Prismatic ray cells versus tetrahedra - numerical tests of interpolation methods

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1 Introduction

The method for interpolation of ray-theory travel times in nodes of 3-D grids presented in the last Report (Bulant & Klimeš 1998) is based on decomposition of ray tubes into prismatic ray cells, i.e. the cells formed by six points on three rays. If the rays interact with structural interfaces, it is sometimes not possible to create such cells, and one to several degenerate cells, which are formed by five or four points, must be generated. As was shown in Bulant & Klimeš (1998), computation of local coordinates within standard prismatic cells formed by six points leads to the solution of a cubic equation, while within degenerate cells formed by five points it leads to a quadratic equation, and finally within cells formed by four points local coordinates may be computed by solving a linear equation.

Because the solution of cubic equation is much more time-consuming than the solution of linear equation, it might be interesting to divide ray tubes into ray cells formed by four points (tetrahedra) and gain from the simplification of the equations. This may be done very easy by splitting each cell with five or six vertices into two or three tetrahedra, see figure 1. The sides of tetrahedra for two neighbouring ray tubes must coincide. This condition is satisfied, e.g., when we create first tetrahedron along the ray with the highest index, and the second tetrahedron along the ray with the second highest index from the indices of the three rays forming the original prismatic cell.

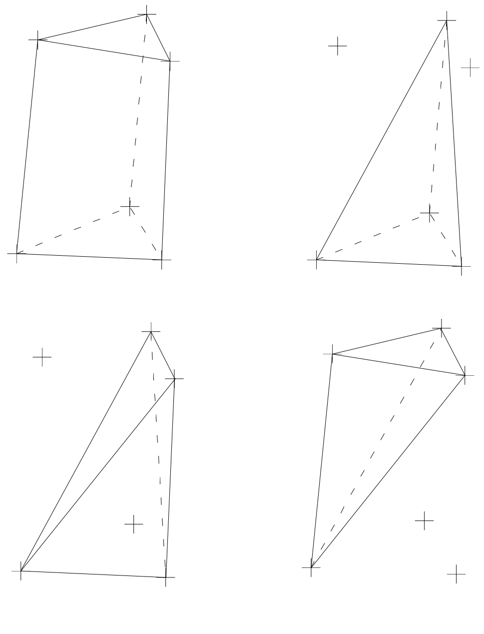

Figure 1: Prismatic ray cell decomposed into three tetrahedra.

In: Seismic Waves in Complex 3-D Structures, Report 8 (Department of Geophysics, Charles University, Prague, 1999), pp. 69–70

Due to the simplification of the equations for local coordinates we could expect, that the interpolation within tetrahedra will be faster.

On the other hand, we know that all the receivers situated inside the smallest box containing the whole ray cell enter into the computation of the local coordinates. The above mentioned boxes are about the same for all the three tetrahedra and for the original prismatic ray cell containing the tetrahedra. Thus, if we use tetrahedra, we will have to solve the equations for local coordinates about three times more than in the case of prismatic cells. This will slow down the tetrahedra computation more in case of irregular (e.g. long and thin) ray cell, and less in case of regular cell, when most of the receivers from the above mentioned box are located within the cell.

Moreover, we know that special care must be taken of the receivers located at the sides of ray cells. The use of tetrahedra means also increased number of sides of the cells and thus more receivers located at the sides.

2 Comparison of the two methods

As the final effect of the use of tetrahedra instead of prismatic cells is not clear, we decided to compare the methods numerically. We took the code for interpolation within prismatic cells, changed the decomposition of ray tubes into tetrahedra instead of former decomposition into prismatic cells, and removed all operations connected with quadratic and cubic equations for computation of local coordinates within prismatic ray cells. The final interpolation of travel times and other quantities in tetrahedra remained the same as in prismatic cells, and it gives the same results for both kinds of cells. Then we tested both the methods in model with lenticular inclusion and in model "98".

The computational time of interpolation in model 98 was 8 minutes 34 seconds for tetrahedra method and 6 minutes 11 seconds for prismatic cells method.

The computational time in model with lenticular inclusion was 1 minutes 52 seconds for tetrahedra and 2 minutes 36 seconds for prismatic cells.

In the next numerical test carried out again in the model with lenticular inclusion we were changing the shape of ray cells. The length of the cells was managed by parameter STORE, which describes the time interval for storing points along the rays. Note that typical velocity in the model is 5 km/s. The width of the cells was influenced by parameter PRM0(4), which describes the maximum width of the cells in the ray-tube metric. Three computations were realized and following computational times were measured:

STORE:	PRM0(4):	computational time for prismatic cells method:	computational time for tetrahedra method:
1.000 sec	10. km	2 min 36 sec	1 min 52 sec
0.175 sec	20. km	1 min 45 sec	1 min 41 sec
0.084 sec	40. km	2 min 02 sec	2 min 19 sec

3 Conclusions

We can conclude, that the interpolation within tetrahedra may be sometimes slower and sometimes faster than the interpolation within prismatic cells. Computational time is not a criterion, according which one of the two methods is preferable in general.

References

Bulant, P. & Klimeš, L. (1998): Interpolation of ray–theory travel times within ray cells. In: Seismic Waves in Complex 3-D Structures, Report 7, pp. 17–32, Department of Geophysics, Charles University, Prague.