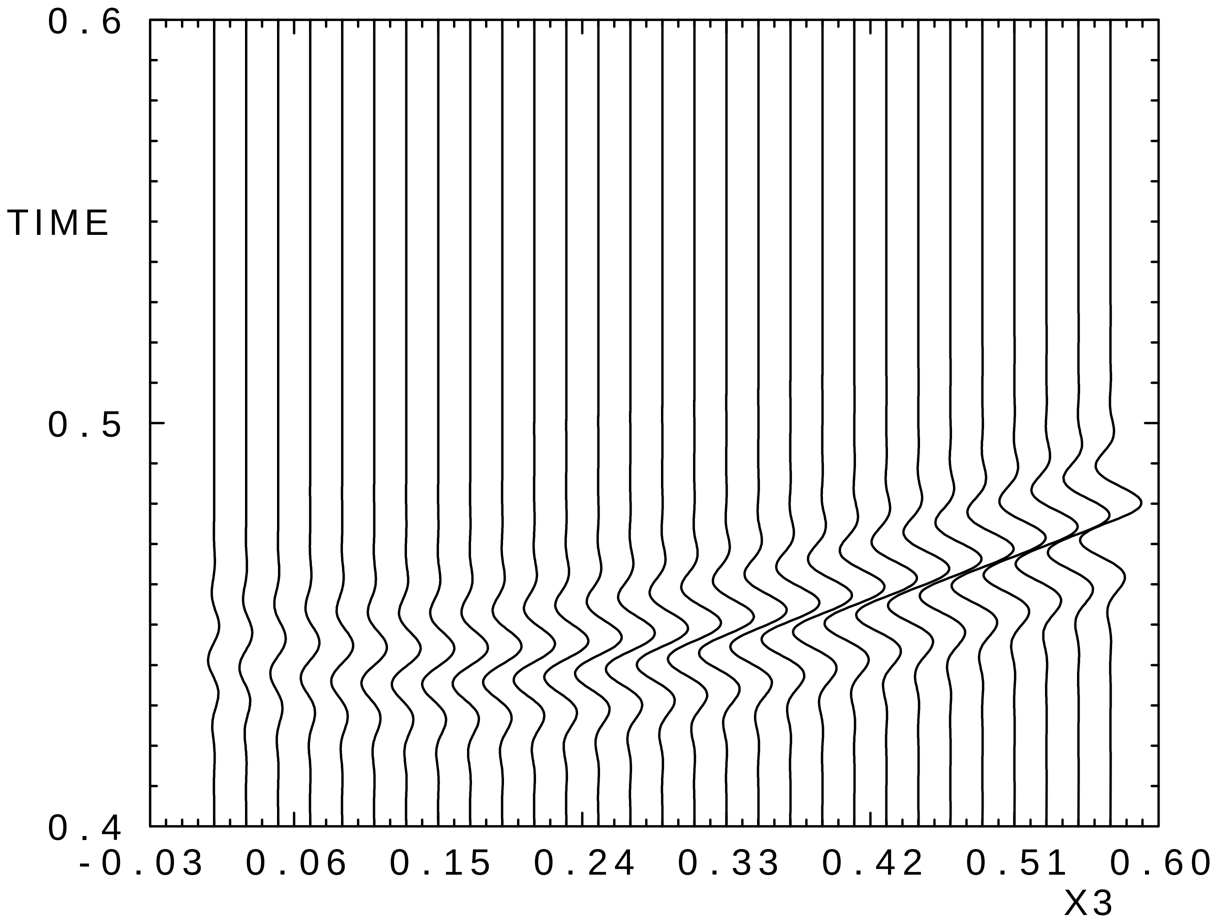

Figure 2. Second (transversal) component of the synthetic seismograms. Note that this component is zero in the one–dimensional reference isotropic model.

and in the depth of 1 length unit are

$$\begin{pmatrix} 22.08963 & 6.90063 & 7.24375 & 0.00000 & 0.00000 & -0.88450 \\ & 22.08953 & 7.24375 & 0.00000 & 0.00000 & -0.88450 \\ & & 23.95775 & 0.00000 & 0.00000 & -0.44225 \\ & & & 7.86138 & -0.26688 & 0.00000 \\ & & & & 7.86138 & 0.00000 \\ & & & & & 7.69363 \end{pmatrix} \quad . \tag{39}$$

The reference isotropic model is given by

$$v_{\mathrm{P}}^2 = 15.00 \quad , \qquad v_{\mathrm{S}}^2 = 5.10 \tag{40}$$

at the surface, and

$$v_{\mathrm{P}}^2 = 23.00 \quad , \qquad v_{\mathrm{S}}^2 = 7.79 \tag{41}$$

in the depth of 1 length unit. All the above values are interpolated linearly with the depth. The homogeneous density is $\rho = 1$. The synthetic seismograms, corresponding to vertical force $\mathbf{F} = (0, 0, 100)^{\mathrm{T}}$ at position $(50, 50, 0)^{\mathrm{T}}$, are calculated at 29 receivers $(51, 50, 0.010)^{\mathrm{T}}$, $(51, 50, 0.030)^{\mathrm{T}}$, $(51, 50, 0.050)^{\mathrm{T}}$, ..., $(51, 50, 0.570)^{\mathrm{T}}$ located in a vertical well. The resulting seismograms, calculated according to history file `qi-ss.h` and shown

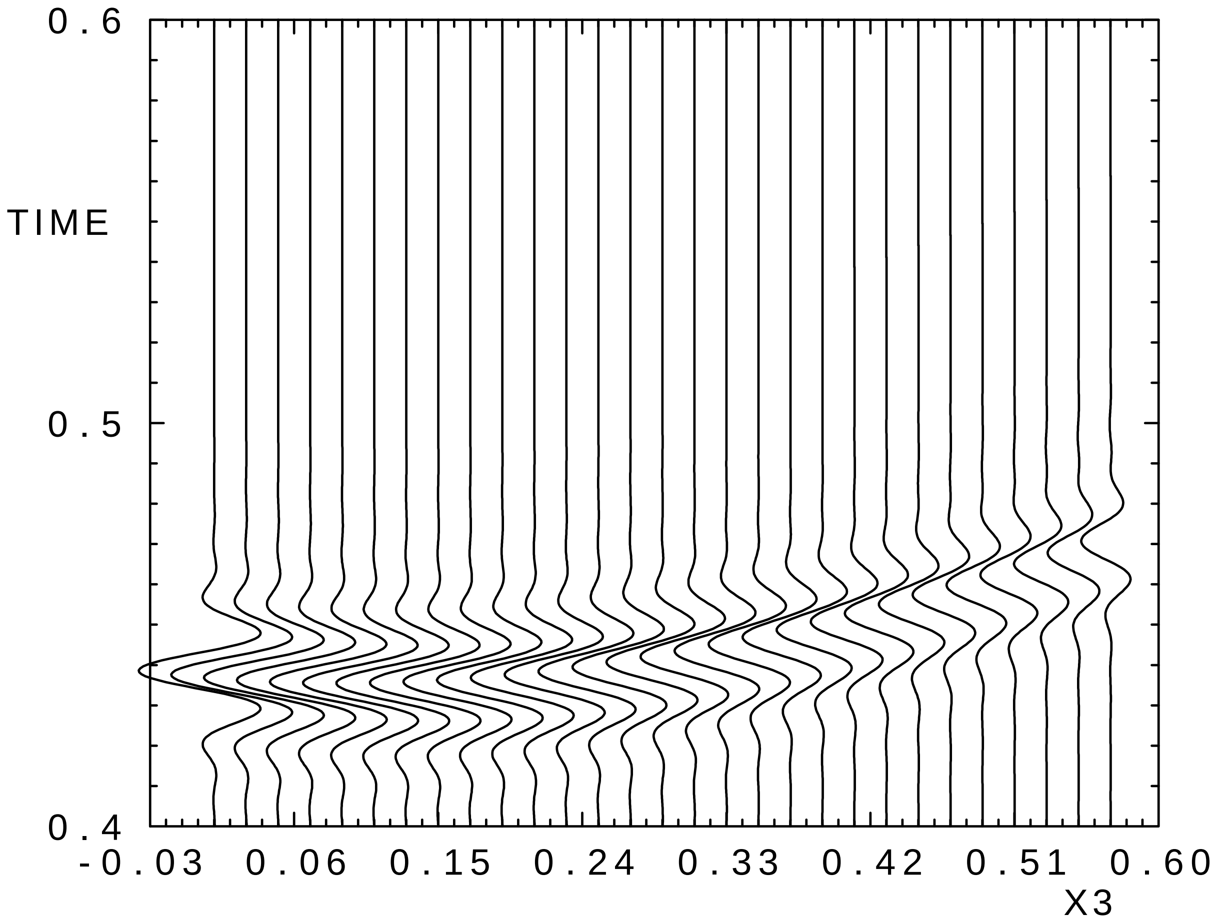

Figure 3. Third (vertical) component of the synthetic seismograms.

in Figures 1, 2 and 3, are slightly different from those calculated by package ANRAY. The differences in vectorial amplitudes are about 10% and will be investigated.

The model is named QI and the data may be found on compact disk SW3D-CD-2. The data for package CRT are located in subdirectory "`crt/qi/`", the data for package ANRAY have extension "`.qi`".

References

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